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A Simplified Elastic Composite Floor Section Analysis with Incomplete Interaction

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ABSTRACT

This paper presents a simplified partial interaction elastic analysis of composite floor panels. A theoretical analysis is carried out on repeating section composing of two elastic beam elements connected longitudinally by linearly elastic connection. The analysis is based on the assumption that a continuous imperfect connection exists between the two separate elements. By applying method of elastic equivalence, expressions for stiffness along the major axis of such composite section is developed. Results obtained from the authors' full scale experimental studies involving profiled steel sheet/ dryboard composite floor panels are compared with results obtained from the simplified expressions, and with theoretical results obtained by other researchers. It is concluded that these expressions are able to predict with reasonable accuracy the behaviour of composite floor panels.

ABSTRAK

Kertas kerja ini memperkenalkan analisis dipermudah interaksi separa bagi panel-panel lantai komposit. Satu teori analisis telah diolah untuk suatu keratan berulang yang terdiri dari dua elemen rasuk kenyal yang dilekatkan secara membujur menggunakan penyambung lelurus kenyal. Analisis dilakukan dengan andaian bahawa penyambung yang tidak sempurna wujud diantara dua elemen berasingan tersebut. Dengan menggunakan kaedah kesetaraan kenyal, ungkapan-ungkapan untuk kekukuhan di sepanjang paksi utama bagi panel lantai komposit tersebut dapat diorakkan. Keputusan yang diperolehi daripada ujikaji berskala penuh yang telah dijalankan oleh penulis dibandingkan dengan keputusan daripada ungkapan-ungkapan yang telah diterbitkan, serta dengan keputusan teori yang diperolehi dari penyelidik lain. Dapatlah disimpulkan bahawa ungkapan yang telah diperolehi boleh meramalkan dengan agak tepat kelakuan suatu panel lantai komposit.

INTRODUCTION

It is a common practice in modern Structural Engineering to arrange different materials in an optimum geometric configuration, with the aim that only the desirable properties of each material will be utilised by virtue of its designated position. The resulting structure is known as a composite structure. Composite structures, in spite of being formed of isotropic materials, may exhibit different elastic properties in two mutually perpendicular directions. The various elastic properties in these cases could be expressed by the different in-plane, flexural, and torsional rigidities of the elements in different directions. This structural orthotropy is the result of the geometrical configuration, rather than the physical properties of the materials.

Compared to its non-composite counterpart, a composite structure tends to have greater stiffness, higher load capacity against material damage, and higher capacity against collapse. Consequently, a composite section is generally smaller than alternative designs to sustain the same load, thus resulting in the saving of material weight and structural depth.

There are various approaches available for the analysis of composite sections. Generally, these approaches can either be classical or numerical ones. This paper will consider a simplified incomplete interaction composite section analysis developed by the present authors, in which two elastic beam elements are connected longitudinally by linearly elastic connections based on classical partial interaction method of analysis.

INCOMPLETE AND FULL INTERACTION BEHAVIOUR

In a composite section, if the beam elements of a composite section are not interconnected, each element acts separately, and thus the load carrying capacity of the beam is not greater than the sum of the individual capacity of the elements (Newmark et al. 1951) On the other hand, if provision is made for the transfer of horizontal shear from one element to the other, the load carrying capacity is considerably increased. Many practical types of shear connectors are not able to transmit all of the horizontal shear, only part of it is transferred, and thus an incomplete interaction behaviour is accomplished.

If the beam elements are joined together by an infinitely stiff shear connection, or the spacing between connectors are very small, then the two members behave as one. Slip and slip strain are everywhere zero, and it can be assumed that plane sections remain plane. This situation represents a full interaction behaviour. However, it would be very economically unviable to try achieving such an ideal situation, as it will be very expensive to provide stiff shear connectors that are very closely spaced. Therefore, this paper focuses on the analysis of composite sections with incomplete interaction. Sections with full interaction, however, can be considered as a special case of incomplete interaction.

BACKGROUND OF THE APPROACH

Originally, the partial interaction approach was first introduced by Newmark et al. (1951). They derived and solved the differential equations governing the behaviour of two elastic beam elements, connected longitudinally by linearly elastic connection, and subjected to point load. The analysis includes the flexibility of the connecting medium in predicting deflections and slips between the surfaces of two elastic beams. Johnson (1975) also obtained a differential equation relating slip to distance along the beam from midspan. This solution was particularly for the case of traditional concrete steel composite structures.

