ON FEKETE-SZEGÖ PROBLEMS FOR A SUBCLASS OF ANALYTIC FUNCTIONS
(Berkenaan Permasalahan Fekete-Szegö bagi Subkelas Fungsi Analisis)

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ABSTRACT

The aim of this paper is to determine the Fekete-Szegö inequalities for a normalised analytic function \( f(z) \) defined on the open unit disc for which \( z(D_{\lambda_1,\lambda_2}^{\lambda_1,\lambda_2,\delta}f(z))' / (D_{\lambda_1,\lambda_2}^{\lambda_1,\lambda_2,\delta}f(z)), \delta, m, b \in \mathbb{N}_0, \lambda_1, \lambda_2 \geq 0 \) lies in a region starlike with respect to 1 and it is symmetric with respect to the real axis by using the operator \( D_{\lambda_1,\lambda_2}^{\lambda_1,\lambda_2,\delta}f(z) \) given recently by the authors. As a special case of this result, Fekete-Szegö inequality for a class of functions defined by fractional derivatives is also obtained.

Keywords: analytic function; starlike function; subordination; Fekete-Szegö inequality; derivative operator

1. Introduction

Let \( \mathcal{A} \) denote the class of functions of the form

\[
f(z) = z + \sum_{n=2}^{\infty} q_n z^n, \quad (z \in U),
\]

which are analytic in the open unit disc \( U = \{ z : z \in \mathbb{C}, |z| < 1 \} \). Also, let \( \mathcal{S} \) be the subclass of \( \mathcal{A} \) consisting of all functions, which are univalent in \( U \). Let \( \phi \in \mathcal{P} \), where \( \phi(z) \) is an analytic function with positive real part on \( \mathcal{A} \) with \( \phi(0) = 1, \phi'(0) > 0 \), and let \( \mathcal{S}^*(\phi) \) be the class of functions in \( f \in \mathcal{A} \) such that

\[
\frac{zf'(z)}{f(z)} < \phi(z), \quad (z \in U),
\]

and \( \mathcal{C}(\phi) \) be the class of functions in \( f \in \mathcal{A} \) for which
\[ 1 + \frac{zf'''(z)}{f'(z)} < \phi(z), \quad (z \in U). \]  

where \(<\) denotes to the subordination between two analytic functions.

Let \( a_n \) be a complex number and \( 0 \leq \mu \leq 1 \). A classical theorem of Fekete and Szegö (1933) states that for \( f \in S \) and given by (1),

\[ |a_3 - \mu a_2^2| \leq 1 + 2 \exp\left(\frac{-2\mu}{1-\mu}\right). \]

The inequality is sharp.

For a brief history of the Fekete-Szegö problem for the class of starlike functions \( S^* \), convex functions \( C \) and close-to-convex functions \( K \), see the papers by Mohammed and Darus (2010), Srivastava et al. (2001), Darus (2002), Al-Abbadi and Darus (2011), Ravichandran et al. (2004) and Al-Shaqsi and Darus (2008). In particular, for \( f \in K \) and given by (1), Keogh and Merkes (1969) showed that

\[
|a_3 - \mu a_2^2| \leq \begin{cases} 
3 - 4\mu & \text{if } 0 \leq \mu \leq \frac{1}{3}, \\
\frac{1}{3} + \frac{4}{9\mu} & \text{if } \frac{1}{3} \leq \mu \leq \frac{2}{3}, \\
1 & \text{if } \frac{2}{3} \leq \mu \leq 1, \\
4\mu - 3 & \text{if } \mu \geq 1,
\end{cases}
\]

and for each \( \mu \) there is a function in \( K \) for which equality holds.

