Based on Equation (4), we modify Equation (2) to obtain the cdf of time to failure which can be given by

$$F_T(t) = \Phi\left(\frac{\frac{t}{D_f - \alpha} - \mu}{\sigma}\right) - 2T\left(\frac{\frac{t}{D_f - \alpha} - \mu}{\sigma}, \lambda\right)$$

By taking the derivative of F with respect to *t*, we get the pdf of time to failure distribution as follows:

$$f_T(t) = \frac{2}{(D_f - \alpha)\sigma}\phi\left(\frac{t - (D_f - \alpha)\mu}{(D_f - \alpha)\,\sigma}\right)\Phi\left(\lambda\left(\frac{t - (D_f - \alpha)\mu}{(D_f - \alpha)\sigma}\right)\right) \quad (5)$$

Now, it is clear that the cdf and the pdf of time to failure distribution are the cdf and the pdf of skew normal distribution with location parameter $\mu^{*} = (D - \alpha)\mu$, scale parameter $\sigma = (D_f - \alpha)\sigma$ and shape parameter λ. To compute the *r* th percentile of the time to failure distribution, denoted as t_r, we determine the inverse cdf of F and solved Equation (2) for t_r to obtain

$$t_r = (D_f - \alpha)\, G_\beta^{-1}(r) \quad (6)$$

Let $T_1, T_2, \ldots, T_n$ denoted a random sample of size n from the time to failure distribution with parameters λ, σ and μ. Given that $T_1 = t_1$, $T_2 = t_1, \ldots$ and $T_n = t_n$, based on Equation (5), we obtain the likelihood function given by

$$L(\lambda, \sigma, \mu\, ; \boldsymbol{t}) = \left(\frac{2}{(D_f - \alpha)\sigma}\right)^n \prod_{i=1}^{n} \phi\left(\frac{t_i - (D_f - \alpha)\mu}{(D_f - \alpha)\,\sigma}\right) \Phi\left(\lambda\left(\frac{t_i - (D_f - \alpha)\mu}{(D_f - \alpha)\,\sigma}\right)\right) \quad (7)$$

BAYESIAN APPROACH FOR SKEW NORMAL DEGRADATION MODEL

In this section, Bayesian approach is considered for skew normal degradation model. Both informative and non-informative priors are represented by gamma and uniform distributions, respectively, with certain known parameters. It is known that if non-informative priors are used to specify the prior distribution, the sample data would dominate the posterior distribution, particularly when the sample size is large. The posterior distribution can be given by

$$\pi(\lambda, \sigma, \mu | \boldsymbol{t}) = \frac{L(\lambda, \sigma, \mu; \boldsymbol{t}) q(\lambda, \sigma, \mu)}{\int \iint L(\lambda, \sigma, \mu; \boldsymbol{t}) q(\lambda, \sigma, \mu) d\lambda d\sigma d\mu} \quad (8)$$

where $L(\lambda, \sigma, \mu; \boldsymbol{t})$ is the likelihood function of the time to failure distribution for the parameters λ, σ and μ and $q(\lambda, \sigma, \mu)$ is the joint prior distribution of λ, σ and μ. These parameters are assumed independent. So, $q(\lambda, \sigma, \mu) = h(\lambda)u(\sigma)\, w(\mu)$, where $h(\lambda)$, $u(\sigma)$ and $w(\mu)$ are the prior distributions.

Gamma distribution is assumed for the prior distribution of λ while the prior distribution of σ and μ are assumed uniform. These prior distributions are given as follows:

i) Given that $\lambda \sim G(a, b)$, then $h(\lambda) = \begin{cases} \frac{\lambda^{a-1}\, e^{-b\lambda}}{\Gamma(a) b^{-a}}, & \lambda > 0 \\ 0, & \text{otherwise} \end{cases}$

ii) Given that $\sigma \sim U(0, c)$, then $u(\sigma) = \begin{cases} \frac{1}{c}, & 0 < \sigma < c \\ 0, & \text{otherwise} \end{cases}$

iii) Given that t $\mu \sim U(0, d)$, then $w(\mu) = \begin{cases} \frac{1}{d}, & 0 < \mu < d \\ 0, & \text{otherwise} \end{cases}$

where a, b, c, and d are assumed known constants.

