Sains Malaysiana 43(3)(2014): 483–489

Axisymmetric Slip Flow on a Vertical Cylinder with Heat Transfer

(Aliran Gelincir Simetri Sepaksi pada Silider Menegak dengan Pemindahan Haba)

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ABSTRACT

The paper is aimed at studying fluid flow heat transfer in the axisymmetric boundary layer flow of a viscous incompressible fluid, along the axial direction of a vertical stationary isothermal cylinder in presence of uniform free stream with momentum slip. The equations governing the flow i.e. continuity, momentum and energy equation are transformed into non-similar boundary layer equations and are solved numerically employing asymptotic series method with Shanks transformation. The numerical scheme involves the Runge-Kutta fourth order scheme along with the shooting technique. The flow is analyzed for both assisting and opposing buoyancy and the effect of different parameters on fluid velocity, temperature distribution, heat transfer and shear stress parameters is presented graphically.

Keywords: Cylindrical surface; heat transfer; non-similarity solution; Runge-Kutta method; shear stress; slip flow

ABSTRAK

Makalah ini bertujuan untuk mengkaji pemindahan haba aliran bendalir dalam aliran lapisan sempadan simetri sepaksi bagi suatu bendalir likat tak termampat, sepanjang arah paksi sebuah silinder isoterma pegun tegak yang berdiri dalam kehadiran strim bebas seragam dengan gelincir momentum. Persamaan yang mengawal aliran contohnya persamaan keselanjaran, persamaan momentum dan persamaan tenaga dijelmakan kepada persamaan lapisan sempadan tak serupa dan diselesaikan secara berangka menggunakan kaedah siri berasimptot dengan penjelmaan Shanks. Skim berangka tersebut melibatkan skim tertib keempat Runge-Kutta dan juga teknik penembakan. Aliran dianalisis daripada dua aspek dalam membantu dan melawan keapungan dan kesan ke atas halaju bendalir akibat parameter yang berbeza: Taburan suhu, pemindahan haba dan parameter tegasan ricih dipersembahkan secara grafik.

Kata kunci: Aliran gelincir; kaedah Runge-Kutta; pemindahan haba; permukaan silinder; persamaan tak serupa; tegasan ricih

INTRODUCTION

The laminar boundary layer flow and heat transfer along a cylinder is studied for its applications in various fields of engineering such as manufacturing fibres, extrusion process involved in glass and polymer industry (Redmond & McDonnell 2004). In addition, in higher atmospheric stratum the air is rarefied and the mean free path can range to the order of satellites and so the heat transfer study in slip regime is important for thermal insulation and safety of satellites (Crane & McVeigh 2010a). In case of nano and micro scale technologies (e.g. MEMS) such studies are important since the mean free path is comparable to characteristic length and so the no slip regime at fluid surface interaction is replaced by slip flow regime (Gad-el-Hak 1999) and the Knudsen Number lies in the range between 0.01 and 0.1 (Martin & Boyd 2006). Earlier studies in laminar boundary layer flow along a cylinder includes the work of Seban and Bond (1951) who presented the first three terms in the series solution which is valid near the leading edge. Kelly (1962) improved upon the solution presented by Seban and Bond (1951). Glauert and Lighthill (1955) considered flow along the entire cylinder. Lin and Shih (1980) studied the heat transfer along a static cylinder. Pop et al. (1995) observed boundary flow and heat transfer along a uniformly moving cylinder with suction/injection. Aziz and Na (1982) presented a method of perturbation solution using Shanks transformation to analyze natural convection along vertical cylinder. Redmond and Spillane (2003) and Spillane and Redmond (2004) studied shear stress along the surface of cylinder using series solution and Padé approximation. Redmond and McDonnell (2004) observed heat transfer in boundary layer flow along cylindrical fibre. Crane and McVeigh (2010b) studied uniform slip flow along a static cylinder for small and long axial distance using series solution and Rayleigh approximation method.

