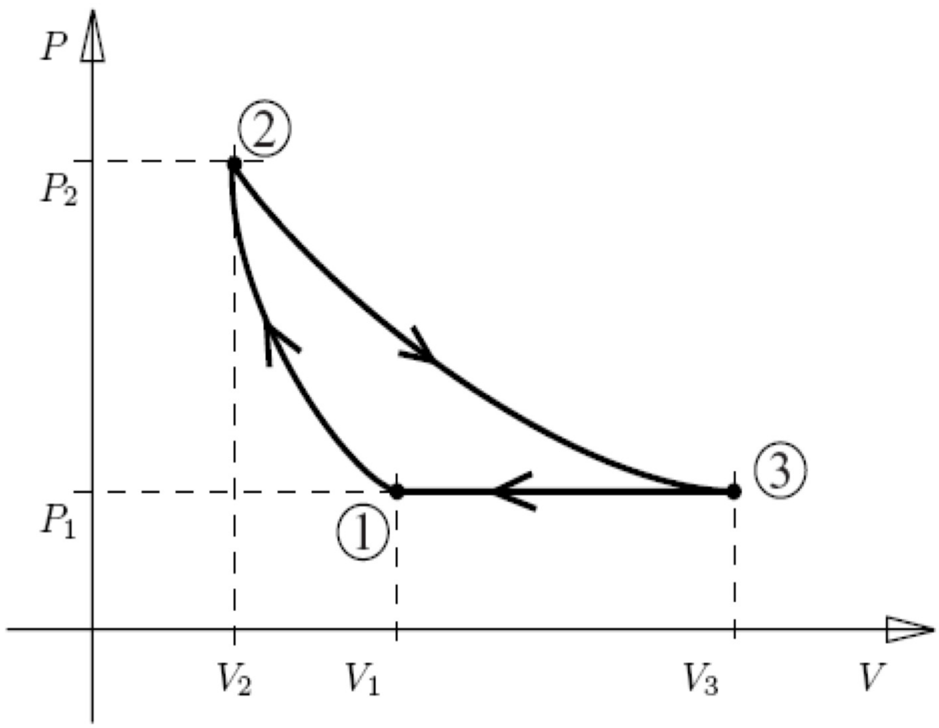
National Physics Competition 2008
Pre-University Level

Answer **ALL** questions in this paper.

1. One mole of a classical ideal gas undergoes the cycle in the PV diagram shown in the figure below; 1-2 is an adiabatic process; 2-3 is an isothermal process; 3-1 is an isobaric process; and $V_3 = 2V_1$.



(a) Define what is meant by adiabatic process. Write an equation relating the work done by the gas, dW and the change in internal energy of the gas, dU during an adiabatic process. [2]

Adiabatic process ; A process that takes place without any transfer of heat between the system and the surroundings. *[1]*

$dU = -dw$ *[1]*

(b) From your equation in part (a), show that the equation for an adiabatic process can be written as

$p V^{\gamma}$ = constant

where $\gamma = C_P/C_V$, and C_P and C_V are the molar heat capacities at constant pressure and volume respectively. [4]

$$dU = -pdV$$

$$nC_v dT = -pdV \qquad [1]$$

$$n(\frac{f}{2}R)dT = -pdV \qquad \text{from} \qquad f = \frac{2}{\gamma - 1}$$

$$\frac{nR}{\gamma - 1}dT = -pdV \quad \text{———————} \quad (1) \qquad [1]$$

From ideal gas equation :

$$pV = nRT$$

$$pdV + Vdp = nRdT \quad \text{———————} \quad (2) \qquad [1]$$

Subt (2) into (1) :

$$\frac{pdV + Vdp}{\gamma - 1} = -pdV$$

$$Vdp = -\gamma \; pdV$$

$$\int\frac{dp}{p} = -\gamma\int\frac{dV}{V} \qquad [1]$$

$$\ln P = -\gamma \ln V$$

$$pV^{\gamma} = const.$$

(c) Find an expression for V_2 in terms of V_1 and γ. [5]

In process 2 → 3 :

$$P_2 V_2 = P_3 V_3 \qquad \boxed{[1]}$$

$$= P_1 V_3$$

$$P_2 V_2 = 2P_1 V_1$$

$$nRT_2 = 2\,nRT_1$$

$$T_2 = T_3 = 2T_1 \qquad [1]$$

From $P_2V_2 = 2\,P_1\,V_1$

$$P_2/P_1 = 2\,(V_1/V_2)$$

$$P_2 V_2^{\gamma} = P_1 V_1^{\gamma} \qquad [1]$$

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma}$$

$$2\left(\frac{V_1}{V_2}\right) = \left(\frac{V_1}{V_2}\right)^{\gamma} \qquad [1]$$

$$V_2 = \left(\frac{1}{2}\right)^{\frac{1}{\gamma-1}} V_1 \qquad [1]$$

(d) Find an expression for P_2 in terms of P_1 and γ. [3]