The basic mechanism in the Bayesian approach involves updating of the prior distribution for the parameter of interest using the current information based on the following relationship:

posterior ∝ prior × likelihood

Based on this relationship, involving informative and non-informative prior distributions explained earlier and the likelihood function given in Equation (7), the joint posterior density function $\pi(\lambda, \sigma, \mu \,|\, \boldsymbol{t})$ is proportional to

$$\frac{\lambda^{a-1}\, e^{-b\lambda}}{\Gamma(a)\, c\, d\, b^{-a}} \left(\frac{2}{(D_f - \alpha)\sigma}\right)^n \prod_{i=1}^{n} \phi\left(\frac{t_i - (D_f - \alpha)\mu}{(D_f - \alpha)\,\sigma}\right) \Phi\left(\lambda\left(\frac{t_i - (D_f - \alpha)\mu}{(D_f - \alpha)\,\sigma}\right)\right) \quad (9)$$

SIMULATION STUDY FOR COMPARING THE PARAMETER ESTIMATION UNDER DIFFERENT PRIOR ASSUMPTIONS

The performance of Bayesian model under informative prior and weakly informative priors is studied based on simulated data which are generated using certain value of the true parameter values and the critical level of degradation D_f. The assumed true parameter values and D_f are respectively given as follows:

$\lambda = 3$; $\sigma = 2$; $\mu = 1$ and $D_f = 20$

In the real application, we have D_f = 5, but in the simulation study we assumed D_f = 20. A much large value of D_f in the simulation study is not surprising since the simulated data depict an ideal situation, which may be related to a very reliable product, where only large values of degradation data represent cases of failure. In addition, D_f = 20has been considered in the study of Rawashdeh, Ebrahem and Momani (2018). The initial value of parameter is assumed 6, while the sample sizes considered are n = 30, 60 and 200.

Based on these assumed values, the data for time to failure distribution are generated using the skew normal distribution with location parameter $\mu_* = (D_f - \alpha)\mu$, scale parameter $\sigma_* = (D_f - \alpha)\sigma$ and shape parameter λ. Since the joint posterior distribution of the parameters cannot be determined in a closed form, MCMC method is applied. In particular, samples from the joint posterior distribution of Equation (9) is generated using the JAGS (Just Another Gibbs Sampler) algorithm. The weakly informative gamma prior is represented based on the assumed values of the shape and scale hyper parameters of 0.1 and 0.01, respectively, i.e., a = 0.1 and b = 0.01, while the informative gamma prior is represented by a = 2 and b = 2. The choice made on the true values of the shape and scale hyper parameters, although may be arbitrary, reflects the level of uncertainty on the gamma priors. The variances for the weakly informative and informative gamma priors are 1000 and 1/2, respectively. In addition, for the uniform priors, we assume the upper limits c = 4 and d = 2. We believe that this is a fair representation of the level of uncertainty that we have on the little information regarding the prior parameters.

The JAGS algorithm which has been developed by Plummer (2003) can be used to carry out MCMC simulation based on Gibbs sampling, making the posterior analysis comparatively easy. For details on the implementation of JAGS algorithm, refer to Albert (2008) and Coro (2017). For computation, the programming language R version 4.0.3 is used.

The results for the estimation of the parameters and certain percentiles based on Bayesian approach of the skew normal linear degradation model under informative and weakly informative priors for the different sample sizes are provided in Table 3. The properties of point estimates are evaluated using bias (B) and mean squared error (MSE) values of the estimators. Estimators with small bias and mean squared error are preferred.

TABLE 3. B and MSE for the parameter estimate and certain percentiles of the time to failure distribution under informative and weakly informative prior for n = 30, 60 and 200

Parameter values	Informative prior						Weakly Informative prior					
	n = 30		n = 60		n = 200		n = 30		n = 60		n = 200	
	B	MSE	B	MSE	B	MSE	B	MSE	B	MSE	B	MSE
$\mu = 1$	0.268	0.178	0.025	0.049	0.069	0.023	0.464	0.363	0.044	0.075	0.063	0.024
$\sigma = 2$	0.469	0.318	0.089	0.071	0.101	0.032	0.617	0.497	0.111	0.0956	0.095	0,032
$\lambda = 3$	0.314	2.808	1.332	4.870	0.554	1.027	0.970	7.535	2.937	80.400	0.523	1.062
$t_{0.05}$	1.894	14.870	1.331	7.845	0.100	2.043	0.312	19.200	1.112	7.946	0.127	2.091
$t_{0.2}$	0.719	7.819	0.410	4.590	0.336	1.578	0.042	9.978	0.104	5.479	0.324	1.594
$t_{0.5}$	1.332	9.300	0.462	5.715	0.014	1.770	1.641	10.510	0.333	6.201	0.003	1.801
$t_{0.75}$	4.027	28.240	1.071	11.290	0.656	3.393	4.664	33.630	1.131	11.800	0.641	3.384
$t_{0.9}$	7.131	75.320	1.697	23.500	1.357	7.976	8.443	95.880	1.925	26.000	1.299	7.910