The studies along cylindrical surface are few in literature and mixed convection in slip regime along axial direction of vertical cylinder has not been studied. Thus, the aim of the present paper was to investigate the effect of slip flow; on the fluid velocity and temperature distribution, heat transfer and shear stress parameter, at the leading edge of a vertical infinite along isothermal cylinder. The present study is valid when Knudsen $number\left\{=\frac{mean\ free\ of\ fluid}{radius\ of\ cylinder}\right\}$ lies in the range 0.01 to 0.1 and at the leading edge (Crane

& McVeigh 2010a) i.e. where the boundary layer thickness is comparable to radius of cylinder. It is important to point that Kundsen number is a deciding factor whether the no slip or slip regime would occur at fluid-surface interaction. When Kundsen number is very small, no slip is observed between the surface and the fluid and is in tune with the essence of continuum mechanics. However, when Kundsen number lies in the range 10^{-3} to 0.1, slip occurs at the surface-fluid interaction and is generally studied under the light of first-order momentum slip boundary conditions.

FORMULATION OF THE PROBLEM

Consider a 2D axisymmetric viscous incompressible flow along a stationary infinite vertical cylinder of radius r_0. The cylindrical coordinates (x, r) are such that the axis is parallel to the uniform free stream flow, U. The origin of coordinates system is at the centre of leading edge of the cylinder. The cylinder surface is maintained at constant temperature T_w while free stream temperature is T_∞. The physical diagram is shown in Figure 1.

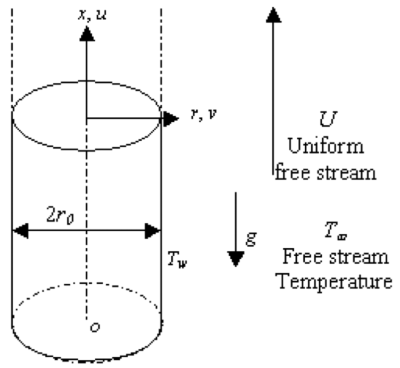


FIGURE 1. Physical model

The governing equations of fluid flow and heat transfer, following Aziz and Na (1982), Crane and McVeigh (2010a) and Pop et al. (1995) are:

$$\frac{\partial}{\partial x}(ru)+\frac{\partial}{\partial x}(rv)=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=\frac{v}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)\pm g\beta\left(T-T_\infty\right), \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right), \tag{3}$$

where u and v are the axial and radial velocity components, β is coefficient of thermal expansion, T is the fluid temperature, ν is the kinematic viscosity and α is thermal diffusivity of the fluid. The buoyancy force term is (+) for assisting flow when $T_w > T_\infty$ and (-) for opposing flow when $T_\infty > T_w$. The boundary conditions of the problem with slip (Crane & McVeigh 2010a, 2010b), are:

$$\begin{aligned} &u=L\left(\frac{\partial u}{\partial r}\right), \quad v=0, \quad T=T_w \quad \text{at} \quad r=r_0,\\ &u\to U, \qquad T\to T_\infty \qquad \text{as} \quad r\to\infty, \end{aligned} \tag{4}$$

where L (meter) is proportionality constant of velocity slip. Equation (1) is satisfied by introducing a stream function ψ, such that $u=\frac{1}{r}\frac{\partial\psi}{\partial r}$ and $v=\frac{1}{r}\frac{\partial\psi}{\partial x}$. Equations (2) and (3) can be transformed into corresponding differential equation using transformation (Crane & McVeigh 2010a; Pop et al. 1995)

$$\begin{aligned} &\xi=\frac{4}{r_0}\sqrt{\frac{vx}{U}}, \qquad \eta=\frac{r^2-r_0^2}{\xi r_0^2},\\ &\psi=\left(Uvx\right)^{1/2} r_0 f\left(\xi,\eta\right), \theta\left(\xi,\eta\right)=\frac{T-T_\infty}{T_w-T_\infty}. \end{aligned} \tag{5}$$