Wright et al. (1989) programmed the resulting equations from Johnson (1975) for a computer solution when composite beams are subjected to uniformly distributed load. This computer program calculates the deflection and slip at any point along the span or the composite beam section. In addition, the longitudinal strains on the outer and inner surfaces of elements are calculated.

In this paper, Newmark's approach is further simplified to obtain simpler expressions. In addition, the authors have applied method of elastic equivalence to derive expression for stiffness along the major axis of the composite sections. The simplified solution is for a single span of the composite section when subjected to a uniformly distributed load. Results obtained from this simplified approach are compared with result from an orthotropic folded plate method (Wan Hamidon 1994, Wright et al. 1989) computer program and the authors experimental results.

THE SIMPLIFIED APPROACH

A composite section built from profiled steel sheeting, connected by screws to dryboard (PSSDB panel) is used in this paper to illustrate the approach (Figure 1). One section of the panel, considered to be made from two elastic members, longitudinally connected by linearly elastic connection is considered. The analysis is based on the following assumptions (Newmark et al. 1951):

1. The shear connection between the upper and the lower section is assumed to be continuous along the length of the beam.
2. The amount of slip permitted by the shear connection is directly proportional to the load transmitted.
3. The distribution of strains throughout the depth of the upper section and the lower section is linear.
4. The upper and lower element are assumed to deflect equal amounts at all points along their length at all times.

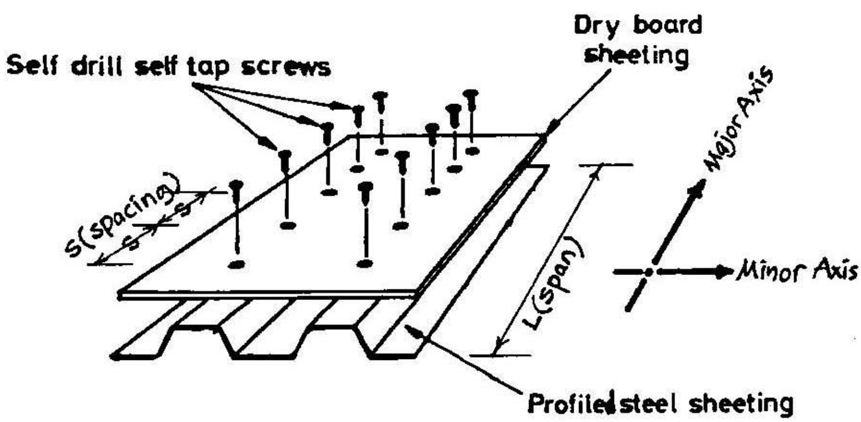
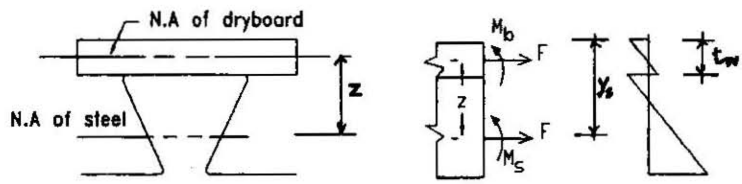


FIGURE 1. Typical profiled steel sheet dry board panel

Referring to Figure 2, the governing differential equation for the above mentioned composite beam as derived by Newmark et al. (1951) is,

$$\frac{d^2F}{dx^2} - c_1 F = -c_2 M \tag{1}$$



(a) Cross-section (b) Internal Forces (c) Strain Distribution

FIGURE 2. Composite beam with imperfect interaction

where, M = external moment.
F = forces acting at the centroids of the board and steel sheeting.
x = distance of the cross-section from the left support.

$$c_1 = \frac{k}{S}\frac{\overline{EI}}{\overline{EA}\sum EI};\ c_2 = \frac{k}{S}\frac{z}{\sum EI};\ M = M_s + M_b + Fz;\ \sum EI = E_b I_b + E_s I_s$$

$$\overline{EI} = \sum EI + \overline{EA}z^2;\ \frac{1}{\overline{EA}} = \frac{1}{E_s A_s} + \frac{1}{E_b A_b}.$$

M_s and M_b = resisting moment by steel and board sections respectively.
E_s and E_b = modulus of elasticities of profiled steel sheet and dry board respectively.
A_s and A_b = areas of profiled steel sheet and board section respectively.
I_s and I_b = second moment of areas of profiled steel and board section about the neutral axes respectively.
z = distance between the neutral axist of profiled steel sheet and dry board section.
k = connector modulus.
S = spacing of connector.

