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A Plasticity Theory and Finite Element Implementation of Friction Model

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ABSTRACT

A friction model based on plasticity theory is presented. An interface element was used in the finite element implementation. An incremental-iterative solution strategy was suggested to simulate the non-linear problem in friction. The model was tested to simulate the friction force of the ejection of a powder compact component from a die. The numerical simulation results were validated and shows good agreement.

ABSTRAK

Satu model geseran berdasarkan teori plastik dibentangkan. Satu unsur antara-muka digunakan dalam implementasi unsur terhingga. Strategi penyelesaian tokokan-lelaran dicadangkan bagi menyelaku masalah tak-lelurus dalam geseran. Model ini diuji untuk penyelakuan daya geseran terhadap daya tolakan komponen padatan serbuk dari dalam acuan. Keputusan penyelakuan berangka telah dibuktikan dan menunjukan keputusan yang baik.

INTRODUCTION

Friction appears as a consequence of the interaction between two bodies. The nature of friction forces developed during contact and sliding is extremely complex and is affected by a number of factors such as the characteristics of the interface, the time scale and the frequency of the contact, the response of the interface to normal forces, inertia and thermal effects, the roughness of the contact surfaces, history of loading, wear and general failure of the interface materials, the presence or absence of lubricants and so on (Oden and Martin, 1985) .

Recently, much attention has been devoted to the numerical analysis of friction and contact in general engineering problems. Despite this, friction modelling is not so well developed as a continuum mechanics modelling, and further work in this area is still needed. In order to take into account the frictional effects, several treatments have been considered in conjunction with finite element modelling. Most of the finite element implementations of frictional phenomena have been based on the classical Coulomb law (Coccoz et al. 1994). A Tresca friction law which is a generalisation of the Coulomb frictional law has also been used (Jinka 1992). More recently, frictional phenomena have been considered within the framework of the theory of plasticity. Elastoplastic analogies have been used to devise the frictional algorithms (Curnier 1984; Schonauer 1993). Also due to its effect at the microscopic scale, micro-mechanical models have also been used in the treatment of friction and their results incorporated into a constitutive theory (Rodic and Owen 1989).

Another issue in this modelling is the treatment of the interface. The interface behaviour has been modelled by a connecting node between the workpiece and die (Park 1985). This has been extensively used in the metal forming processes especially in sheet metal simulation. In this type of treatment, common connected nodes were used for different materials which may not represent the real situation when a thin layer of oxide or lubricant lies in between them. Another approach for representing this interface behaviour is to use a contact layer which has a very small thickness (Vakhroucher et al. 1992). In this case, another type of domain was considered as an interface material. A more robust treatment is associated with using an interface element (Rodic and Owen 1989) with one side representing the die and the other representing the workpiece. The work presented in this paper will address the macromechanical model of the friction incorporating the plasticity anology and the used of an interface element in the finite element procedure.

PLASTICITY THEORY OF FRICTION

One of the first descriptions of frictional behaviour which can be derived from the classical theory of plasticity can be found in Fredriksson (1976). The formulation can be achieved from an analogy between frictional and plastic phenomena. Table 1 presents the related formulation to a plasticity model. The formulation of frictional phenomena are discussed as itemised below.

TABLE 1. Plasticity and fricion theory

Plasticity	⇔	Friction
elastic	⇔	stick
elastic-plastic	⇔	stick-slip
yield criterion	⇔	friction criterion
hardening/softening rules	⇔	tearing/wearing rules
flow rules	⇔	slip rules

STICK REGION

The physical meaning of the stick region is that sticking is caused by the elastic deformation of the asperities at the contact surface. The sticking behaviour in the tangential direction is described by

$$\tau = c_t g_t^c + c_n g_t^c \tag{1}$$

The tangent stress, τ leads to a tangential shear displacement, g_t^c with coefficient of sticking, c_t. In a closed or rigid die, it is assumed for simplicity that there is no movement in the normal direction because there will be no penetration of workpiece into the tools, so the normal displacement, $g_n^c = 0$. Figure 1 shows a clear view of the shear force and displacement in a quasistatic compaction or more obviously during ejection of the component from the die.

