Point Forecast Markov Switching Model for U.S. Dollar/ Euro Exchange Rate
(Ramalan Titik Menggunakan Model Peralihan Markov untuk Kadar Pertukaran Wang Dolar US/Euro)

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ABSTRACT
This research proposes a point forecasting method into Markov switching autoregressive model. In case of two regimes, we proved the probability that h periods later process will be in regime 1 or 2 is given by steady-state probabilities. Then, using the value of h-step-ahead forecast data at time \( t \) in each regime and using steady-state probabilities, we present an h-step-ahead point forecast of data. An empirical application of this forecasting technique for U.S. Dollar/ Euro exchange rate showed that Markov switching autoregressive model achieved superior forecasts relative to the random walk with drift. The results of out-of-sample forecast indicate that the fluctuations of U.S. Dollar/ Euro exchange rate from May 2011 to May 2013 will be rising.

Keywords: Exchange rate; Markov switching; point forecast

INTRODUCTION
Engle and Hamilton (1990) found that Markov switching model of exchange rate generates better forecasts than random walk. Yuan (2011) proposed an exchange rate forecasting model which combines the multi-state Markov-switching model with smoothing techniques. In this paper, we present a point forecasting method into Markov switching autoregressive model. Usually, two or three regimes were defined in this model. In case of two regimes, regime 1 describes the periods of downtrend of exchange rates and regime 2 denotes the periods of uptrend of exchange rates. In case of two regimes, we showed the probability that \( h \) periods later process will be in regime 1 or 2 is given by steady-state probabilities. Then, using the value of h-step-ahead forecast data at time \( t \) in each regime and using steady-state probabilities, we generate an h-step-ahead point forecast of data.

Markov Switching models by a change in their regimes themselves will up to date, when jumps arise in time series data. Therefore, these models will offer a better statistical fit to the data with jumps than the linear models.

DATA
In this study, we employed the U.S. Dollars to One Euro, which are collected monthly from January 2003 to April 2011. The data were obtained from the Board of Governors of the Federal Reserve System (http://research.stlouisfed.org). The variable under investigation is exchange rate returns in percentage:

\[ \text{Exchange rate returns} = \frac{\text{Current price} - \text{Previous price}}{\text{Previous price}} \times 100 \]
\begin{equation}
y_t = 100 \times [\ln(r_t) - \ln(r_{t-1})].
\end{equation}

where \( r_t \) represent the monthly exchange rates.

THE MARKOV SWITCHING METHODOLOGY

The Markov switching model was introduced by Hamilton (1989). A Markov switching autoregressive model (MS-AR) of two states with an AR process of order \( p \) is written as:

\begin{equation}
y_t = \begin{cases} 
c_1 + \alpha_1 y_{t-1} + \ldots + \alpha_p y_{t-p} + \varepsilon_t, & S_t = 1 
c_2 + \alpha_1 y_{t-1} + \ldots + \alpha_p y_{t-p} + \varepsilon_t, & S_t = 2,
\end{cases}
\end{equation}

where regimes in model (2) are index by \( s_t \). In this model, the parameters of the autoregressive part and intercept are depended on the regime at time \( t \). The regimes are discrete unobservable variable. Regime 1 describes the periods of downtrend of exchange rates and regime 2 denotes the periods of uptrend of exchange rates. The transition between the regimes is governed by a first order Markov process as follows:

\( p_e = \Pr(s = j | s_{i-1} = i) \) \quad \forall i, j = 1, 2, \sum_i p_e = 1.

It is normal to collect the transition probabilities in a matrix \( P \) known as the transition matrix:

\[
P = \begin{pmatrix} p_{11} & p_{12} 
p_{21} & p_{22} \end{pmatrix}.
\]

Note that \( p_{11} + p_{12} = 1 \) and \( p_{21} + p_{22} = 1 \).

We estimated the parameters of MS-AR model by MLE. The log likelihood function is given by:

\[
\ln L = \sum_{i=1}^{T} \ln \left( \sum_{\psi_{i-1}} f(y_i | \psi_{i-1}) P(\psi_i | \psi_{i-1}) \right). 
\]

Where \( \psi_{i-1} = \{y_{i-1}, \ldots, y_{1} \} \). In Eq. (3), \( P(s_i | \psi_{i-1}) \) are filtered probabilities. Using \( \gamma_t \) as observed at the end of the t-th iteration, we calculated filtered probabilities as:

\[
P(s_i = j | \psi_{i-1}) = \sum_{i=1}^{T} P(s_t = j | s_{t-1} = i | \psi_t).
\]

The next step, using all the information in the sample i.e. \( \psi_T = \{y_T, \ldots, y_1\} \), we calculated smoothed probabilities:

\[
P(s_i = j | \psi_T) = \sum_{i=1}^{T} P(s_t = j | s_{t-1} = k | \psi_T).
\]

In addition, \( P(s_i | \psi_T) \) at the last iteration of filter is calculated.

