2007 National Physics Competition
Pre-University Level

Answer ALL questions in this paper.

1. (a) A bar shaped n-type semiconductor is placed in a uniform magnetic field B in z-direction as shown in Figure 1. When a constant current I flows along the x-axis in the semiconductor, a Hall voltage, V_H is generated across the semiconductor.

(i) Which surface of the semiconductor is charged positively, due to the Hall voltage?

The surface whose normal is parallel to the positive y axis.
Alternatively, the back surface along the y axis. /diagram [2]

Candidates should be allowed to show the surface using a diagram.

(ii) Derive an expression for V_H in terms of the drift velocity of charge carriers v, charge carrier density, n, magnetic flux density B and the dimensions of the sample.

$F_B = Bev$ [2]

$F_E = \frac{V_H e}{w}$ [2]

At equilibrium, $F_B = F_E$ [2]

$V_H = Bvw$ [2]

(iii) Hence, deduce an expression for V_H in terms of the current I, the magnetic field B, the sample thickness, d, the width w and the carrier (free electron) density n.

$I = neAv = newdv$ [2]

$v = I/newd$

$V_H = B \times \frac{I}{newd} \times w$

$= \frac{BI}{ned}.$ [2]

(iv) Explain the effect on the polarity of Hall voltage if a p-type semiconductor is used in place of the n-type in the same configuration, other factors remaining the same.

The polarity will be reversed because the charge carriers are now holes (positive) and they will collect at the front surface along the y axis. [4]

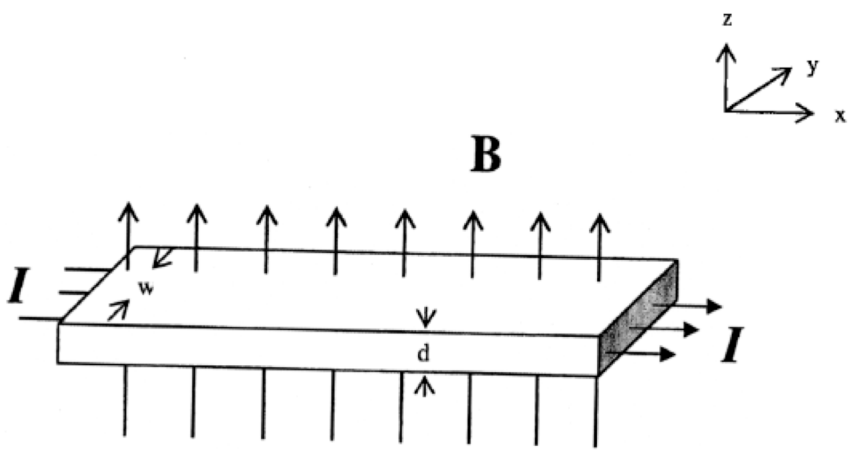

Figure 1

(b) MAGNETOHYDRODYNAMIC (MHD) GENERATOR

The production of electrical power through the use of a conducting fluid moving through a magnetic field is referred to as magnetohydrodynamic, or MHO, power generation**.**

As shown in Figure 2, a horizontal rectangular plastic pipe of width w and height h, which closes upon itself, is filled with mercury of electrical conductivity, σ. An overpressure P is produced by a turbine which drives this fluid with a constant speed v_0. The two opposite vertical walls of a section of the pipe with length L are made of copper.

