

ON THE SECOND HANKEL DETERMINANT OF SOME ANALYTIC FUNCTIONS

(Mengenai Penentu Hankel Kedua bagi Beberapa Fungsi Analisis)

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ABSTRACT

Let the function f be analytic in $z \in D = \{z : |z| < 1\}$ and be given by $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$. For $0 \leq \alpha \leq 1$, denote by $V(\alpha)$ and $U(\alpha)$, the sets of functions analytic in D , satisfying $\operatorname{Re} \left((1-\alpha) f'(z) + \alpha \left(1 + \frac{zf''(z)}{f'(z)} \right) \right) > 0$ and $\operatorname{Re} \left((1-\alpha) \frac{f(z)}{z} + \alpha \frac{zf'(z)}{f(z)} \right) > 0$ respectively, so that $f \in V(\alpha) \Leftrightarrow zf' \in U(\alpha)$. We give sharp bounds for the Hankel determinant $H_2 = |a_2 a_4 - a_3^2|$ for $f \in V(\alpha)$ and $f \in U(\alpha)$.

Keywords: univalent functions; starlike and convex functions; Hankel determinant

ABSTRAK

Andaikan fungsi f analisis dalam $z \in D = \{z : |z| < 1\}$ dan diberi oleh $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$. Untuk $0 \leq \alpha \leq 1$, set fungsi analisis dalam D yang dilambangi oleh $V(\alpha)$ dan $U(\alpha)$, masing-masing memenuhi $\operatorname{Re} \left((1-\alpha) f'(z) + \alpha \left(1 + \frac{zf''(z)}{f'(z)} \right) \right) > 0$ dan $\operatorname{Re} \left((1-\alpha) \frac{f(z)}{z} + \alpha \frac{zf'(z)}{f(z)} \right) > 0$ sehinggakan $f \in V(\alpha) \Leftrightarrow zf' \in U(\alpha)$. Batas terbaik bagi penentu Hankel $H_2 = |a_2 a_4 - a_3^2|$ untuk $f \in V(\alpha)$ dan $f \in U(\alpha)$ diberi.

Kata kunci: fungsi univalen; fungsi bak bintang dan cembung; penentu Hankel

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