FURTHER DISCUSSION OF MARKOV SWITCHING MODELS

The Markov switching autoregressive models applied a great variety of specifications. These models can be applied where the autoregressive parameters, the mean or the intercepts, are regime-dependent (see Krolzing 1998 for further details). The Markov switching-mean according to the notation introduced by Krolzing (1998):

\[
y_t = \mu + \alpha_1 y_{t-1} + \ldots + \alpha_p y_{t-p} + \varepsilon_t.
\]

In this model, only the mean is depended on regime. Andel (1993) showed that Markov switching-mean and ARMA the processes have similar properties than a long memory.
process. Kuswanto and Sibbertsen (2008) discussed and showed that model (4) is a candidate for a Markov switching process which is able to create a spurious long memory. Charfeddine and Guegan (2009) applied models that have changes in mean like the Markov switching model and the structural change model. They showed that when the data are weakly dependent with changes in mean, the hypothesis of long memory is accepted with a high power. Therefore, often Markov switching-mean model needs a survey for apparent of long memory.

Ismail and Isa (2006) used structural change test to detect nonlinear feature in three ASEAN countries exchange rates. They find that the null hypothesis of linearity is rejected and there is evidence of structural breaks in the exchange rates series. Therefore, they apply regime-switching model in their study.

**PARAMETER ESTIMATION**

We followed Psaradakis and Spagnolo (2003) for selecting the number of regimes, who propose to use the value of the Akaike Information Criterion (AIC). Then, we compared the different types of Markov switching autoregressive models. Our comparison strategy follows Cologni and Manera (2009), who compared Markov switching models using value of the log-likelihood function, values of means or intercepts estimated in any regime and estimated matrix of transition probabilities. Using these selection strategies, the best performance was obtained for model (2) with two regimes and one-lag autoregressive component. The details of the model fitted for MS-AR is presented in Table 1. All estimated coefficients were statistically significant at conventional significance levels. The transition probabilities suggest that regime 1 is highly persistent. When the process was in regime 1, there was a low probability that it switches to regime 2 \( p(s_t = 2|x_{t-1}=1) = 0.06 \). The average duration of the two regimes were 17.81 and 7.50 months, respectively (Table 1).

Figure 3 shows time series of smoothed probabilities for fluctuations of the U.S. Dollar/ Euro exchange rate by MSAR model. This figure shows the probability of being in regime 1 or 2 at a specific time. In December 2008 and July 2010, the fluctuation for U.S. Dollar/ Euro exchange rate is ascendant (Figure 2), which causes the process in regime 2 with a high probability (Figure 3). In to be other years since, the fluctuations for exchange rate is low (Figure 2), therefore the process is in regime 1 with a high probability (Figure 3).

**TABLE 1. Estimated of MS-AR model with details**

<table>
<thead>
<tr>
<th>Regime</th>
<th>Coefficient</th>
<th>Stand. Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>a₀</td>
<td>0.5418</td>
<td>0.0303 (0.00)***</td>
</tr>
<tr>
<td>a₁</td>
<td>0.2333</td>
<td>0.0225 (0.00)***</td>
</tr>
<tr>
<td>σ</td>
<td>1.9312</td>
<td></td>
</tr>
<tr>
<td>a₀</td>
<td>-0.4067</td>
<td>0.0307 (0.00)***</td>
</tr>
<tr>
<td>a₁</td>
<td>0.2699</td>
<td>0.0385 (0.00)***</td>
</tr>
<tr>
<td>σ</td>
<td>3.3842</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Regime</th>
<th>Expected Duration</th>
<th>Steady-state Probability</th>
<th>Log. Likelihood</th>
<th>AIC</th>
<th>BIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime 1</td>
<td>17.81</td>
<td>0.6842</td>
<td>-224.3386</td>
<td>468.6773</td>
<td>494.5270</td>
</tr>
<tr>
<td>Regime 2</td>
<td>7.50</td>
<td>0.3158</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

P-values are reported in the parenthesis.***, **, * denotes significance of the coefficient at the 0.1%, 1%, 5% level.
FIGURE 2. Percent changes series in the log of the U.S. Dollars to One Euro (Jan 2003-Apr 2011)

FIGURE 3. Smoothed probabilities of (a) Regime 1 and (b) Regime 2
FORECAST

Yuan (2011) predicted fluctuations of the dollar by Markov switching model of K regimes, which implied according to the following formula:

\[ \hat{y}_{ts} = E(y_{ts} | \psi_t) = \hat{\pi}_4 \cdot P^t \hat{\mu}, \]

where

\[ y_t = \mu + \sigma \varepsilon_t, \text{ and } \hat{\pi}_4 = \left[ pr(s_t = 1 | \psi_t), \ldots, pr(s_t = k | \psi_t) \right]. \]

In the following lemma, we prove \( \hat{\pi}_4 \cdot P^t \) is equal to \( \hat{\pi}_4 \) when Markov chain has two regimes.