The transformed equations are:

$$\left(1+\xi\eta\right)\frac{\partial^3 f}{\partial\eta^3}+\left(\xi+\eta\right)\frac{\partial^2 f}{\partial\eta}\pm Gr\theta=\xi\left(\frac{\partial f}{\partial\eta}\frac{\partial^2 f}{\partial\xi\partial\eta}-\frac{\partial f}{\partial\xi}\frac{\partial^2 f}{\partial\eta^2}\right), \tag{6}$$

$$\frac{\left(1+\xi\eta\right)}{Pr}\frac{\partial^2\theta}{\partial\eta^2}+\left(\xi+\eta\right)\frac{\partial\theta}{\partial\eta}=\xi\left(\frac{\partial\theta}{\partial\xi}\frac{\partial f}{\partial\eta}-\frac{\partial f}{\partial\xi}\frac{\partial\theta}{\partial\eta}\right), \tag{7}$$

and the boundary conditions for $\xi \geq 0$ are:

$$\begin{aligned} &\lambda f''\left(\xi,0\right)=\xi f'\left(\xi,0\right), f\left(\xi,0\right)+\xi\frac{\partial f\left(\xi,0\right)}{\partial\xi}=0,\\ &\theta\left(\xi,0\right)=1, \quad f'\left(\xi,\infty\right)=2, \quad \theta\left(\xi,\infty\right)=0, \end{aligned} \tag{8}$$

where $Gr\left\{=8\,\frac{g\beta\left(T_w-T_\infty\right)x^3}{v^2}\frac{1}{\left(Ux/v\right)^2}\right\}$ is the buoyancy parameter, $Pr\left\{=\frac{v}{\alpha}\right\}$ is Prandtl number and $\lambda\left\{=\frac{2L}{r_0}\right\}$ is the momentum slip parameter.

METHOD OF SOLUTION

Following Seban and Bond (1951), for ξ in the neighborhood of origin, let

$$f(\xi,\eta)=f_0(\eta)+\xi f_1(\eta)+\xi^2 f_2(\eta)+\ldots, \tag{9}$$

and

$$\theta(\xi,\eta)=\theta_0(\eta)+\xi\theta_1(\eta)+\xi^2\theta_2(\eta)+\ldots, \tag{10}$$

be taken in expansion form.

Therefore (6), (7) and (8), for expansion up to ξ^2, would take the form:

$$\xi^0: \begin{aligned} &f_0'''+f_0 f_0''+Gr\theta_0=0,\\ &\theta_0''+Prf_0\theta_0'=0,\\ &f_0(0)=0, f_0''(0)=0, \ f_0'(\infty)=2, \theta_0(0)=1\\ &and\ \theta_0(\infty)=0, \end{aligned} \tag{11}$$

$$\xi^1: \begin{aligned} & f_1''' + f_0 f_1'' - f_0' f_1' + 2 f_1 f_0'' + \eta f_0''' + Gr\theta_1 = 0, \\ & \theta_1'' + Pr\left(f_0\theta_0' - f_0'\theta_1 + 2f_1\theta_0'\right) + \eta\theta_0'' + \theta_0' = 0, \\ & f_1(0) = 0, f_1''(0) = \frac{f_0'(0)}{\lambda},\ f_1'(\infty) = 0, \theta_1(0) = 0 \\ & \text{and } \theta_1(\infty) = 0, \end{aligned} \tag{12}$$

$$\xi^2: \begin{aligned} & f_2''' + f_0 f_2'' - 2 f_0' f_2' + 2 f_1 f_1'' + 3 f_2 f_0'' - f_1'^2 + \eta f_1''' + Gr\theta_2 = 0, \\ & \theta_2'' + Pr\left(f_0\theta_2' - 2f_1\theta_1' + 3f_2\theta_0' - \theta_1 f_2' - 2f_2'\theta_2\right) + \eta\theta_1'' + \theta_1' = 0, \\ & f_2(0) = 0, f_2''(0) = \frac{f_1'(0)}{\lambda},\ f_2'(\infty) = 0, \theta_2(0) = 0 \\ & \text{and } \theta_2(\infty) = 0, \end{aligned} \tag{13}$$