**Definition 1.1** (El-Yagubi & Darus 2013) Let \( f \) be in the class \( \mathcal{A} \). For \( \delta, m, b \in \mathbb{N}_0 \) and \( \lambda_2 \geq \lambda_1 \geq 0 \), we define the differential operator as follows:

\[
\mathcal{D}^{m, b}_{\lambda_1, \lambda_2, \delta} f(z) = z + \sum_{n=2}^{\infty} \left[ \frac{1 + \delta + (\lambda_1 + \lambda_2)(n-1) + b}{1 + \lambda_2(n-1) + b} \right]^m C(\delta, n) a_n z^n,
\]

where \( C(\delta, n) = \binom{\delta+n-1}{\delta} = (\delta + 1)_{n-1}/(n-1)! \) and \( (\delta)_n \) denotes the Pochhammer symbol defined by

\[
(\delta)_n = \begin{cases} 
1 & \text{if } n = 0, \\
\delta(\delta+1)(\delta+2)...(\delta+n-1) & \text{if } \delta \in \mathbb{C}, n \in \mathbb{N}.
\end{cases}
\]
Using the operator $D_{\lambda_1,\delta_2}^{m,f}(z)$, we define the class $M_{\lambda_1,\delta_2}^{m,f}(\phi)$ as follows:

**Definition 1.2.** Let $\phi \in P$ be a univalent starlike function with respect to 1, which maps the unit disc $U$ onto a region in the right half plane and symmetric with respect to the real axis, $\phi(0) = 1$ and $\phi'(0) > 0$. A function $f \in A$ is in the class $M_{\lambda_1,\delta_2}^{m,f}(\phi)$ if

$$\frac{z(D_{\lambda_1,\delta_2}^{m,f}(z))'}{D_{\lambda_1,\delta_2}^{m,f}(z)} \preceq \phi(z),$$

(5)

where $\delta, m, b \in \mathbb{N}_0, \lambda_2 \geq \lambda_1 \geq 0$ and $D_{\lambda_1,\delta_2}^{m,f}(z)$ denotes the differential operator (4).

The motivation of this paper is to generalise the Fekete-Szegö inequalities proved by Srivastava and Mishra (2000) for functions in the class $M_{\lambda_1,\delta_2}^{m,f}(\phi)$. We also give some applications of our results for certain functions defined by fractional derivatives.

To prove our main results, the following lemma is required.

**Lemma 1.1** (Ma & Minda 1994). If $p_1(z) = 1 + c_1(z) + c_2 z^2 + \ldots$ is an analytic function with positive real part in $U$, then

$$|c_2 - vc_1^2| \leq \begin{cases} -4v + 2 & \text{if } v \leq 0, \\ 2 & \text{if } 0 \leq v \leq 1, \\ 4v + 2 & \text{if } v \geq 1. \end{cases}$$

When $v < 0$ or $v > 1$, the equality holds if and only if $p_1(z)$ is $\frac{1+z}{1-z}$ or one of its rotations. If $0 < v < 1$, then equality holds if and only if $p_1(z)$ is $\frac{1+z^2}{1-z^2}$ or one of its rotations. If $v = 0$, the equality holds if and only if

$$p_1(z) = \left(\frac{1}{2} + \frac{1}{2} \gamma\right)\frac{1+z}{1-z} + \left(\frac{1}{2} - \frac{1}{2} \gamma\right)\frac{1-z}{1+z}, \quad (0 \leq \gamma \leq 1),$$

or one of its rotations. If $v = 1$, the equality holds if and only if $p_1(z)$ is the reciprocal of one of the functions such that the equality holds in the case of $v = 0$. Also the above upper bound is sharp, it can be improved as follows when $0 < v < 1$:

$$|c_2 - vc_1^2| + v|c_1^2| \leq 2, \quad (0 < v \leq \frac{1}{2})$$

and

$$|c_2 - vc_1^2| + (1-v)|c_1^2| \leq 2, \quad (\frac{1}{2} < v \leq 1).$$
2. Main Results

Our first result is contained in the following theorem.

**Theorem 2.1.** Let \( \phi(z) \) be an analytic function with positive real part on \( \mathcal{A} \) and \( \phi(z) = 1 + B_1 z + B_2 z^2 + \ldots \). If \( f(z) \) is given by \( (1) \) and belongs to \( \mathcal{M}_{\lambda_1, \lambda_2}^{m,b} \phi(z) \), then