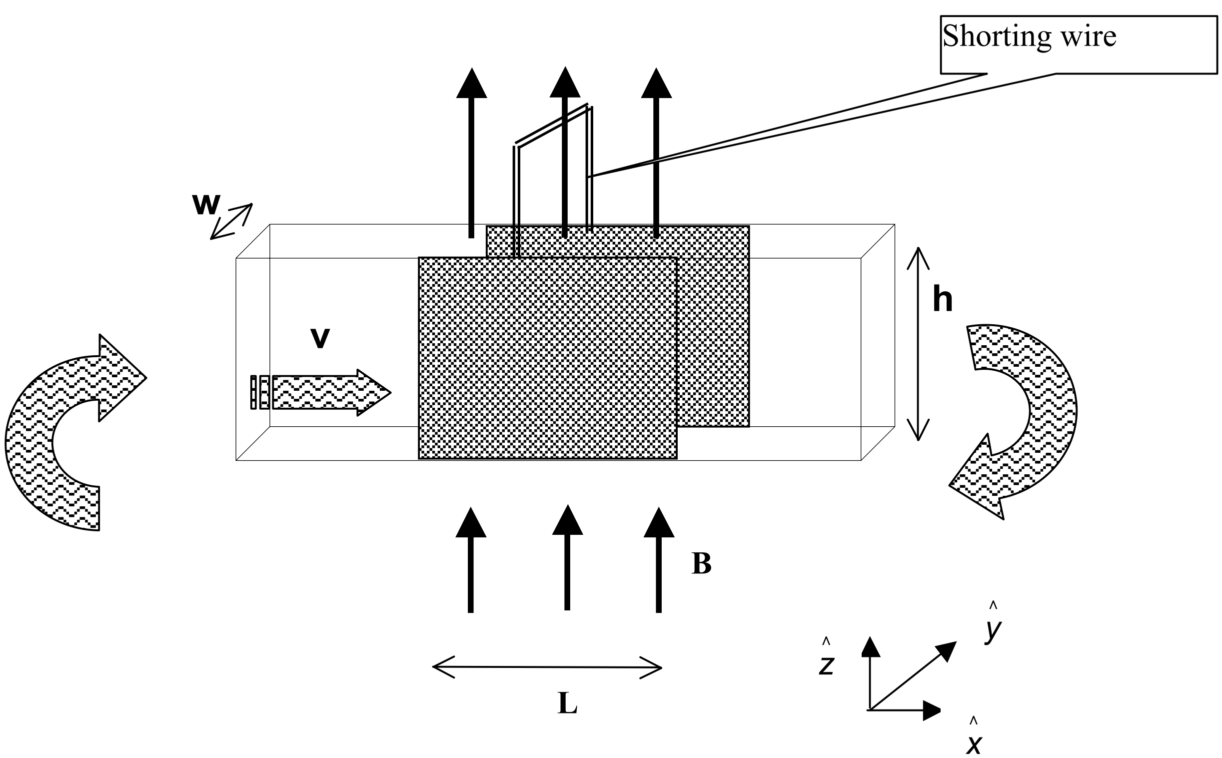

Figure 2

The motion of a real fluid is very complex. To simplify the situation we assume the following:

- Although the fluid is viscous, its speed is uniform over the entire cross section.
- The speed of the fluid is always proportional to the net external force acting upon it.
  $F_{net} \propto v$
- The fluid is incompressible.

These walls are electrically shorted externally and a uniform, magnetic field **B** is applied vertically upward only in this section. The velocity of the fluid is affected when this is done. The new velocity of the fluid is now *v*. The set up is illustrated in Figure 2 , with the unit vectors $\hat{x}$, $\hat{y}$, $\hat{z}$ to be used in the solution.

i) Derive an expression for the voltage V_H produced across the copper electrodes in terms of v. Hence prove that the current flowing in the shorting wire is given by I=BLhσ *v.*

Assuming the flowing mercury represents a rod of length w continuously cutting across the magnetic field B with velocity v,

$V_H = Bwv$ [2]

Alternatively, assuming that the free electrons in mercury are all moving with velocity v along with the mercury,

$V_H = Bwv$, based on the derivation in 1 (a) (ii).

$I = V_H/R$

$R = \rho w/A = \rho w/Lh$ [2]

$$I = \frac{Bwv}{\rho w/Lh}$$

$= BLh\sigma v$ [2]

ii) Find the force acting on the fluid due to the magnetic field (in terms of *L, B, h, w,* σ and the new velocity *v).* State the direction of the force.