where, superscript ´ means derivative with respect to η. The system of equations (11), (12) and (13) is solved using Runge-Kutta fourth order method with shooting technique (Conte & Boor 1981; Sharma & Singh 2010). The numerical scheme is programmed in Turbo C and no commercial numerical solver is used. To validate the scheme, the system of equation is solved for $Gr = 0.0$ and $\lambda = 1$, which reduces to the system solved by Crane and McVeigh (2010). The results obtained by Crane and McVeigh (2010) are $f_0'(0) = 2.0, f_1'(0) = -1.1283, f_2'(0) = 0.43169$ and the results obtained by the present scheme are $f_0'(0) = 2.0, f_1'(0) = -1.1283, f_2'(0) = 0.43171$. This represents an excellent agreement.

WALL SHEAR STRESS PARAMETER

The non-dimensional shear stress parameter (*SSP*) is defined as:

$SSP = \frac{r_0\tau}{\mu U}$ where $\tau = \mu\left(\frac{\partial u}{\partial r}\right)_{r=r_0}$ and μ is viscosity of fluid. So,

$$SSP = \frac{1}{\xi}\frac{\partial^2 f}{\partial\eta^2}\bigg|_{\eta=0} = \frac{1}{\xi}\left(f_0''(0) + \xi f_1''(0) + \xi^2 f_2''(0) + \xi^3 f_3''(0)\right) = \frac{1}{\lambda}\left(f_0'(0) + \xi f_1'(0) + \xi^2 f_2'(0)\right). \tag{14}$$

Further, the use of Shanks transformation is explained. Let the partial sums be given as:

$S_{n-1} = \frac{1}{\lambda} f_0'(0),\ S_n = \frac{1}{\lambda}\left(f_0'(0) + \xi f_0'(0)\right)$ and

$$S_{n+1} = \frac{1}{\lambda}\left(f_0'(0) + \xi f_1'(0) + \xi^2 f_2'(0)\right)$$

then Shanks transformation $e(S_n) = \frac{S_{n+1}S_{n-1} - S_n^2}{S_{n+1} + S_{n-1} - 2S_n}$. The value of *SSP* is evaluated using the Shanks transformation.

HEAT TRANSFER PARAMETER

The local Nusselt number is defined as:

$$Nu = \frac{Q}{\kappa\left(T_w - T_\infty\right)}$$

where $Q = -2\pi r_0 \kappa \frac{\partial T}{\partial r}\Big|_{r=r_0}$ and κ is thermal conductivity of fluid.

Therefore the heat transfer parameter (*HTP*) following Pop et al. (1995) is given as:

$$HTP = \left(\frac{\xi}{4\pi}\right)Nu = \frac{\partial\theta}{\partial\eta}\bigg|_{\eta=0} = \left(\theta_2'(0) + \xi\theta_1' + \xi^2\theta_2'\right). \tag{15}$$

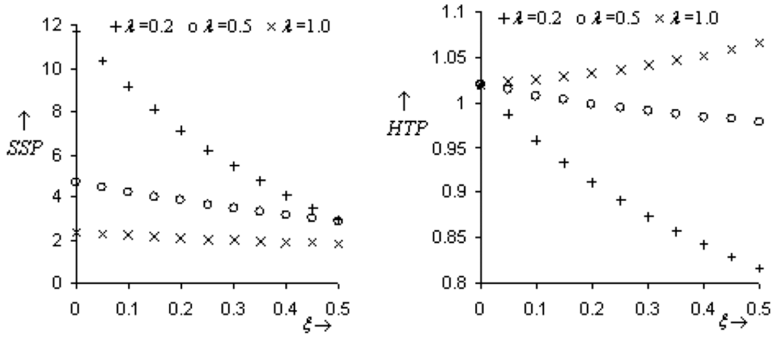
As explained above, to evaluate *HTP* Shanks transformation is employed.