Considering the beam as simply supported, and subjected to a uniformly downward loading, p; Equation (1) becomes,

$$\frac{d^2F}{dx^2} - c_1 F = -C_2 \frac{px}{2}(l - x) \tag{2}$$

where, l is the span of the beam, and other terms are as defined before.

The general solution of this equation is given by,

$$F = K_1\, Cosh\left(\sqrt{c_1}x\right) + K_2 i Sinh\left(\sqrt{c_1}x\right) + \frac{c_2}{c_1} p\left(\frac{lx}{2} - \frac{1}{c_1} - \frac{x^2}{2}\right) \tag{3}$$

To find the constants of integration, the following boundary conditions for F must be satisfied:

At $x=0$, $F=0$, and, at $x=\frac{l}{2}, \frac{dF}{dx}=0;$

Applying these boundary conditions into Equation (3) gives,

$$K_1=\frac{c_2}{c_1^2}p \text{ and } K_2=-\frac{c_2}{c_1^2}p\frac{Sinh\left(\sqrt{c_1}\frac{l}{2}\right)}{iCosh\left(\sqrt{c_1}\frac{l}{2}\right)}$$

Therefore,

$$\frac{dF}{dx}=\frac{c_2}{c_1^2}\sqrt{c_1}p\left[Sinh\left(\sqrt{c_1}x\right)-tanh\left(\sqrt{c_1}l/2\right)\times Cosh\left(\sqrt{c_1}x\right)\right]+\frac{c_2}{c_1}p\left(\frac{l}{2}-x\right) \quad (4)$$

It is now possible to find slip of the composite beam by putting the value of dF/dx in the above equation, where,

$$Slip=\left(\frac{S}{k}\right)\frac{dF}{dx} \quad (5)$$

According to the moment curvature relationship, the relation between moment and curvature for the composite beam is given by,

$$\frac{d^2y}{dx^2}=-\frac{M}{\sum EI}+\frac{Fz}{\sum EI} \quad (6)$$

where y = deflection of the beam section.
or,

$$\sum EI\frac{d^2y}{dx^2}=-\frac{px}{2}(l-x)+z\begin{bmatrix}\frac{c_2}{c_1^2}p\times Cosh\left(\sqrt{c_1}x\right)-\frac{c_2}{c_1^2}p\times tanh\left(\sqrt{c_1}\frac{l}{2}\right)\times Sinh\left(\sqrt{c_1}x\right)\\+\frac{c_2}{c_1}p\left(\frac{lx}{2}-\frac{1}{c_1}-\frac{x^2}{2}\right)\end{bmatrix} \quad (7)$$

After double integration, Equation (7) becomes,

$$\sum EIy=-\frac{plx^3}{12}+\frac{px^4}{24}+zp\begin{bmatrix}\frac{c_2}{c_1^3}Cosh\left(\sqrt{c_1}x\right)-\frac{c_2}{c_1^3}\times tanh\left(\sqrt{c_1}\frac{l}{2}\right)\times Sinh\left(\sqrt{c_1}x\right)\\+\frac{c_2}{c_1}\left(\frac{lx^3}{12}-\frac{x^2}{2c_1}-\frac{x^4}{24}\right)\end{bmatrix}+K_3x+K_4 \quad (8)$$

Applying the following boundary conditions for y, i.e.,
at $x=0$, $y=0$, and, at $x=l/2, dy/dx=0$;
the general equation for deflection becomes,

$$\Sigma EIy = -\frac{plx^3}{12} + \frac{px^4}{24} + zp\frac{c_2}{c_1^3}\left[Cosh\left(\sqrt{c_1}x\right) - tanh\left(\sqrt{c_1}\frac{l}{2}\right) * Sinh\left(\sqrt{c_1}x\right) - 1\right] + zp\frac{c_2}{c_1}\left(\frac{lx^3}{12} - \frac{x^4}{24} - \frac{l^3x}{24}\right) + zp\frac{c_2}{c_1^2}\left(\frac{lx}{2} - \frac{x^2}{2}\right) + \frac{pl^3x}{24} \quad (9)$$

Computer programs are developed to solve Equations (5) and (9) to obtain the slip and deflection along the span of any composite beams with incomplete interaction.

METHOD OF ELASTIC EQUIVALENCE

Method of elastic equivalence (Huffington 1956) has been applied by the present authors to find the general expression for bending stiffness in the longitudinal direction of an equivalent uniform thickness orthotropic plate, representing the original composite section with the same stiffness characteristics. For the particular case of the PSSDB panel considered in this paper, taking the x-coordinate to be parallel to the longitudinal direction of the profiled steel sheet, the orthotropic flexural rigidity associated with bending in that direction shall be denoted by D_{xc}.