$F_B = BIw = B(BLh\sigma v)w = B^2Lh\sigma wv$ [2]

The force acts in the negative x direction. [2]

iii) Assuming the speed of the fluid is always proportional to the net external force acting upon it, derive an expression for the new speed v of the fluid (in terms of v_0, P, L, B and σ) after the magnetic field is applied.

Net external force = $Phw - B^2Lh\sigma wv = kv$ [2]

$$Phw = kV_o$$ [2]

$$Phw - B^2Lh\sigma wv = \frac{Phw}{V_o} v$$

$$v\left(\frac{Phw}{V_o} + B^2Lhw\sigma\right) = Phw$$

$$v = \frac{PV_o}{P + B^2L\sigma V_o}.$$ [2]

iv) By energy conservation or otherwise, derive an expression for the additional power that must be supplied by the turbine to increase the speed to its original value v_0.

Additional power is equal to the power dissipated in the mercury due to the transverse current.

Additional power = I^2R [2]

$$= (BLh\sigma v_o)^2 \frac{w}{\sigma Lh}$$ [2]

$$= B^2Lh\sigma wv_o^2$$ [2]

[40 marks]

2. (a) A particle of mass m is released from rest at a point of height h as shown in Figure 3. The particle then slides down an inclined track of negligible friction. At the floor level, it moves up a circular loop of radius R. Find the minimum starting height h in term of R such that the particle is always in contact with the track at all times.

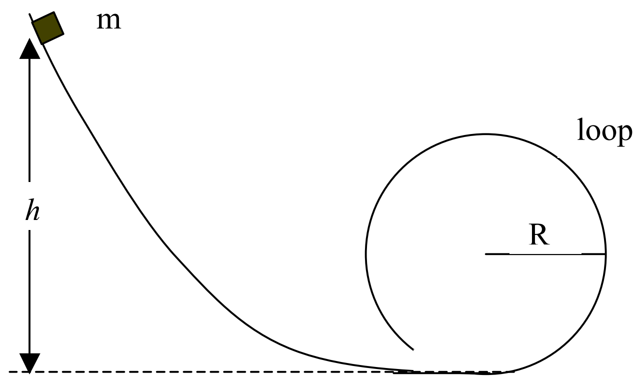

Figure 3

For minimum height h, on reaching the highest point in the circular loop,

$$\frac{mv^2}{R} = mg \qquad [2]$$

$$v^2 = Rg$$

Using conservation of energy,

$$mg(h-2R) = \tfrac{1}{2} m v^2 \qquad [2]$$

$$h - 2R = \frac{v^2}{2g} = \frac{Rg}{2g} = \frac{R}{2} \qquad [2]$$

$$h = 2.5\ R \qquad [2]$$

(b) The particle in (a) is now replaced with a disk of radius r (moment of inertia $I = \frac{1}{2} mr^2$ about a normal axis through its centre of mass) and is released from rest at a point where its center of mass is at height h (Figure 4). The disk then rolls without slipping down the inclined track. At floor level, it rolls into the circular loop of radius R. Find the minimum starting height h needed to ensure that the disk will remain in contact with the track at all times for the special case $r = \frac{1}{3}$ R. Express your answer in term of R.

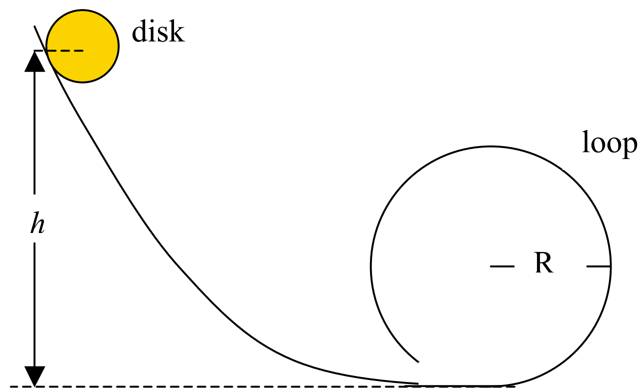

Figure 4

[20 marks]