RESULTS AND DISCUSSION

It is seen from Figure 2(a) that the shear stress experienced by cylinder is more for lower values of slip parameter λ, which is true because the higher the surface velocity the more is the drag experienced by the surface. However, as axial distance increases the shear stress decreases and dip is more in the case when value of λ is small. It is interesting to note that at increased axial distance the cylinder surface tends to experience almost the same shear stress even for different values of λ. Hence it is seen that slip parameter has more effect on shear stress at the leading edge. Figure 2(b) depicts that the heat transfer for $\lambda = 0.2$ decreases with the increase in axial distance ξ, but as the value of λ increase a change in the trend is observed and heat transfer for $\lambda = 1$ is seen to increase slightly with increasing axial distance. It is also noted that heat transfer is more in case of higher value of slip parameter.

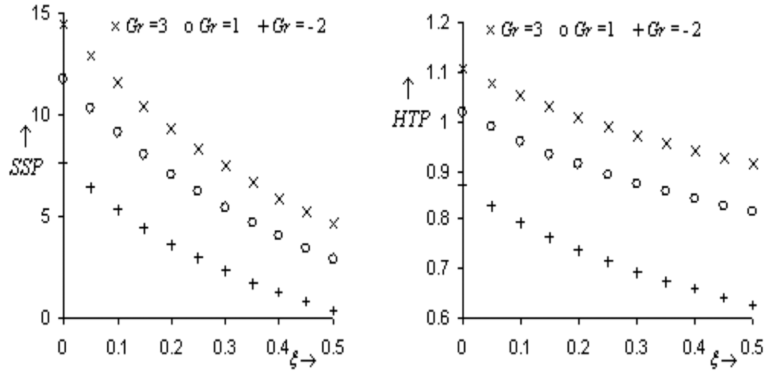
It is seen from Figure 3(a) that for higher value of buoyancy parameter, the shear stress at cylinder is more. This is true, because high buoyancy parameter means increased buoyancy force which assists the axial direction flow i.e. increased fluid velocity and therefore more drag is experienced by the surface. It is also seen that as the axial distance increases the shear stress reduces. The inferences drawn through Figure 3(a) explain the trends observed in Figure 3(b). At higher buoyancy parameter fluid velocity is higher and therefore the heat transfer is also higher. The decline in the heat transfer with the increase in axial distance is attributed to decrease in the shear stress with increasing axial distance.

Figure 4(a) shows that shear stress is more for the lower value Prandtl number and shear stress reduces with the increase in axial distance. Also the effect of difference in the value of Prandtl number on shear stress tends to die down with the increase in axial distance. It is seen from Figure 4(b) that heat transfer is higher for lower value of Prandtl number. In addition, for $Pr = 0.2$ the heat transfer decreases with increasing axial distance, however for $Pr = 5.0$ it almost the same (decreases slightly) with increasing axial distance. Hence, for high value of Prandtl number the heat transfer remains almost uniform along the axial distance.