\[
|a_i - \mu a_i| \leq \left\{ \begin{array}{ll}
\frac{(1 + 2 \lambda_1 + b)^n B_1}{(\delta + 2)(\delta + 1)(1 + 2 \lambda_1 + b)^n} - \frac{(1 + 2 \lambda_2 + b)^n B_2}{(\delta + 1)^2 (1 + \lambda_2 + b)^n} & \text{if } \mu \leq \sigma_1, \\
+ \frac{(1 + 2 \lambda_1 + b)^n B_1}{(\delta + 2)(\delta + 1)(1 + 2 \lambda_1 + b)^n} & \text{if } \sigma_1 \leq \mu \leq \sigma_2, \\
- \frac{(1 + 2 \lambda_2 + b)^n B_2}{(\delta + 2)(\delta + 1)(1 + 2 \lambda_2 + b)^n} + \frac{(1 + 2 \lambda_2 + b)^n B_2}{(\delta + 1)^2 (1 + \lambda_2 + b)^n} & \text{if } \mu \geq \sigma_2,
\end{array} \right.
\]

where

\[ \sigma_1 := \frac{(\delta + 1)^2 (1 + \lambda_1 + \lambda_2 + b)^{2m}}{(\delta + 2)(\delta + 1)(1 + 2 \lambda_1 + b)^m (1 + 2 \lambda_1 + b) B_1^2} \]

and

\[ \sigma_2 := \frac{(\delta + 1)^2 (1 + \lambda_1 + \lambda_2 + b)^{2m}}{(\delta + 2)(\delta + 1)(1 + 2 \lambda_2 + b)^m (1 + 2 \lambda_2 + b) B_2^2}. \]

**Proof.** For \( f \in \mathcal{M}_{\lambda_1, \lambda_2}^{m,b} \phi(z) \), let

\[ p(z) = \frac{z(D_{\lambda_1, \lambda_2, \delta}^{m,b} f(z))'}{D_{\lambda_1, \lambda_2, \delta}^{m,b} f(z)} = 1 + b_1 z + b_2 z^2 + \ldots. \]

Substituting (4) in (9) and comparing the coefficients of \( z^2 \) and \( z^3 \) on both sides in equation (9), we have

\[
\left[ \frac{1 + \lambda_1 + \lambda_2 + b}{1 + \lambda_2 + b} \right]^m (\delta + 1) a_2 = b_1
\]

and

\[
\left[ \frac{1 + 2(\lambda_1 + \lambda_2) + b}{1 + 2 \lambda_2 + b} \right]^m (\delta + 2)(\delta + 1) a_3 = \left[ \frac{1 + \lambda_1 + \lambda_2 + b}{1 + \lambda_2 + b} \right]^{2m} (\delta + 1)^2 a_2^2 + b_2. \]
Now, we want to find out the values for \( b_1 \) and \( b_2 \). Since \( \phi(z) \) is univalent and \( p \prec \phi \), the function

\[
p_1(z) = \frac{1 + \phi^{-1}(p(z))}{1 - \phi^{-1}(p(z))} = 1 + c_1z + c_2z^2 + \ldots
\]

is analytic and has a positive real part in \( U \). Thus, we have

\[
p(z) = \phi \left( \frac{p_1(z) - 1}{p_1(z) + 1} \right).
\]

From the equations (9) and (11), we obtain

\[
1 + b_1z + b_2z^2 + \ldots = \phi \left( \frac{c_1z + c_2z^2 + \ldots}{2 + c_1z + c_2z^2 + \ldots} \right)
\]

\[
= \phi \left[ \frac{1}{2} c_1z + \frac{1}{2}(c_2 - \frac{1}{2}c_1^2)z^2 + \ldots \right]
\]

\[
= 1 + B_1 \frac{1}{2} c_1z + B_1 \frac{1}{2}(c_2 - \frac{1}{2}c_1^2)z^2 + B_2 \frac{1}{4} c_1^2z^2 + \ldots,
\]

and this implies

\[
b_1 = \frac{1}{2} B_1 c_1 \quad \text{and} \quad b_2 = \frac{1}{2} B_1 (c_2 - \frac{1}{2}c_1^2) + \frac{1}{4} B_2 c_1^2.
\]

By substituting the values of \( b_1 \) and \( b_2 \) in equation (10), we have

\[
a_2 = \frac{B_1 c_1 (1 + \lambda_2 + b)^m}{2(1 + \lambda_1 + \lambda_2 + b)^m(\delta + 1)}
\]

and

\[
a_3 = \frac{(\frac{1}{4} B_2^2 c_1^2 + \frac{1}{2} B_1 (c_2 - \frac{1}{2}c_1^2) + \frac{1}{4} B_2 c_1^2)(1 + 2\lambda_2 + b)^m}{(1 + 2(\lambda_1 + \lambda_2) + b)^m(\delta + 2)(\delta + 1)}.
\]