For minimum height h, on reaching the highest position in the circular loop,

$$m\frac{v^2}{(R-r)} = mg \qquad [2]$$

$$v^2 = g(R-r) \qquad [2]$$

By conservation of energy,

$$mg(h-(2R-r)) = \tfrac{1}{2} mv^2 + \tfrac{1}{2} I\,\omega^2 \qquad [2]$$

$$= \tfrac{1}{2} mv^2 + \tfrac{1}{4}mr^2\,\omega^2 \qquad [2]$$

$$mg(h-2R+r) = \tfrac{3}{4} mv^2$$
$$mg(h-2R+r) = \tfrac{3}{4}\, m\,g\,(R-r)$$

For $r = \tfrac{1}{3}R$, $\quad h - 5/3\,R = \tfrac{1}{2}R \qquad [2]$

$$h = 13/6\,R \qquad [2]$$

3. Ruchardt's tube is an apparatus that uses mechanics to measure the ratio of specific heat capacity ($\gamma = Cp/Cv$) for a gas. The apparatus is a long glass tube with cross-sectional area A connected to a large gas volume as shown in Figure 5. A steel ball of mass m fits precisely into the long tube. When the ball in the tube is at equilibrium, the pressure and volume of the enclosed gas are P_{eq} and V_{eq} respectively. The atmospheric pressure is P_o.

(a) Write an expression for P_{eq} in terms of P_o , m , A and gravitational acceleration g.

$$P_{eq} = P_o + \frac{mg}{A} \qquad [2]$$

(b) When the ball in the tube is displaced away from the equilibrium position by a small distance, dy, there is a total volume change dV and pressure change dP due to adiabatic change of the gas in the volume. Assume dP and dV are much smaller than the equilibrium pressure and volume P_{eq} and V_{eq} , and the gas behaves as an ideal gas. Show that

$$dP = -\frac{k\gamma}{V^{\gamma+1}}dV$$ where k is a constant.

For an adiabatic change,

$$P\,V^{\gamma} = k \qquad [2]$$

$$P = k\,V^{-\gamma}$$

$$dP = k\,(-\gamma)\,V^{-\gamma-1}\,dV \qquad [2]$$

$$= \frac{-k\gamma}{V^{\gamma+1}}\,dV$$

(c) Show that the perturbed ball will undergo simple harmonic motion and solve for the period of the motion in terms of γ , m , A, P_{eq} and V_{eq}.

[20 marks]

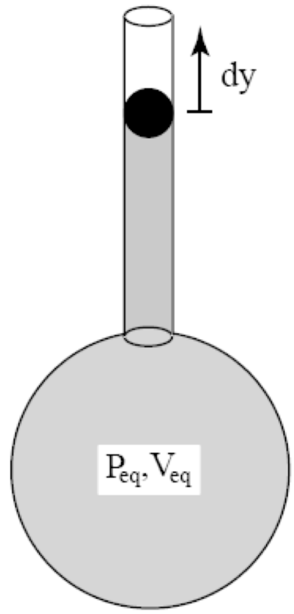

Figure 5

$dP = F/A$ where F is the restoring force on the ball. [2]

$dV = A\,dy$ [2]

$$dP = \frac{-k\gamma}{V^{\gamma+1}}\,dV$$

$$F/A = \frac{-k\gamma}{V^{\gamma+1}}\,A\,dy$$

$$m\frac{d^2(d\,y)}{dt^2} = \frac{-k\gamma}{V^{\gamma+1}}\,A^2\,dy$$ [2]

$$\frac{d^2(d\,y)}{dt^2} = \frac{-k\gamma}{m\;V^{\gamma+1}}\,A^2\,dy$$ [2]

This is shm.

$$\text{Period } T = \frac{2\pi}{\sqrt{\dfrac{A^2 k\gamma}{mV^{\gamma+1}}}}$$

$$= \frac{2\pi \sqrt{mV^{\gamma+1}}}{A\sqrt{k\gamma}}$$ [2]