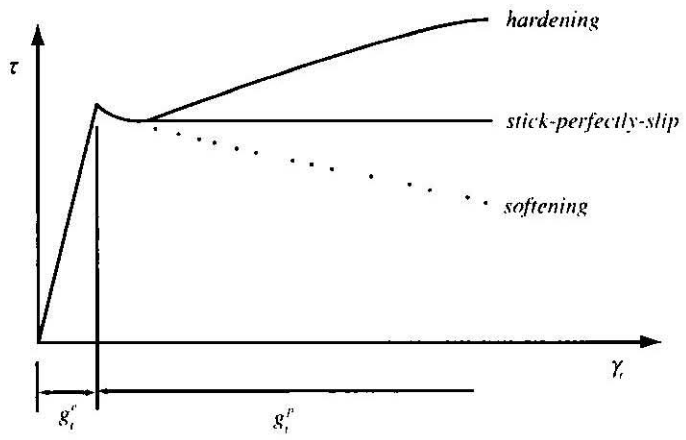


FIGURE 1. Shear stress - shear strain

STICK-SLIP DECOMPOSITION

The shear displacement is decomposed into two parts; stick and slip, which is in principle the same as the decomposition of elastic and plastic behaviour. The stick is reversible and the slip is irreversible. As mentioned earlier, the normal displacement is assumed to be zero so only a tangential displacement will be considered which consists of stick and slip decompositions.

$$g_t = g_t^e = g_t^p \tag{2}$$

where g_t^e and g_t^p are the tangential shear displacement for stick and slip respectively.

FRICTION CRITERION

A friction criterion is the indicator whether tangential sticking or slipping occurs. If the friction force reaches a certain threshold, called the slip limit, then relative shearing occurs. The general form is given as

$$F_f(\tau,\sigma_N)\begin{cases} < 0 & \text{stick} \\ = 0 & \text{slip} \end{cases} \tag{3}$$

The most common friction criterion for a perfect friction state is the Coulomb law which can be expressed mathematically as

$$F_f(\tau,\sigma_N) = |\tau| + \mu\sigma_N - c_f \tag{4}$$

where τ is frictional shear stress, σ_N is normal stress, μ is friction coefficient and c_f is friction cohesion. The normal stress should in under compression condition or there will be no friction between the workpiece and the die if the stress is under tensile condition. Or more generally as suggested by Curnier (1984) which includes wear and tear model.

$$F_f(\tau,\sigma_N) = |\tau|^a + \mu\sigma_N + b \tag{5}$$

where exponent a and constant b denotes the tear and wear phenomena.

WEAR AND TEAR

The wear and tear is the phenomena of hardening and softening during sliding. If this phenomenon is ignored, the frictional behaviour is analogous to a perfectly plastic material in plasticity. The kinematic variables of stick and slip are associated with the friction force and also wear and tear forces. These two forces were introduced as different forms of the same phenomenon (i.e. grinding-in) (Curnier, 1984; Rodic and Owen, 1989) and both processes were assumed to produce a reduction in the force of friction. The shear stress increases when the hardening mechanisms occur and conversely, when the shear stress decreases, the softening mechanism takes place.

The development of shear force during shearing is dependent on the length of the compacted part which has been found in several references (Ernst and Barnekow, 1994), which describe typical examples for this behaviour. In ejection, the shear force decreases for the shorter component. On the other hand, the shear force increases for longer component.