Therefore, we have

\[
a_3 - \mu a_2^2 = \frac{B_1 (1 + 2\lambda_2 + b)^m}{2(1 + 2(\lambda_1 + \lambda_2) + b)^m(\delta + 2)(\delta + 1)} \left( c_2 - \frac{1}{2} (1 - \frac{B_1}{B_2}) + \frac{1}{2} B_1 \right) \left[ (\delta + 2)(\delta + 1)(1 + 2\lambda_2 + b)^m(1 + 2(\lambda_1 + \lambda_2) + b)^m \mu - (1 + \lambda_1 + \lambda_2 + b)^{2m}(\delta + 1)^2 \right].
\]
which implies

\[ a_3 - \mu a_2^2 = \frac{B_1(1 + 2\lambda_2 + b)^m}{2(1 + 2(\lambda_1 + \lambda_2) + b)(\delta + 1)} [c_2 - v c_1^2], \]

where

\[ v = \frac{1}{2} \left( 1 - \frac{B_1}{B_2} \right) \]

\[ + \frac{(\delta + 2)(\delta + 1)(1 + 2\lambda_2 + b)^m(1 + 2(\lambda_1 + \lambda_2) + b)^m \mu - (1 + \lambda_1 + \lambda_2 + b)^{2m}(\delta + 1)^2 B_1}{(1 + \lambda_1 + \lambda_2 + b)^{2m}(\delta + 1)^2 (\delta + 2)}. \]

If \( \mu \leq \sigma_1 \), then by Lemma 1.1 we obtain

\[ |a_3 - \mu a_2^2| \leq \frac{(1 + 2\lambda_2 + b)^m B_2}{(\delta + 2)(\delta + 1)(1 + 2(\lambda_1 + \lambda_2) + b)^m} - \frac{(1 + 2\lambda_2 + b)^m \mu B_2^2}{(\delta + 1)^2 (1 + \lambda_1 + \lambda_2 + b)^{2m}} \]

\[ + \frac{(1 + 2\lambda_2 + b)^m B_1^2}{(\delta + 2)(\delta + 1)(1 + 2(\lambda_1 + \lambda_2) + b)^m}. \]

If \( \mu \geq \sigma_2 \), then we get

\[ |a_3 - \mu a_2^2| \leq -\frac{(1 + 2\lambda_2 + b)^m B_2}{(\delta + 2)(\delta + 1)(1 + 2(\lambda_1 + \lambda_2) + b)^m} + \frac{(1 + 2\lambda_2 + b)^m \mu B_2^2}{(\delta + 1)^2 (1 + \lambda_1 + \lambda_2 + b)^{2m}} \]

\[ - \frac{(1 + 2\lambda_2 + b)^m B_1^2}{(\delta + 2)(\delta + 1)(1 + 2(\lambda_1 + \lambda_2) + b)^m}. \]

Similarly if \( \sigma_1 \leq \mu \leq \sigma_2 \), we get

\[ |a_3 - \mu a_2^2| \leq \frac{(1 + 2\lambda_2 + b)^m B_1}{(\delta + 2)(\delta + 1)(1 + 2(\lambda_1 + \lambda_2) + b)^m}. \]
3. Application of Fractional Derivatives

For fixed \( g \in A \), let \( \mathcal{M}_{\lambda_1,\lambda_2,\delta}^{m,b}(\phi) \) be the class of functions \( f \in A \) for which \((f * g) \in \mathcal{M}_{\lambda_1,\lambda_2,\delta}^{m,b}(\phi)\). Note that, for any two analytic functions \( f(z) = z + \sum_{n=2}^{\infty} a_n z^n \) and \( g(z) = z + \sum_{n=2}^{\infty} b_n z^n \), their convolution is defined by
\[
(f * g)(z) = f(z) * g(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n.
\]

**Definition 3.1** (Owa & Srivastava 1987). Let \( f \) be analytic in a simply connected region of the \( z \)-plane containing the origin. The functional derivative of \( f \) of order \( \gamma \) is defined by
\[
D_z^\gamma f(z) = \frac{1}{\Gamma(1-\gamma)} \frac{d}{dz} \int_0^z \frac{f(\zeta)}{(z-\zeta)^\gamma} d\zeta, \quad (0 \leq \gamma < 1),
\]
where the multiplicity of \((z-\zeta)^\gamma\) is removed by requiring that \( \log(z-\zeta) \) is real for \( z-\zeta > 0 \).