Considering an infinitely long strip of the equivalent orthotropic plate, having simply supported boundaries at $x=0$, and at $x=l$ (Figure 3), which is loaded by uniform pressure p, the expression for deflection (Timoshenko & Gere 1961), w, is given as,

$$w = \frac{p}{24D_{xc}}\left(x^4 - 2lx^3 + l^3x\right) \quad (10)$$

and, the maximum deflection will occur at $x=l/2$.

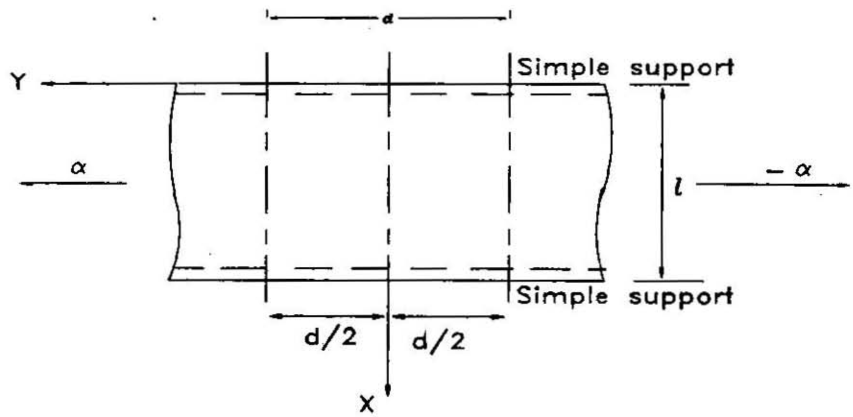


FIGURE 3. Geometry of infinitely long strips

Therefore,

$$w_{max} = \frac{5pl^4}{384D_{xc}} \tag{11}$$

Considering the actual composite member, the maximum deflection can be found by putting $x=l/2$ in the general equation for deflection, thus, Equation (9) becomes,

$$\sum Ely = \frac{5pl^4}{384}\left(1 - z\frac{c_2}{c_1}\right) + zp\frac{c_2}{c_1^3}\left[Sech\left(\sqrt{c_1}l/2\right) - 1\right] + zp\frac{c_2}{c_1^2}\frac{l^2}{8}$$

Putting the values of c_1 and c_2 into the above Equation gives,

$$\begin{aligned}\sum EIy_{max} &= \frac{5pl^4}{384}\left(1 - z^2\frac{\overline{EA}}{\overline{\overline{EI}}}\right) + z^2\left(\frac{\overline{EA}}{\overline{\overline{EI}}}\right)^2 . \frac{pl^2}{8\left(\frac{k}{S}\right)}(\sum EI) \\ &+ z^2\left(\frac{\overline{EA}}{\overline{\overline{EI}}}\right)^3 \frac{(\sum EI)^2}{\left(\frac{k}{S}\right)^2} p\left[Sech\left(\sqrt{\frac{k}{S}\frac{\overline{\overline{EI}}}{\overline{EA}\sum EI}}\frac{l}{2}\right) - 1\right]\end{aligned} \tag{12}$$

where y_{max} is the maximum deflection of the composite section.
For an infinitely stiff connection, Equation (12) takes the form as follows,

$$\sum EIy_{max} = \frac{5pl^4}{384}\left(1 - z^2\frac{\overline{EA}}{\overline{\overline{EI}}}\right)$$

or,

$$y_{max} = \frac{5pl^4}{384\sum EI}\left(1 - z^2\frac{\overline{EA}}{\overline{\overline{EI}}}\right) \tag{13}$$

Equating the maximum deflections of Equation (11) and Equation (13), the expression for stiffness for fully composite action is given by,

$$D_{xc} = \frac{\sum EI}{1 - z^2\frac{\overline{EA}}{\overline{\overline{EI}}}} \tag{14}$$

Equating the maximum deflections of Equations (11) and (12), the expression for stiffness for incomplete interaction is given by,

$$\frac{1}{D_{xc}} = \frac{1}{\sum EI}\left(1 - z^2\frac{\overline{EA}}{\overline{\overline{EI}}}\right) + \frac{384}{5l^4}\left[z^2\left(\frac{\overline{EA}}{\overline{\overline{EI}}}\right)^2\frac{l^2S}{8k} + z^2\left(\frac{\overline{EA}}{\overline{\overline{EI}}}\right)^3\frac{\sum EI}{\left(\frac{k}{S}\right)^2}\left\{sech\left(\sqrt{c_1}l/2\right) - 1\right\}\right] \tag{15}$$

The modulus of the shear connector, k, the only term in the derived equations which is not known from the dimension of the composite beam, and the properties of the materials, may be determined experimentally from relatively simple push-out tests.