SLIP RULES

The direction of slip, is governed by slip rules deriving from a slip potential, Z

$$dg_t^p = \varphi \frac{\partial Z}{\partial \tau} \text{ and } dg_n^p = \varphi \frac{\partial Z}{\partial \tau_s} \tag{6}$$

where φ is a constant expressing the colinearity of the slip increment with the outward normal to the slip potential. If Z is replaced by the slip criterion F_f, the slip rule becomes associated. Physically, this is not acceptable because it will create a gap (separation) as shearing process proceeds. The non-associated slip rule is therefore more applicable to model the friction problem. Figure 2 shows a better view of the associated and non-associated slip rule applied to contact behaviour. It clearly shows that the direction of

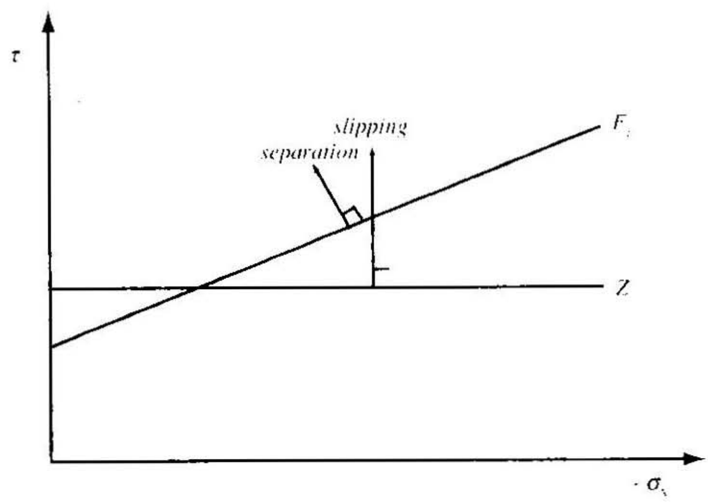


FIGURE 2. Slip surface criterion and non-associated slip rules

slipping is parallel to the direction of shearing stress. The non-associated slip rule, by analogy, is the same as associated flow rule in the von Mises criterion in plasticity.

COMPUTATION OF THE FRICTIONAL PROBLEM

A special six-noded isoparametric interface element was used in the finite element discretization procedure and this is shown in Figure 3. The first three nodes of this element represent the workpiece material on one side and the tool on the other side. After giving the definition of displacement and strain by using the standard finite element, the stress-strain relationships for this interface element have to be defined which represent the normal and shear relationships. The normal is to capture the asperity deformation in the workpiece material or the tooling, whereas the shear relationship is dependent on the yield criterion. The frictional stiffness matrix was then calculated from these relationships to give the stress-strain matrix at a particular element.

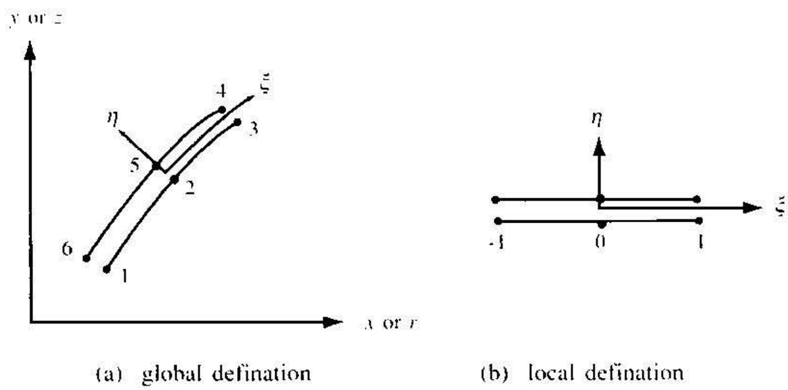


FIGURE 3. Global and local definations of interface element

FINITE ELEMENT APPROXIMATIONS

By using shape functions as the usual way in the finite element procedure, the displacement at any point ξ on the powder side is given by the vector $\{u_p\}$ and on the tooling by $\{u_t\}$.

