A methodology based on the non-linear accumulation and probabilistic modelling for stress-life prediction at variable amplitude loading

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Abstract

In 1924, Palmgren [1] suggested for the first time the concept of the linear damage rule (LDR). About two decades later (1945), Miner [2] proposed the first mathematical expression for the damage computation. Fatigue damage (M) can be calculated in terms of the number of cycles applied at a given stress range (n_i) divided by the corresponding number of cycles required to produce failure at the same stress range (N_{fi}). Miner's model is proposed in the majority of design codes. The sequential loading effects not properly accounted using the Miner's model. Therefore, the non-linear damage accumulation models are required when sequential effects are important. Due to deficiencies associated with LDR, Marco and Starkey [3] proposed the first non-linear load-dependent damage theory, in 1954. In 1960, the two-stage linear damage approach suggested by Langer and Grover [4] considered cycle ratios for two separate stages, in the fatigue damage process of constant amplitude stressing: 1) Damage due to crack initiation, $N_I = N_f$; and, 2) Damage due to crack propagation, $N_{II} = (1-\alpha)N_f$ where α is a life fraction factor for the initiation stage. In 1966 and 1981, a double linear damage rule (DLDR) for treating cumulative fatigue damage was proposed by Manson & Halford [5]. Recently, Huffman & Beckman [6] proposed a non-linear damage accumulation fatigue model in which is modelled by calculating the damage of each cycle based on the state of damage when that cycle occurs, using the assumption that damage accumulation behaves like crack growth. The effect is that for any particular reversal to a tensile stress in a variable amplitude stress history, that stress reversal will cause more damage later in a history than it will at the beginning of the history. An important issue of fatigue damage accumulation theories is their development on a deterministic form, when it is well recognized the probabilistic nature of the fatigue lifetime. Some attempts have been presented in the literature concerning probabilistic fatigue approaches, but fundamentally they are based on linear damage summation approaches [7,6,8,10]. In this research work, a methodology based on the non-linear accumulation and probabilistic modelling for stress-life prediction at variable amplitude loading using fatigue data at constant amplitude loading. The probabilistic modelling used in the non-linear damage accumulation fatigue model is based on probabilistic S-N fields for constant amplitude fatigue data proposed by Castillo and Fernández-Canteli [11,12]. This proposed methodology is validated using experimental variable amplitude fatigue data for riveted joints made of puddle iron from the Fão Bridge under stress-controlled conditions [13,14]. This methodology can be generalized for the local fatigue damage models and various probabilistic approaches.

Keywords

Fatigue, non-linear damage accumulation, constant and variable amplitudes, probabilistic modelling

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