EXPERIMENTAL STUDY AND COMPARISON OF RESULTS

An extensive study has been conducted by Ehsan (1996) using 1 mm thick Bondek profiled steel sheet, attached to dryboard made of either 18 mm plywood, or chipboard, or cemboards of various thicknesses (either 12, 16 or 24 mm thick). The panels are having dimensions of 600 mm by 2400 mm. The two components, i.e. the board and steel sheet are connected by self tapping screws at various spacings in each rib. The behaviour observed in these tests, and the measured results could then be used to establish the performance of the system as a whole, and as a basis for comparison with analytical method developed in this paper.

RELIABILITY TEST OF SIMPLIFIED APPROACH

The derived equations in the preceding sections are programmed for a computer solution. The slip and deflection at various points along the span are calculated using Equations (5) and (9) respectively. The stiffness values of the composite panels along the major axis of the section are calculated using Equations (14) and (15) for full and incomplete interaction cases respectively.

The Chosen Tests To check the reliability of the simplified approach, and the newly developed computer program, three sample tests involving composite PSSDB sections as shown in Table 1 have been used. Figure 4 shows a typical test arrangement, and one repeating cross section of the panel. The composite section is subjected to a uniformly distributed downward load of 5 N/mm. The span chosen for all tests is 2.2 m.

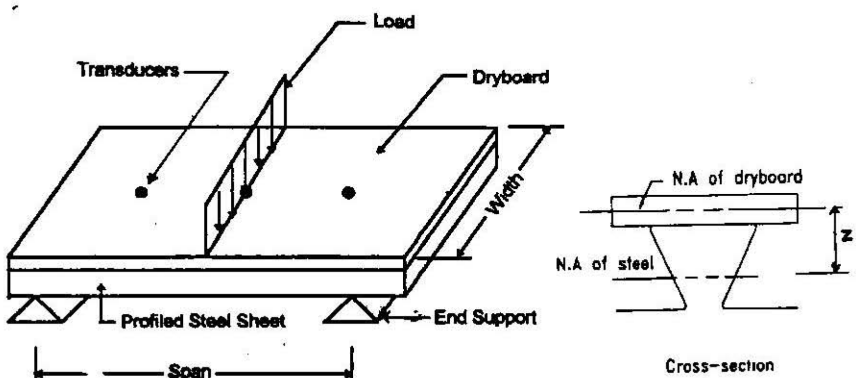


FIGURE 4. Typical test arrangement and one repeating cross-section of the panel

TABLE 1. Sample tests using Bondek II/1.0 mm thick, 200 mm connector spacings, and 2.2 m span

Test No.	Board
1	18 mm thick Plywood
2	18 mm thick Chipboard
3	16 mm thick Cemboard

The Computer Program Input Data The input parameters required for the newly developed computer program to solve Equations (12), (13), (14) and (15) to calculate slip and deflection are as follows:

Young's modulus of steel is, E_{steel}	= $210x10^3$ N/mm².
Young's modulus of chipboard, $E_{chipboard}$	= 1950 N/mm².
Young's modulus of plywood, $E_{plywood}$	= 5300 N/mm².
Young's modulus of cemboard, $E_{cemboard}$	= 5250 N/mm².
Area of profiled steel sheet	= $1.633x10^3$ mm²/m.
Area of chipboard/plywood	= $18x10^3$ mm²/m.
Area of cemboard	= $16x10^3$ mm²/m.
Second moment of area of profiled steel sheet	= 63.68 cm⁴/m.
Second moment of area of chipboard/plywood	= $0.486x10^6$ mm⁴/m.
Second moment of area of cemboard	= $0.341x10^6$ mm⁴/m.
Distance between centroids of upper and lower sections	= 48.57 mm (for ply/chip board) and 47.57 mm (for cemboard).
Number of studs through thickness	= 5 (single connector is used on the top flange of each of the five bays in the 1 m width section).
Stud stiffness	= 730, 625 and 470 N/mm², for plywood, cemboard and chipboard respectively.
Spacing of studs	= 200 mm.
Span of composite beam	= 2.2 m in all the tets.
Uniform transverse loading	= 5 N/mm.