**Lemma:** For a 2-regime Markov chain, we have:

\[ \hat{\pi}_4 \cdot P^t = \hat{\pi}_4. \]

**Proof:** For a two-state Markov chain, the transition matrix is:

\[ P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}. \]

Then, the matrix of m-period-ahead transition probabilities for an ergodic two-state Markov chain is given by:

\[ P^m = \begin{bmatrix} (1 - p_{12}) + \lambda_{1}^m (1 - p_{11}) + \lambda_{2}^m (1 - p_{21}) \\ 2 - p_{11} - p_{22} \\ (1 - p_{22}) - \lambda_{2}^m (1 - p_{21}) + \lambda_{1}^m (1 - p_{11}) \\ 2 - p_{11} - p_{22} \end{bmatrix}. \]

Thus, the steady-state probabilities is given by

\[ \pi = \frac{1 + p_{22}}{2 - p_{11} - p_{22}} \begin{bmatrix} \pi_1 \\ \pi_2 \end{bmatrix}. \]

The first element of (6) becomes

\[ \pi_1 p_{11}^t + \pi_2 p_{22}^t = \frac{1 - p_{22}}{2 - p_{11} - p_{22}} \times \frac{(1 - p_{11}) + \lambda_{1}^t (1 - p_{22})}{2 - p_{11} - p_{22}} + \frac{1 - p_{22}}{2 - p_{11} - p_{22}} \times \frac{(1 - p_{22}) - \lambda_{2}^t (1 - p_{11})}{2 - p_{11} - p_{22}} \]

By similar reasoning, the second element of (6) becomes

\[ \pi_2 p_{12}^t + \pi_1 p_{21}^t = \pi_2. \]

Next step, we show the true of above lemma by using our empirical finding. Using details of Table 1, the matrix of steady-state probabilities is estimated as

\[ \hat{\pi} = \begin{bmatrix} 0.6842 \\ 0.3158 \end{bmatrix}. \]

Using (5), the matrix of 2-period-ahead transition probabilities is estimated as

\[ \hat{\pi}^2 = \begin{bmatrix} (1 - 0.94) - 0.81^t (1 - 0.94) \\ 2 - 0.94 - 0.87 \end{bmatrix} \]

Hence, for two periods ahead

\[ \hat{\pi}^2 = \begin{bmatrix} 0.8914 & 0.1086 \\ 0.2353 & 0.7647 \end{bmatrix}. \]

A similar result holds for three periods ahead:

\[ \hat{\pi}^3 = \begin{bmatrix} (1 - 0.94) - 0.81^t (1 - 0.94) \\ 2 - 0.94 - 0.87 \end{bmatrix} \]

Thus with this details, we have

\[ \pi \cdot P^t = \begin{bmatrix} \pi_1, \pi_2 \\ \pi_1, \pi_2 \end{bmatrix} \begin{bmatrix} p_{11}^t & p_{12}^t \\ p_{21}^t & p_{22}^t \end{bmatrix} = \begin{bmatrix} \pi_1 p_{11}^t + \pi_2 p_{22}^t, \pi_1 p_{12}^t + \pi_2 p_{21}^t \end{bmatrix}. \]
Finally, for $n$ periods ahead

$$\hat{\pi}^n \hat{\pi} = \begin{bmatrix} 0.6842 & 0.3158 \\ 0.6842 & 0.3158 \end{bmatrix}$$

$$= \begin{bmatrix} 0.6842 & 0.3158 \end{bmatrix}.$$
This paper outlines techniques for point forecasting into Markov switching autoregressive model. In case of two regimes, using the value of $h$-step-ahead forecast data at time $t$ in each regime and using steady-state probabilities, we present an $h$-step-ahead point forecast of data.

Our applications focused on fluctuations of U.S. Dollar/ Euro exchange rate. The fluctuations of U.S. Dollar/ Euro exchange rate have jumps in their behavior. Markov Switching models by a change in their regimes themselves will up to date, when jumps arise in time series data. Hence, this model can be useful for modeling and forecasting this data, which is also confirmed by this study. Our finding demonstrated that MS-AR achieved superior forecasts relative to the random walk with drift. The results of out-of-sample forecast indicated that the fluctuations of U.S. Dollar/ Euro exchange rate from May 2011 to May 2013 will be rising.

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