Fr $$\frac{P_2}{P_1} = 2\left(\frac{V_1}{V_2}\right)$$

$$= 2 \times 2^{\frac{1}{\gamma-1}} \qquad [1]$$

$$= 2^{1+\frac{1}{\gamma-1}}$$

$$= 2^{\frac{\gamma}{\gamma-1}} \qquad [1]$$

$$P_2 = 2^{\frac{\gamma}{\gamma-1}} P_1 \qquad [1]$$

(e} In terms of P_1, V_1 and the ratio of molar heat capacities, γ,

(i) find the heat absorbed by the gas and the work done by the gas in each of the three steps 1-2, 2-3, and 3-1. [12]

1→2 : Q = 0 [1]

$$W = \frac{P_2V_2 - P_1V_1}{1-\gamma} \quad [1]$$

$$= \frac{2P_1V_1 - P_1V_1}{1-\gamma} \quad [1]$$

$$= \frac{P_1V_1}{1-\gamma} \quad [1]$$

2→3 :

$$Q = W \quad [1]$$

$$= nRT_2 \ln\left(\frac{V_3}{V_2}\right) \quad [1]$$

$$= 2RT_1 \ln\left(2^{\frac{\gamma}{\gamma-1}}\right) \quad [1]$$

$$= 2P_1V_1 \ln\left(2^{\frac{\gamma}{\gamma-1}}\right) \quad [1]$$

3→1 :

$$Q = C_p dT \quad [1]$$

$$= (C_v + R)dT$$

$$= \left(\frac{R}{\gamma - 1} + R\right)(T_1 - T_3)$$

$$= -\left(\frac{\gamma}{\gamma - 1}\right)RT_1 = -\frac{\gamma P_1V_1}{\gamma - 1} \quad [1]$$

$$W = P_1(V_1 - V_3) \quad [1]$$

$$= P_1(V_1 - 2V_1)$$

$$= -P_1V_1 \quad [1]$$

(ii) find the internal energy of the gas at each of the three states 1, 2, and 3.

[6]

$$U_1 = \frac{f}{2} RT_1 \qquad [1]$$

$$= \frac{f}{2} P_1V_1$$

$$= \frac{P_1V_1}{\gamma - 1} \qquad [1]$$

$$U_2 = \frac{f}{2} P_2V_2 = \frac{f}{2}(2P_1V_1\} \qquad [1]$$

$$= \frac{2P_1V_1}{\gamma - 1} \qquad [1]$$

$$U_3 = U_2 \qquad [1]$$

$$= \frac{2P_1V_1}{\gamma - 1} \qquad [1]$$

(f) Calculate the efficiency of a reversible engine utilizing this cycle. [8]

$$W_{12} = \frac{P_1V_1}{1-\gamma} \qquad [1]$$

$$W_{23} = \frac{2P_1V_1\gamma}{\gamma - 1}\ln 2 \qquad [1]$$

$$W_{31} = -P_1V_1 \qquad [1]$$

$$W_{net} = W_{12} + W_{23} + W_{31} \qquad [1]$$

$$= \frac{P_1V_1\gamma}{\gamma - 1}(2\ln 2 - 1) \qquad [1]$$

$$Q = Q_{12} + Q_{23} \qquad [1]$$

$$= 0 + 2P_1V_1 \times \frac{\gamma}{\gamma - 1}\ln 2$$

$$= \frac{2P_1V_1\gamma \ln 2}{\gamma - 1} \qquad [1]$$

$$\eta = \frac{W_{net}}{Q}$$

$$0.279 \quad or \quad 27.9\% \qquad [1]$$

2. (a) A geostationary satellite is a satellite which is orbiting the Earth above the equator with the same angular velocity as the Earth. The satellite appears stationary to an observer on the Earth's surface. Calculate the altitude of the satellite. [5]