For the special element shown in Figure 3, the displacement of node i in the local ξ and η axes needs to be defined. The shape functions N_i associated with node i are expressed in terms of the local ξ coordinate as

$$N_i = \frac{1}{2}\xi(\xi - 1) \quad ; \; i = 1,6 \tag{7}$$

$$N_i = 1 - \xi^2 \quad ; \; i = 2,5 \tag{8}$$

$$N_i = \frac{1}{2}\xi(\xi + 1) \quad ; \; i = 3,4 \tag{9}$$

The unit tangent and vector normal to the interface are respectively given by

$$n=\frac{1}{J}\left[\frac{\partial x}{\partial \xi}i+\frac{\partial y}{\partial \xi}j\right] \tag{10}$$

and

$$s=\frac{1}{J}\left[\frac{\partial y}{\partial \xi}i+\frac{\partial x}{\partial \xi}j\right] \tag{11}$$

where the Jacobian J is related to the length L of the interface

$$J=\left[\left(\frac{\partial x}{\partial \xi}\right)^2+\left(\frac{\partial y}{\partial \xi}\right)^2\right]^{1/2} \tag{12}$$

The transformation from global to local displacement (or strain) is then given by the following matrix and is introduced to deal with the arbitrarily orientated surface geometry of the tooling i.e.

$$T=\begin{bmatrix} n \\ s \end{bmatrix} \tag{13}$$

Therefore, the local displacement is obtained from the following transformation

$$u_p'=Tu_p \tag{14}$$

$$u_t'=Tu_t \tag{15}$$

The local relative displacements are defined as the difference between the local displacement of the workpiece and the tool, i.e

$$\begin{Bmatrix} g_t \\ g_n \end{Bmatrix}=\begin{Bmatrix} u_p' \\ v_p' \end{Bmatrix}-\begin{Bmatrix} u_t' \\ v_t' \end{Bmatrix} \tag{16}$$

From equations (14) and (15), then (16) leads to the relationship

$$\begin{Bmatrix} g_t \\ g_n \end{Bmatrix}=B_f\begin{Bmatrix} u_p \\ v_p \end{Bmatrix} \qquad \text{where} \qquad B_f=TN \tag{17}$$

Since the interface is a zero thickness element, the definition of strain is taken directly from the relative displacement in equation (16).

CONSTITUTIVE RELATIONSHIP

The normal strain is considered negligible and is governed mainly by asperity deformation at the contact. Since the normal stress is significant, the small strain is achieved by prescribing a large modulus coefficient E_f. Also this is chosen since deformation in this normal direction does not have any significance in the forming process (Burr and Donachie 1963). Thus, the normal stress-strain relationship can be achieved by means of a linear equation

$$\sigma_n=E_f g_n \tag{18}$$

where E_f can be chosen to capture asperity contact as an appropriately chosen large number. In this work, the value used is 105. Equation (1) expressed the tangent stress which has been decomposed into stick and slip part. Considering the stick region first, the incremental form of the shear stress-strain relationship can be given as

$$\Delta\tau = c_t^e \Delta g_t \tag{19}$$

where the sticking coefficient is proportional to the stick shear modulus of the workpiece and tool as shown in Figure 1. The shear force then, is limited by the slip criterion as in equation (4) or (5). However, because of the availability of experimental data (Gethin et al, 1994), the shear stress-strain relationship for the stick-slip region can be obtained.

The stiffness matrix at the interface is defined using the standard finite element method, i.e

$$K = \int_{L_e} B_f^T D_f B_f \, dL \tag{20}$$

where L_e is the length of the interface element. D_f is the stress-strain matrix in the local coordinate system. So that the stress-strain can be written in incremental form as

$$\Delta\sigma = \begin{Bmatrix} \Delta\tau \\ \Delta\sigma_n \end{Bmatrix} = \begin{bmatrix} G_F & 0 \\ 0 & E_f \end{bmatrix} \begin{Bmatrix} \Delta g_t \\ \Delta g_n \end{Bmatrix} \tag{21}$$

$$D_f \Delta\varepsilon \tag{22}$$

The frictional non-linearity is produced by the modulus of slip shear which can be obtained in terms of the gradient of relationship between shear stress and shear strain.

The non-linearity of the friction problem needs the application of numerical techniques. The incremental-iterative solution strategy is used, incorporating the frictional parameters. The total computational procedure is summarised in Appendix I.