Using Definition 3.1 and the well known extension involving fractional derivatives and fractional integrals, Owa and Srivastava (1987) introduced the operator \( \Omega^\gamma : A \to A \), which is defined by
\[
\Omega^\gamma f(z) = \Gamma(2-\gamma)z^\gamma D_z^\gamma f(z), \quad (\gamma \neq 2,3,4,\ldots).
\]

The class \( \mathcal{M}_{\lambda_1,\lambda_2,\delta}^{m,b,\gamma}(\phi) \) consists of functions \( f \in A \) for which \( \Omega^\gamma f \in \mathcal{M}_{\lambda_1,\lambda_2,\delta}^{m,b}(\phi) \). Note that \( \mathcal{M}_{\lambda_1,\lambda_2,\delta}^{m,b,\gamma}(\phi) \) is the special case of the class \( \mathcal{M}_{\lambda_1,\lambda_2,\delta}^{m,b}(\phi) \) when
\[
g(z) = z + \sum_{n=2}^{\infty} \frac{\Gamma(n+1)\Gamma(2-\gamma)}{\Gamma(n+1-\gamma)} z^n.
\]

Let
\[
g(z) = z + \sum_{n=2}^{\infty} g_n z^n, \quad (g_n > 0).
\]

Since \( D_{\lambda_1,\lambda_2,\delta}^m f(z) \in \mathcal{M}_{\lambda_1,\lambda_2,\delta}^{m,b,\gamma}(\phi) \) if and only if \( D_{\lambda_1,\lambda_2,\delta}^m f(z) * g(z) \in \mathcal{M}_{\lambda_1,\lambda_2,\delta}^{m,b,\gamma}(\phi) \), we obtain the coefficient estimate for functions in the class \( \mathcal{M}_{\lambda_1,\lambda_2,\delta}^{m,b,\gamma}(\phi) \), from the corresponding estimate for functions in the class \( \mathcal{M}_{\lambda_1,\lambda_2,\delta}^{m,b}(\phi) \). Applying Theorem 2.1 for the function
we get the following result after an obvious change of the parameter \( \mu \).

**Theorem 3.1.** Let \( g(z) = z + \sum_{n=0}^{\infty} g_n z^n \), \((g_n > 0)\) and let the function \( \phi(z) \) be given by \( \phi(z) = 1 + \sum_{n=1}^{\infty} B_n z^n \). If \( D_{\alpha_1,\alpha_2,\delta}^{m,b} f(z) \) given by (4) belongs to \( M_{\alpha_1,\alpha_2,\delta}^{m,b} (\phi) \), then

\[
|a_j - \mu a_j^2| \leq \begin{cases} 
\frac{(1+2\lambda_1+b)^m B_j}{(\delta+2)(\delta+1)(1+2(\lambda_1+\lambda_2)+b)^m g_3} - \frac{(1+2\lambda_1+b)^m \mu B_j^2}{(\delta+1)^2(1+\lambda_1+\lambda_2+b)^m g_2} & \text{if } \mu \leq \sigma_1, \\
\frac{(1+2\lambda_1+b)^m B_j}{(\delta+2)(\delta+1)(1+2(\lambda_1+\lambda_2)+b)^m g_3} - \frac{(1+2\lambda_1+b)^m \mu B_j^2}{(\delta+1)^2(1+\lambda_1+\lambda_2+b)^m g_2} & \text{if } \sigma_1 \leq \mu \leq \sigma_2, \\
\frac{(1+2\lambda_1+b)^m B_j}{(\delta+2)(\delta+1)(1+2(\lambda_1+\lambda_2)+b)^m g_3} + \frac{(1+2\lambda_1+b)^m \mu B_j^2}{(\delta+1)^2(1+\lambda_1+\lambda_2+b)^m g_2} & \text{if } \mu \geq \sigma_2,
\end{cases}
\]

where

\[
\sigma_1 := \frac{g_3^2(\delta+1)^2(1+\lambda_1+\lambda_2+b)^2 m (B_1 - B_2) + B_j^2}{g_3(\delta+2)(\delta+1)(1+2\lambda_1+b)^m (1+2(\lambda_1+\lambda_2+b)^m B_j^2},
\]

\[
\sigma_2 := \frac{g_3^2(\delta+1)^2(1+\lambda_1+\lambda_2+b)^2 m (B_1 + B_2) + B_j^2}{g_3(\delta+2)(\delta+1)(1+2\lambda_1+b)^m (1+2(\lambda_1+\lambda_2+b)^m B_j^2}.
\]