$$\frac{GMm}{r^2} = mr\omega^2 \qquad [1]$$

$$\frac{gR^2}{r^3} = \left(\frac{2\pi}{T}\right)^2 \qquad [1]$$

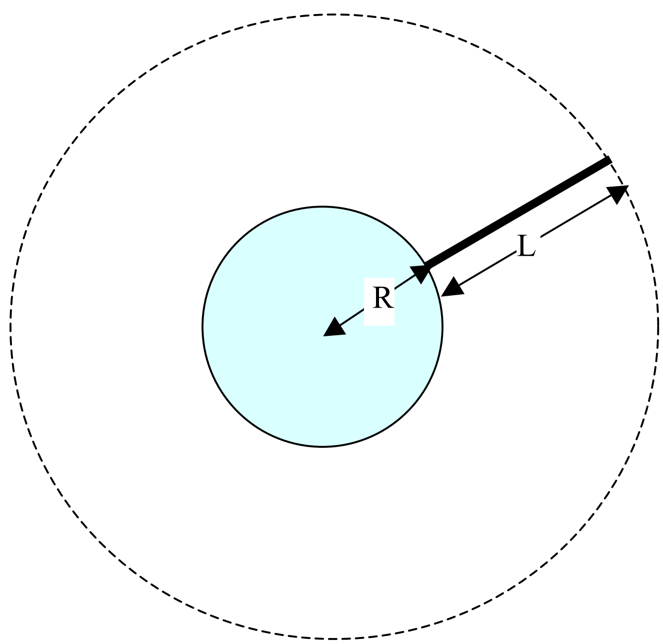
$$r^3 = \frac{gR^2T^2}{4\pi^2}$$

$$= \frac{9.81\times(6.4\times10^6)^2\times(24\times3600)^2}{4\pi^2} \qquad [1]$$

$$= 4.24\times10^7\,m \qquad [1]$$

$$h = 36{,}000km \qquad [1]$$

(b) The science-fiction writer R.A. Heinlein describes the "skyhook" geostationary satellite to consist, in its simplest form, of a uniform cable of mass per unit length, λ placed in the equatorial plane, oriented radially and extending from just above the surface of the planet to a distance L above the planet as shown in the diagram below.



By considering an infinitesimal segment of the cable, find an expression for

(i) the total gravitational force acting on the "skyhook", W.

[1]

$$dW = \frac{GMdm}{r^2}$$

$$= \frac{gR^2\lambda\ d\lambda}{r^2}$$ [1]

[1]

$$W = \int_R^{R+L} \frac{gR^2\lambda}{r^2} dr$$

$$= gR^2\lambda\left[-\frac{1}{r}\right]_R^{R+L}$$ [1]

$$= \frac{\lambda gRL}{R+L}$$ [1]

(ii) the total centripetal force acting on the "skyhook", F_c. [10]

$$dF_c = dmr\omega^2$$ [1]

$$= \lambda dr\ r\omega^2$$

$$= \lambda r\omega^2 dr$$ [1]

$$F_c = \int_R^{R+L} \lambda r\omega^2 dr$$ [1]

$$= \frac{\lambda\omega^2 L}{2}(L+2R)$$ [2]

(c) Hence, deduce the required length of "skyhook" for it to orbit around the Earth as a geostationary satellite. [5]

(The radius of the Earth = 6,400 km;
The gravitational acceleration at the surface of the Earth = 9.81 m s^{-2})

$$F_c = W$$ [1]

$$\frac{\lambda\omega^2 L}{2}(L+2R) = \frac{\lambda gRL}{R+L}$$ [1]

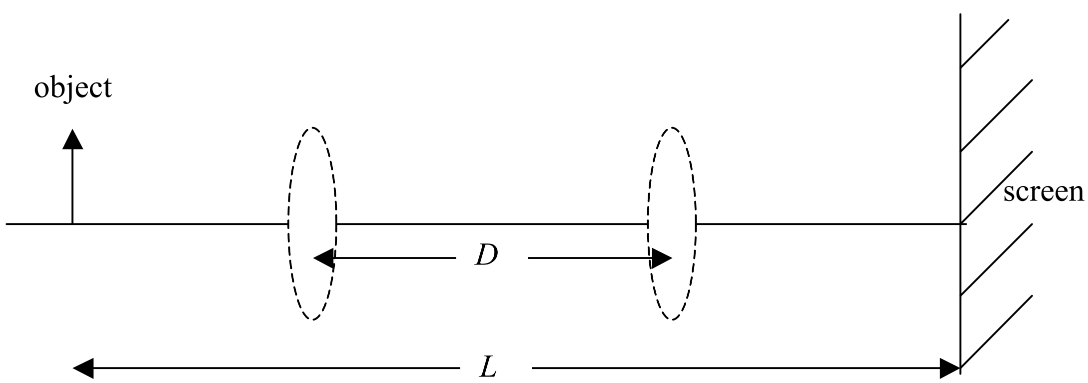
$$L^2 + 3RL + \frac{2R(R\omega^2 - g)}{\omega^2} = 0 \qquad [1]$$