(using $PV^{\gamma} = k$ and $V \sim V_{eq}$)

$$= \frac{2\pi \sqrt{mkV_{eq}/P_{eq}}}{\sqrt{k\gamma}}$$ [2]

$$= \frac{2\pi\sqrt{mV_{eq}}}{A\sqrt{(\gamma P_{eq})}}$$. [2]

4. Circular refraction is a phenomenon observed in the atmosphere of certain planets in which light can propagate around the planet along the circumference in the atmosphere. This phenomenon is caused by the variation of the refractive index with height from the surface of the planet.
The index of refraction n of the atmosphere of a planet decreases with the altitude h as $n = n_o - \alpha h$, where α is a constant, n_o is the refractive index at the surface of the planet. The radius of the planet is R. Circular refraction occurs at altitude h_o as shown in Figure 6. Consider a narrow beam of light of width Δh where $\Delta h << h_o$, the speed of light at altitude h_o and $h_o + \Delta h$ are v_1 and v_2 respectively.

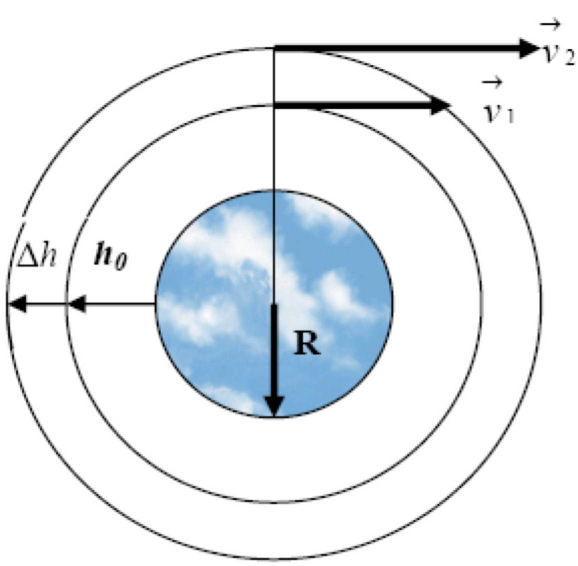

Figure 6

(a) Derive an expression for v_1 in terms of the speed of light in vacuum, c, h_o, n_o and α.

$v_1 = c/n$ [2]

$= c/(n_o - \alpha h_o)$ [2]

(b) Find an expression for the time interval for the light to travel around the planet at altitude h_o in terms of c, h_o, n_o, R and α.

$$T = \frac{2\pi(R+h_o)}{v_1}$$ [2]

$$= \frac{2\pi(R+h_o)(n_o - \alpha h_o)}{c}$$. [2]

(c) By considering the wavefront of the narrow beam of light, determine an expression for the altitude h_o in which the light can propagate around the planet along the circumference? Express your answer in terms of n_o, R and α.

[20 marks]

For the inner edge of the wavefront, time for one complete revolution is

$$T = \frac{2\pi(R+h_o)(n_o - \alpha h_o)}{c}$$.

For the outer edge of the wavefront, time for one complete revolution is

$$T' = \frac{2\pi(R+h_o+\Delta h)(n_o - \alpha(h_o+\Delta h))}{c}$$ [2]

$T = T'$ if the wavefront goes around the planet. [2]

$$(R+h_o)(n_o - \alpha h_o) = (R + h_o + \Delta h)(n_o - \alpha(h_o+\Delta h)) \quad [2]$$

$$= (R+h_o)(n_o-\alpha h_o) - (R+h_o)\alpha\Delta h + \Delta h(n_o - \alpha(h_o+\Delta h))$$

$$(R+h_o)\alpha\Delta h = \Delta h(n_o - \alpha(h_o+\Delta h)) \quad [2]$$

$$R\alpha + h_o\alpha = n_o - \alpha h_o - \alpha\Delta h$$

$$2h_o\alpha = n_o - \alpha(R + \Delta h) \quad [2]$$

Δh is negligible compared to R.

$$h_o = \frac{n_o - \alpha R}{2\alpha} \quad [2]$$