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NUMERICAL CONFORMAL MAPPING OF DOUBLY CONNECTED REGIONS ONTO A DISC WITH A CIRCULAR SLIT

(Pemetaan Konformal Berangka bagi Rantau Berkait Ganda Dua
ke Seluruh Cakera dengan Belahan Membulat)

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ABSTRACT

An integral equation method based on the Neumann kernel for conformal mapping $f(z)$ of doubly connected regions onto a unit disc with a circular slit of radius $\mu < 1$ is presented. The theoretical development is based on the boundary integral equation for conformal mapping of doubly connected region in an earlier work of the authors. In this paper, a related system of integral equations is constructed that is satisfied by $f'(z)$ and μ. For numerical experiment, the integral equation is discretised which leads to a system of nonlinear equations. The system obtained is solved simultaneously using Gauss-Newton method. Numerical implementation on a circular annulus is also presented.

Keywords: conformal mapping; integral equation; doubly connected region; Neumann kernel; Gauss-Newton method

ABSTRAK

Satu kaedah persamaan kamiran berdasarkan inti Neumann untuk pemetaan konformal $f(z)$ bagi rantau berkait ganda dua ke seluruh unit cakera dengan belahan membulat berjejari $\mu < 1$ dipersembahkan. Pembangunan teori adalah berdasarkan persamaan kamiran sempadan bagi pemetaan konformal rantau berkait ganda dua dalam penyelidik pengarang sebelum ini. Dalam kertas kerja ini, suatu perhubungan sistem persamaan kamiran telah dibina yang memenuhi $f'(z)$ dan μ. Untuk kajian berangka, persamaan kamiran berkenaan telah didiskretkan menghasilkan sistem persamaan tak linear. Sistem yang diperoleh diselesaikan secara serentak dengan menggunakan kaedah Gauss-Newton. Pelaksanaan berangka terhadap anulus juga dipersembahkan.

Kata kunci: pemetaan konformal; persamaan kamiran; rantau berkait ganda dua; inti Neumann; Kaedah Gauss-Newton

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