$$L = \frac{-3R + \sqrt{9R^2 - 8R(R\omega^2 - g)}}{\omega^2} \qquad [1]$$

$$= 1.45 \times 10^5\, km \qquad [1]$$

3. (a) **Bessel method of focal length determination**

The diagram below shows a lens being moved along the optical axis between a fixed object and a fixed image screen. The object and image positions are separated by a distance L that is more than 4 times the focal length of the lens. Two positions of the lens are found for which an image is in focus on the screen, magnified in one case and reduced in the other. If the separation between the two positions of the lens is D, derive an expression for the focal length , f in terms of L and D.



[8]

u + v = L [1]

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \qquad [1]$$

$$\frac{1}{u} + \frac{1}{L-u} = \frac{1}{f}$$

$$u^2 - Lu + Lf = 0 \qquad [1]$$

$$u = \frac{L \pm \sqrt{L^2 - 4Lf}}{2} \qquad [2]$$

$D = u_2 - u_1$ [1]

$= \frac{L + \sqrt{L^2 - 4Lf}}{2} - \frac{L - \sqrt{L^2 - 4Lf}}{2}$

$= \sqrt{L^2 - 4Lf}$ [1]

$f = \frac{L^2 - D^2}{4L}$ [1]

(b) **Image formed by thick lens**

An object is placed 5 *cm* from the front surface of a thick symmetrical biconcave lens with surfaces of radius 10 *cm*. The lens is made of glass with *n*=1.5, and it is 3 *cm* thick at the centre. Using the thick lens approach calculate

(i) the image location with respect to the object . [4]

$\frac{n_1}{u} + \frac{n_2}{v} = \frac{n_2 - n_1}{r}$ [1]

$\frac{1}{5} + \frac{1.5}{v_1} = \frac{1.5 - 1}{-10}$ [1]

$v_1 = -6.0cm$

$u_2 = 9cm$ [1]

$\frac{1.50}{9} + \frac{1}{v} = \frac{1 - 1.50}{10}$ [1]

$v = -4.62cm$

Image is formed 3.38cm from the object between the object and the lens.

(ii) state whether the image is real or virtual, upright or inverted. [2]

The image is virtual and upright. [1,1]

(c) **Lens-maker's equation**

A thin convex lens with refractive index of 1.50 has a focal length of 30 cm in air. When immersed in a certain transparent liquid, it becomes a negative lens with a focal length of 188 cm. Determine the refractive index of the liquid

[6]

$$\frac{1}{f} = (n-1)(\frac{1}{r_1} - \frac{1}{r_2}) = \frac{1}{30}$$ [1]

$$(1.5-1)(\frac{1}{r_1} - \frac{1}{r_2}) = \frac{1}{30}$$ [1]

$Inliquid,$

$$\frac{1}{f} = \left(\frac{n}{n'} - 1\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$ [1]

$$-\frac{1}{188} = \left(\frac{1.5}{n'} - 1\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$ [1]

Solve the equations above;

$$\frac{1}{-188} = \left(\frac{1.5}{n'} - 1\right)\left(\frac{1}{15}\right)$$ [1]

$$n' = 1.63$$ [1]

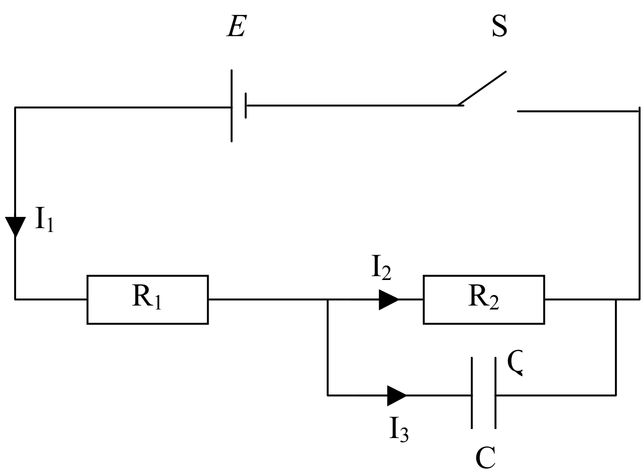
4. (a) State Kirchoff's laws for circuits. [2]

Kirchoff's first Law: The total current flowing into a junction equals to the total current flow out from the junction.