NUMERICAL EVALUATION AND VALIDATION

In the literature review, friction is well known as an important parameter where it may affect the applied forces, stresses and density variation throughout the component. Figure 4 illustrates the variation of experimental ejection force as a compact component drawn out from the die. There are two regions which may be defined as stick friction and follows by slip friction. The curves also show clearly that the level of ejection force is very dependent on the length. Details of this phenomena has been explained (Gethin et al. 1994) where the ejection stage is considered. These information data is used in this work for numerical input data and the experimental validation.

The incremental form of the shear stress-strain relationship under this stick region can be given by;

$$\Delta\tau = G_A \Delta g_t \tag{23}$$

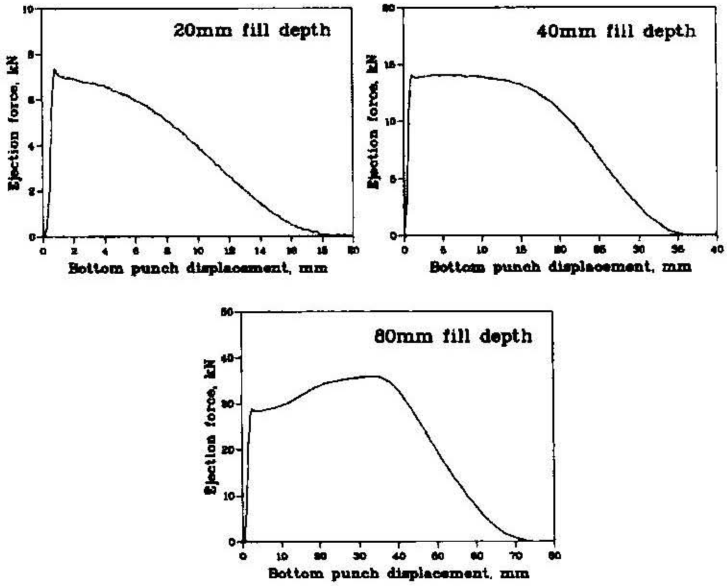


FIGURE 4. Ejection force for iron powder

where G_s is the stick shear modulus which is given by the slope in this region. In the slip region, the slip shear modulus, G_f is appropriate. This slip modulus is given by the slope in the slip region. The values of stick and slip modulus summarised in Table 2 are used in this work.

TABLE 2. Stick and slip modulus

G_s (N/mm²)	G_f (N/mm²)	
	20 mm	40 mm
150	-2.10	0.05

Figure 5 shows the effect of friction modulus associated with the stick and slip behaviour during ejection phase. Figure 5(a) shows the variation in ejection force over a range of stick modulus (110 - 180 N/mm2). It is evident that when a higher stick modulus is applied, the ejection force to break the static friction is higher. Figure 5(b) shows the effect of slip modulus changes (-2.0- 2.0 N/mm²) and clearly the modulus has a significant effect on the level of the ejection force.

Figure 6 shows the simulation of the ejection force which is compared with the experimenal results for 20 mm and 40 mm length component. Good agreement has been achieved when using the friction modulus imput data in Table 2 which incorporates the softening function established from experimental work. In comparing the numerical stability during ejection, the Newton- Cotes integration was found to be more stable than Gauss integration. This is attributed ot the higher order error for Gauss integration for the same number of integration point used.