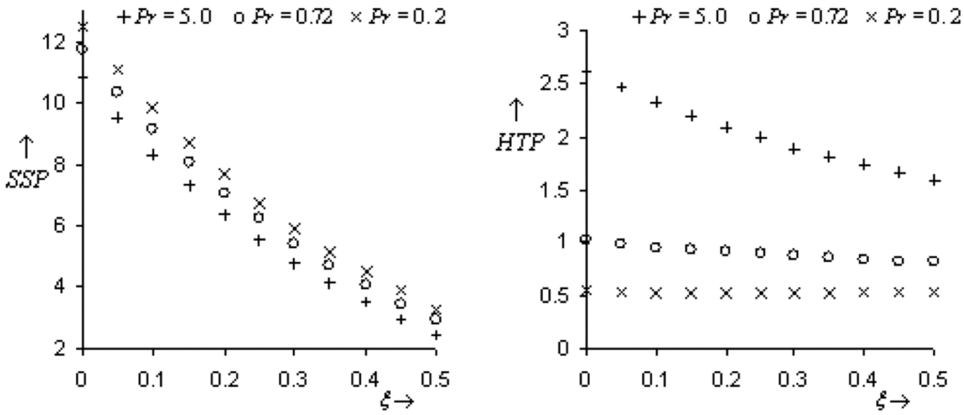
(a) (b)

FIGURE 2. Values of *SSP* and *HTP* versus ξ for *Pr* = 0.72 and *Gr* = 1.0



(a) (b)

FIGURE 3. Values of *SSP* and *HTP* versus ξ for *Pr* = 0.72 and λ = 0.2

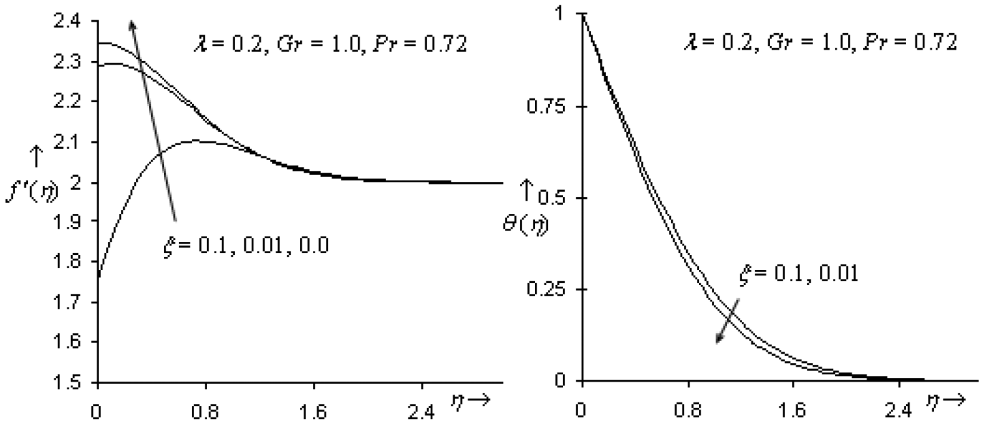


(a) (b)

FIGURE 4. Values of *SSP* and *HTP* versus ξ for *Gr* = 1.0 and λ = 0.2

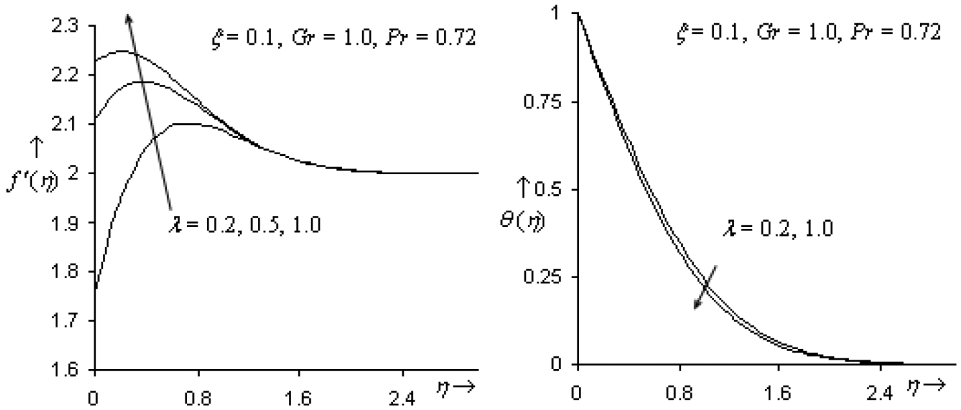
It is seen from Figure 5(a) that surface fluid velocity and the fluid velocity decrease along the increasing axial distance ξ. The decrease in the fluid velocity at the surface would mean that shear stress at the surface (i.e. $\eta = 0$) decreases, the same is observed from Figure 2(a). The decrease in fluid velocity would increase the fluid temperature, which is seen in Figure 5(b).