Kirchoff's Second Law:

The algebraic sum of the potential difference in a closed loop is zero.

The diagram below shows a circuit in which a battery of emf E and negligible internal resistance is connected in series with resistors R_1 and R_2 and a switch S. A capacitor of capacitance, C which is initially uncharged is connected across resistor R_2. After the switch is closed for time t, the currents flowing through resistors R_1, R_2 and the capacitor are I_1, I_2 and I_3 respectively and the charge stored in the capacitor is Q.



(b) In terms of E, R_1 and R_2, find an expression for current I_1

(i) at the moment the switch is closed, t=0.

At t = 0, Q=0

$I_2 = 0$ [1]

$\therefore\ E = I_1R_1$

$I_1 = \frac{E}{R_1}$ [1]

(ii) after the switch is closed for a long time, t = ∞

At t=∞, $I_3 = 0$ [1]

$$I_1 = I_2 = \frac{E}{R_1 + R_2}$$ [1]

[4]

(c) After the switch is closed for time t,

(i) write an equation relating the current I_1, I_2 and I_3 by applying Kirchoff's current law.

$I_1 = I_2 + I_3$ ----------- (a) [1]

(ii) Write two equations relating current I_1, I_2 , and I_3 by applying Kirchoff's voltage law. (Hint: $I = \frac{dQ}{dt}$)

For big loop:

$E = I_1R_1 + I_2R_2$ ----------- (b) [1]

For small loop:

$$I_2R_2 = V_c = \frac{Q}{C}$$ [1]

$$Q = I_2R_2C$$

$$\frac{dQ}{dt} = R_2C\frac{dI_2}{dt}$$ [1]

$$I_3 = R_2C\frac{dI_2}{dt} \text{ ----------- (c)}$$

Or

$$\int I_3 dt = I_2R_2C$$

(iii) deduce a differential equation for current I_2.

Subt. (a) and (c) into (b):

$$\begin{aligned} E &= (I_2 + I_3)R_1 + I_2R_2 \\ &= I_2(R_1 + R_2) + I_3R_1 \end{aligned}$$ [1]

$$= (R_1 + R_2)I_2 + R_1R_2C\frac{dI_2}{dt}$$ [1]

$$R_1R_2C\frac{dI_2}{dt} + (R_1 + R_2)I_2 = E ------(d)$$ [1]

Or $$\frac{dI_2}{dt} + \frac{R_1 + R_2}{R_1R_2C}I_2 = \frac{E}{R_1R_2C}$$

[7]

(d) By assuming that the solution of the differential equation in part (c) (iii) is given by

$$I_2 = a + b e^{-\frac{t}{\tau}}$$ where a, b and τ are constants,

find in terms of E, R_1, R_2, and C an expression for

(i) a

(ii) b

(iii) τ [5]

(i) From equation

$$I_2 = a + be^{-\frac{t}{\tau}}$$

$$\frac{dI_2}{dt} = -\frac{b}{\tau}e^{-\frac{t}{\tau}} \quad ---------(e)$$ [1]

Subst. (e) into (d)

$$a(R_1 + R_2) + b\left[(R_1 + R_2) - \frac{R_1 R_2 C}{\tau}\right]e^{-\frac{t}{\tau}} = E \quad ------(f)$$

From equation (f);

When t → ∞

$$a(R_1 + R_2) = E$$

$$a = \frac{E}{R_1 + R_2}$$ [1]

(ii) From equation

$$I_2 = a + be^{-\frac{t}{\tau}}$$

$$when\ t = 0 \quad I_2 = 0$$ [1]

$$b = -a$$

$$= -\frac{E}{R_1 + R_2}$$ [1]

(iii) From equation (f)

$$\frac{E}{R_1 + R_2}(R_1 + R_2) + b\left[(R_1 + R_2) - \frac{R_1 R_2 C}{\tau}\right]e^{-\frac{t}{\tau}} = E$$

$$\tau = \frac{R_1 R_2 C}{R_1 + R_2} \qquad [1]$$

(e) Write an equation for current I_2 in terms of t. [2]

$$I_2 = \frac{E}{R_1 + R_2} - \frac{E}{R_1 + R_2}\exp(-\frac{R_1 + R_2}{R_1 R_2 C}t)$$

$$= \frac{E}{R_1 + R_2}\left[1 - \exp(-\frac{R_1 + R_2}{R_1 R_2 C}t)\right] \qquad [2]$$