Since

\[
(\Omega^{C} D_{\alpha_1,\alpha_2,\delta}^{m,b}) f(z) = z + \sum_{n=2}^{\infty} \frac{\Gamma(n+1) \Gamma(2-\gamma)}{\Gamma(n+1-\gamma)} \left[ \frac{1+(\lambda_1+\lambda_2)(n-1)+b}{1+\lambda_2(n-1)+b} \right]^n C(\delta,n)a_n z^n,
\]

we have

\[
g_2 := \frac{\Gamma(3) \Gamma(2-\gamma)}{\Gamma(3-\gamma)} = \frac{2}{(2-\gamma)}
\]

and
\[ g_3 := \frac{\Gamma(4)\Gamma(2-\gamma)}{\Gamma(4-\gamma)} = \frac{6}{(2-\gamma)(3-\gamma)}. \]

**Proof.** By using the same technique as in the proof of Theorem 2.1, the required result is obtained.

For \( g_2 \) and \( g_3 \) given by above equalities, Theorem 3.1 reduces to the following result.

**Corollary 3.1.** Let \( g(z) = z + \sum_{n=2}^{\infty} g_n z^n, (g_n > 0) \) and let the function \( \phi(z) \) be given by

\[ \phi(z) = 1 + \sum_{n=1}^{\infty} B_n z^n. \]

If \( D_{\lambda_1,\lambda_2,\delta}^{m,b} f(z) \) given by (3) belongs to \( \mathcal{M}_{b_1,b_2}^{m,b} (\phi) \). Then,

\[
\left| a_n - \mu a_2 \right| \leq \begin{cases} 
\frac{(2-\gamma)(3-\gamma)(1+2\lambda_2 + b)^m B_2}{6(\delta + 2)(\delta + 1)(1+2(\lambda_1 + \lambda_2) + b)^m} - \frac{(2-\gamma)^2(1+2\lambda_2 + b)^m \mu B_2^2}{4(\delta + 1)^2(1+\lambda_1 + \lambda_2 + b)^{2m}} & \text{if } \mu \leq \sigma_1, \\
\frac{(2-\gamma)(3-\gamma)(1+2\lambda_2 + b)^m B_2}{6(\delta + 2)(\delta + 1)(1+2(\lambda_1 + \lambda_2) + b)^m} & \text{if } \sigma_1 \leq \mu \leq \sigma_2, \\
\frac{(2-\gamma)(3-\gamma)(1+2\lambda_2 + b)^m B_2}{6(\delta + 2)(\delta + 1)(1+2(\lambda_1 + \lambda_2) + b)^m} + \frac{(2-\gamma)^2(1+2\lambda_2 + b)^m \mu B_2^2}{4(\delta + 1)^2(1+\lambda_1 + \lambda_2 + b)^{2m}} & \text{if } \mu \geq \sigma_2,
\end{cases}
\]

where

\[ \sigma_1 := \frac{2(3-\gamma)(\delta + 1)^2(1+\lambda_1 + \lambda_2 + b)^{2m}(B_2 - B_1) + B_1^2}{3(2-\gamma)(\delta + 2)(\delta + 1)(1+2\lambda_2 + b)^m(1+2(\lambda_1 + \lambda_2) + b)^{2m} B_1^2}, \]

and

\[ \sigma_2 := \frac{2(3-\gamma)(\delta + 1)^2(1+\lambda_1 + \lambda_2 + b)^{2m}(B_2 + B_1) + B_1^2}{3(2-\gamma)(\delta + 2)(\delta + 1)(1+2\lambda_2 + b)^m(1+2(\lambda_1 + \lambda_2) + b)^{2m} B_1^2}. \]

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References


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