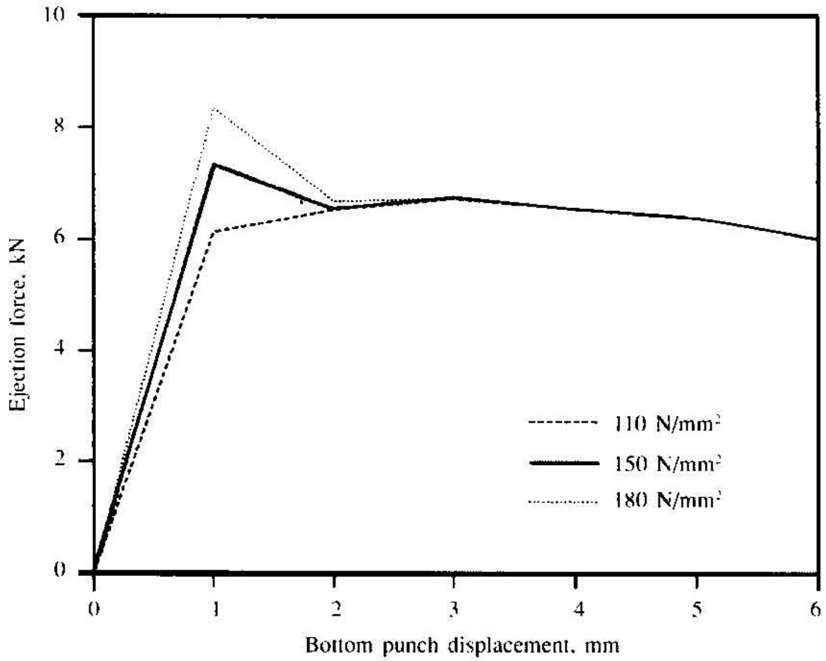


FIGURE 5(a). Variation of ejection force for different stick friction modulus

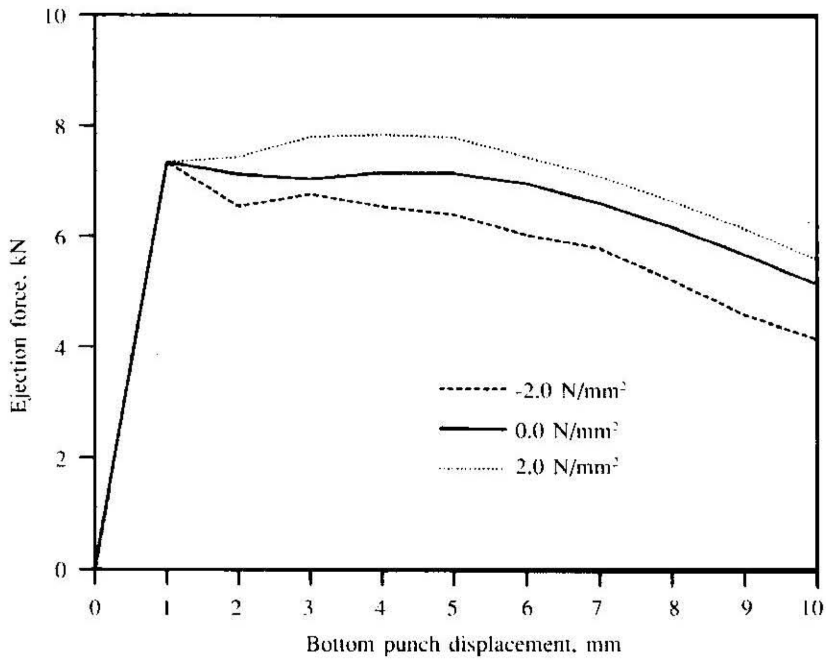
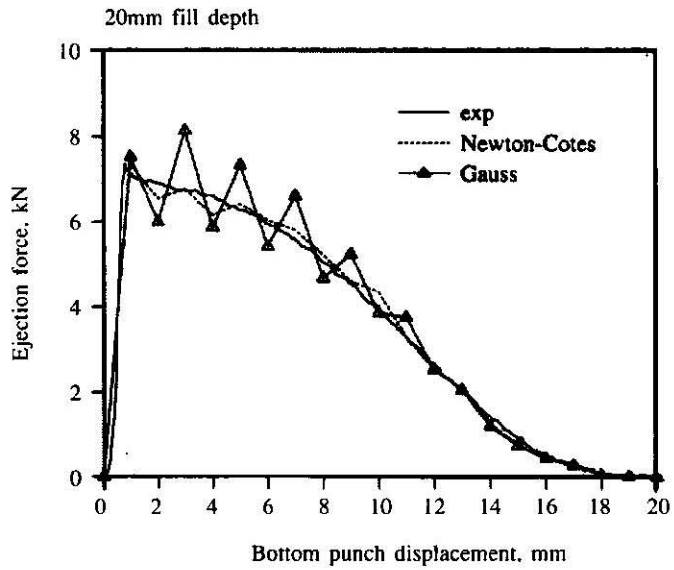