Check on Defletions, End Slips, and Stiffnesses Table 2 shows the deflections at mid-span, mid-width obtained from the author's computer program compared to that obtained by the folded plate method using dummy plate modeling technique (Wan Hamidon, 1994), Wright et al's (1989) computer program based on Johnson's analysis (1975), and the experimental results described earlier.

Table 3 shows the comparison of end slip values for the above mentioned three different test. Here, comparison is made between values obtained from Equation (5) and the values from Wright et al's (1989) computer program.

TABLE 2. Comparison of mid-span, mid-width vertical deflections

Test	Author's Equation	Wan Hamidon Dummy Plate	Wright et al's Programe	Author's Experiment
1	10.85 mm	10.40 mm	10.85 mm	10.81 mm
2	11.10 mm	10.95 mm	11.10 mm	11.05 mm
3	10.97 mm	10.59 mm	10.97 mm	10.74 mm

TABLE 3. Comparison of end slip (mm)

Test	Wright et al's Program	Author's Equation
1	0.749	0.749
2	0.757	0.757
3	0.743	0.743

The stiffness values of the composite panels along the major axis of the section have been calculated using Equations (14) and (15) for complete and partial interaction cases respectively. Table 4 shows the comparison between results obtained by using the derived equations, results from experimental tests, and also results from simple all steel equivalent sections.

TABLE 4. Comparison of stiffness values (kNm^2/m) along the major axis

	Full interaction EI values		Partial interaction EI values	
Test	Elastic analysis based on all steel equivalent section	Values using equation (14)	Experimental values	Values using equation (15)
1	312.0	312.0	141.3	140.5
2	210.3	210.3	138.2	137.3
3	288.1	288.1	141.9	138.9

It can be concluded from the above comparison as shown in Tables 2, 3, and 4, that the derived expressions can predict the overall behaviour of the composite PSSDB panel section with reasonable accuracy.

FULL INTERACTION AS A SPECIAL CASE OF PARTIAL INTERACTION

As mentioned earlier in this paper, full interaction condition is a special case of partial interaction. This condition occurs when slip between the dryboard and profiled steel sheeting under loading is prevented due to very stiff, or very closely spaced connectors. To achieve this condition, the connector modulus, k, and the connector spacing, S, must be chosen in such a way that no slip will occur at the surface in between the dryboard and steel sheeting.

It will be shown that full interaction can be achieved by changing the connector spacing, S, for a fixed value of connector modulus.

The system chosen is the PSSDB system using Bondek profiled steel sheeting 1.0 mm thick, connected to 18 mm thick plywood. The span of the structure is 2.2 m. The connector modulus, k, used in this example is 0.73×10^3 N/mm^2, and the structure is simply supported at its end only (Figure 4). A one meter width of section is considered for the analysis. The composite beam section is subjected to a uniformly distributed downward load of 5 N/mm.

Table 5 shows the deflection at mid-span, mid-width, end slip along the span, and stiffness of the composite panels. It can be seen that, for a fixed connector modulus, k, a reduction of spacing of the connectors will reduce the end slip, and increase the stiffness value. For a spacing of 0.05 mm, simulating a full interaction behaviour, the end slip reduced to zero, and the stiffness value is almost identical to the full interaction case.

TABLE 5. Simulating full-interaction behaviour using author's partial-interaction program for a fixed value of connector modulus, k=730 N/mm^2.

Spacing of Connectors, S (mm)	Vertical deflection, y (mm)	end Slip, s (mm)	Stiffness, EI (kNm^2/m)	Full Interaction Stiffness (kNm^2/m)
300.0	10.97	0.76	139.0	312.0
200.0	10.85	0.74	140.5	312.0
100.0	10.56	0.71	144.4	312.0
50.0	10.04	0.65	151.7	312.0
10.0	7.87	0.38	193.6	312.0
1.0	5.39	0.07	282.5	312.0
0.5	5.15	0.03	296.0	312.0
0.1	4.93	0.01	309.0	312.0
0.05	4.92	0.00	311.5	312.0

CONCLUSION

From the above discussions, it can be concluded that the theoretical approach proposed in this paper is capable of predicting with reasonable accuracy the deflections, slips, and stiffnesses of composite structures. Results obtained by using this simplified elastic approach are found to be in good agreement with results obtained from both experimental, and the folded plate approach. Therefore, the derived equations can be used in the design of any composite floor panel involving two elastic beam elements.

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