Figure 6(a) depicts that with the increase in the slip parameter λ, the surface fluid velocity and the fluid velocity increases. This is true, because in slip flow, fluid does not sticks to the fluid surface and therefore attains higher surface velocity for higher value of λ. For small value of λ the fluid temperature is high which is observed looking at Figure 6(b).



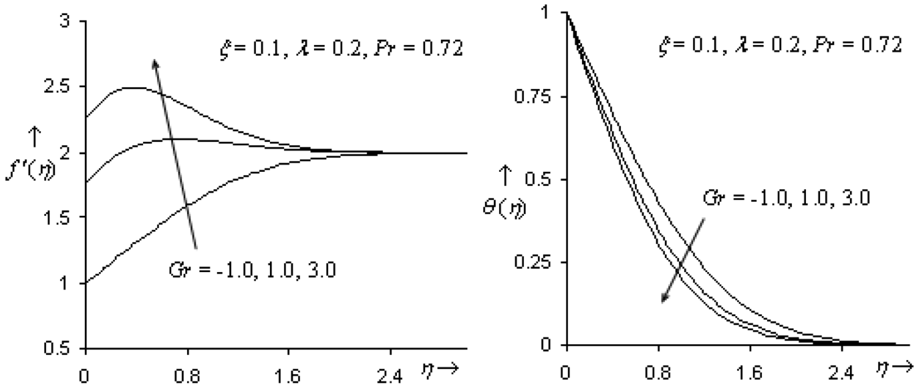
(a) (b)

FIGURE 5. (a) Velocity distribution (b) Temperature distribution versus η



(a) (b)

FIGURE 6. (a) Velocity distribution (b) Temperature distribution versus η



(a) (b)

FIGURE 7. (a) Velocity distribution (b) Temperature distribution versus η

The increase in buoyancy parameter increases the surface fluid velocity and fluid velocity. This is seen in Figure 7(a). Now, as fluid velocity increases the heat is convected readily and so for high buoyancy parameter fluid temperature decreases. The same is seen in Figure 7(b).

It is observed from Figure 8(a) that with the increase in Prandtl number *Pr*, the surface fluid velocity and fluid velocity decreases. The velocity boundary layer is comparatively thicker for lower value of *Pr*. Figure

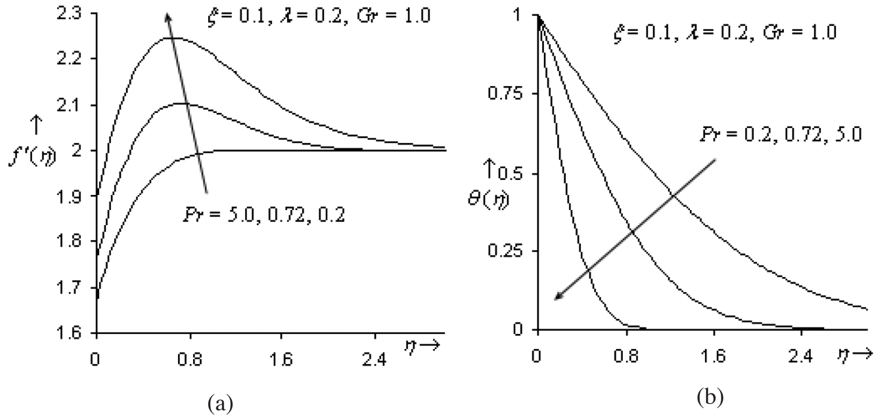


FIGURE 8. (a) Velocity distribution (b) Temperature distribution versus η

8(b) depicts for lower value of *Pr* the fluid temperature is higher, which is true since, in general, fluid with low Prandtl number has higher thermal conductivity. The higher thermal conductivity means fluid has affinity for heat and so low Prandtl fluid attains comparatively higher temperature. The higher fluid temperature increases the buoyancy force thus low Prandtl fluid has higher fluid velocity, which is seen in Figure 8(a).