FIGURE 5(b). Variation of ejection force for different slip friction modulus



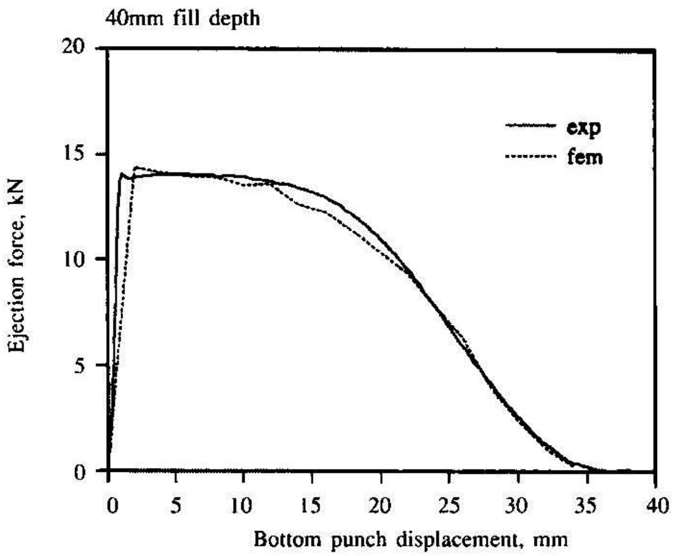


FIGURE 6. Experiment and numerical results on ejection

CONCLUSION

A friction model is completed based on a plasticity theory. Basic ingredient of the model consist of stick-slip decomposition, friction criterion, wear and tear rules and slip rules. In implementing the theory, the numerical procedure incorporating finite element was used. Good agreement has been achieved between simulation and experimental observation.

NOTATION

a, b	Wear and tear constant
c_f	Cohesion of friction
c_n	Coefficient of slipping
c_t	Coefficient of sticking
D	Elasticity matrix
E	Young modulus
F_f	Friction function
g_n	Stick/slip normal displacement
g_t	Stick/slip tangen displacement
u,v	Displacement tensor
Z	Slip function
φ	Plastic multiplier
μ	Friction coefficeint
σ	Stress tensor
τ_f	Friction shear stress
x, y or z, r	Plane or axisymmetric coordinate system for global defination
ζ and η	Local defination coordinate system

APPENDIX I: Computational process for friction

(1) The total prescribed shear displacement, u is divided equally to n small increments, Δu_p^n.

For each increment

(a) For each increment, calculate the tangent stiffness matrix[K] using appropriate G_F modulus. For the first step and first iteration i, set G_F=G_A. For subsequent iterations G_t is defined in computational step (6).

(3) Compute displacement as $\Delta u_n = \mathbf{K}^{-1}\psi_n^{i-1}$

(4) Compute $\begin{Bmatrix} \Delta g_t \\ \Delta g_n \end{Bmatrix} \mathbf{B}_f = \begin{Bmatrix} \Delta u_p \\ \Delta v_p \end{Bmatrix}$.

(5) Evaluate τ at n+1 according to the slip criterion.

(6) Evaluate the frictional shear stress, τ

$$\tau_{n+1}^i = \tau_n^{i-1} + G_F \Delta g_{n+1}^i$$

where $G_F = \dfrac{\tau_E}{|\Delta g_t|}$

(7) Evaluate f$\leq$ 0 continue step (9)

F > 0 continue step (8)

(8) Refine and correct the shear stress state and go to step (5).

(9) Check convergence for each iteration using convergence criteria at each gaussian point

$$\left| \frac{\Delta\tau_n^r - \Delta\tau_n^{r-1}}{\Delta\tau_n^1} \right| \leq \text{TOLLER}$$

if not satisfied calculate frictional stress τ at step (6).

(10) Again n=n+1 for next increment step (1)

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