CONCLUSION

Uniform slip flow along a vertical cylinder is numerically studied using series solution. From the discussion above it can be suitably concluded that the wall shear stress decreases with the increase in slip parameter, Prandtl number and axial distance while it increases with the increase in buoyancy parameter. The heat transfer at the surface increases with the increase in slip parameter, Prandtl number and buoyancy parameter. However, the heat transfer tends to increase or decrease with the increasing axial distance depending on the value of slip parameter. The fluid velocity increases with the increase in slip parameter and buoyancy parameter while it decreases with the increase in axial distance and Prandtl number. The fluid temperature increases with the increase in axial distance while it decreases with the increase in slip parameter, buoyancy parameter and Prandtl parameter.

REFERENCES

Aziz, A. & Na, T.Y. 1982. Improved perturbation solutions for laminar natural convection on a cylindrical cylinder. *Wärme- und Stoffübertragung* 16: 83-87.

Crane, L.J. & McVeigh, A.G. 2010a. Uniform slip flow on a cylinder. *Proc. App. Math Mech.* 10: 477-478.

Crane, L.J. & McVeigh, A.G. 2010b. Slip flow on microcylinder. *ZAMP* 61: 579-582.

Conte, S.D. & Boor, C. 1981. *Elementary Numerical Analysis*. New York: McGraw-Hill Book Co.

Gad-el-Hak, M. 1999. The fluid mechanics of microdevices-The Freeman Scholar Lecture. *J. Fluid Engg*. 121(1): 5-33.

Glauert, M.B. & Lighthill, M.J. 1955. The axisymmetric boundary layer on a long thin cylinder. *Proc. Royal Soc. London A* 230: 188-203.

Kelly, H.R.1962. A note on the laminar boundary layer flow on a circular cylinder in axial incompressible flow. *J. Aero. Sc*. 21(9): 634.

Lin, H.T. & Shih, Y.P. 1980. Laminar boundary layer heat transfer along static and moving cylinders. *J. Chinese Inst. Engs*. 3: 73-79.

Martin, M.J. & Boyd, D.I. 2006. Momentum and heat transfer in a laminar boundary layer with slip flow. *J. Ther. Heat Transfer* 20(4): 710-719.

Pop. I., Watanabe, T. & Taniguchi, H. 1995. Laminar boundary layer flow and heat transfer along a moving cylinder with suction and Injection. *Technische Mechanik* 15(2): 99-106.

Redmond, B. & McDonnell, D. 2004. Heat transfer through the boundary layer on a moving cylindrical fiber. *Proc. App. Math. Mech*. 10: 475-476.

Redmond, B. & Spillane, S. 2003. Shear stress on the surface of a circular cylinder at rest in a moving fluid in axially symmetric flow using Padé approximation technique. *Proc. App. Math Mech*. 3: 360-361.

Seban, R.A. & Bond, R. 1951. Skin friction and heat transfer characteristics of a laminar boundary layer on a circular cylinder in axial incompressible flow. *J. Aero. Sc*. 18: 671-675.

Sharma, P.R. & Singh, G. 2010. Effects of variable thermal conductivity, viscous dissipation on steady MHD flow natural convection flow of low Prandtl fluid on an inclined porous plate with Ohmic heating. *Meccanica* 45: 237-247.

Spillane, S. & Redmond, B. 2004. Shear stress analysis in axially symmetric flow using Padé approximation technique. *Proc. App. Math. and Mech*. 4: 452-453.

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Received: 19 January 2013
Accepted: 12 June 2013