The estimators are evaluated based on bias and MSE. From Table 3, we have found the following results: In most cases, B and MSE values of the estimated parameters and the associated percentiles in the Bayesian approach of skew normal linear degradation model under informative and weakly informative gamma prior decrease as n increase. The MSE of the r^{th} percentiles of the time to failure distribution increases as r increase for all sample sizes, except in the case of the low value of the r^{th} percentile. In most cases, the B values of the estimated parameters and the associated percentiles under informative gamma prior are smaller than the B values found under weakly informative gamma prior for small sample size and these results are quite close for large sample size.

Generally, in the small sample size, the performance of the Bayesian approach of skew normal linear degradation model under informative gamma prior is better than the performance of the Bayesian approach of skew normal linear degradation model under weakly informative gamma prior, while in the case of large sample size, the results found based on the two different prior assumptions are close.

COMPARISON BETWEEN LOG-LOGISTIC AND SKEW NORMAL DEGRADATION MODEL UNDER THE BAYESIAN APPROACH

In this section, the performance of Bayesian approach for modeling the time to failure distribution and its percentiles based on linear degradation model with the degradation parameter following the log-logistic distribution and skew normal distribution is compared in terms of bias and mean squared error.

Rawashdeh, Ebrahem and Momani (2018) have estimated the time to failure distribution based on linear degradation path $D = +X$ with random degradation rate X which follows log-logistic distribution. The cdf of time to failure distribution is given as

$$F_{T_L}(t_L;\alpha_L,\beta_L,\emptyset) = \frac{1}{1+\left(\frac{t_L}{\alpha_*}\right)^{-\beta_L}},\ \alpha_* = \frac{D-\emptyset}{\alpha_L}$$

where $\emptyset$ is the degradation level at at $t_L = 0$; α_L and β_L are the scale and the shape parameters of log-logistic distribution, respectively, and D is the critical degradation level at which failure is declared. It has been shown that the time to failure distribution is also log-logistic with scale parameter α_* and the shape parameter β_L. The percentiles of the time to failure distribution based on Equation (10) can be shown as

$$t_{L-r} = \frac{D-\emptyset}{\alpha_L}\left(\frac{r}{1-r}\right)^{\frac{1}{\beta_L}} \qquad (11)$$

The Bayesian method has been applied for grouped and non-grouped data and the results found are compared. The prior distributions of parameters and are assumed to be non-informative prior. The non-informative prior that has been considered by them are uniform and Jeffery's priors.

In the conclusion of this study, they have stated that the performance of the Bayesian model for the non-grouped method is better due to the smaller value of mean squared error. They have shown that the joint posterior distribution of non-grouped data is given as

$$\pi(\alpha_L,\beta_L,\emptyset|t_L,D) \propto \frac{{\beta_L}^n}{D}\left(\prod_{i=1}^{n} t_{L_i}\right)^{\beta_L-1} {\alpha_L}^{n\beta_L-1}(D-\emptyset)^{-n\beta_L}\frac{1}{\prod_{i=1}^{n}\left(1+\left(\frac{\alpha_L t_{L_i}}{D-\emptyset}\right)^{\beta_L}\right)^2} \qquad (12)$$

In this work, a slight modification is done in the work by Rawashdeh, Ebrahem and Momani (2018) where we consider the linear degradation path model $D = \emptyset + \frac{t}{X}$ which involves reciprocal of the degradation rate instead of the degradation rate. Here X is assumed to follow log-logistic distribution. Thus, the cdf of time to failure distribution can be shown as

$$F_{T_L}(t_L;\alpha_L,\beta_L) = \frac{1}{1+\left(\frac{t_L}{(D-\emptyset)\alpha_L}\right)^{-\beta_L}} \qquad (13)$$

Then, the pdf of time to failure distribution is given as

$$f_{T_L}(t_L;\alpha_L,\beta_L) = \frac{\beta_L}{(D-\emptyset)\alpha_L}\left(\frac{t_L}{(D-\emptyset)\alpha_L}\right)^{\beta_L-1}\left(1+\left(\frac{t_L}{(D-\emptyset)\alpha_L}\right)^{\beta_L}\right)^{-2} \qquad (14)$$

Note that the time to failure distribution is also log-logistic distribution with scale parameter $(D\text{-}\emptyset)\alpha_L$ and the shape parameter β_L. Based on Equation (13), it can be shown that the percentiles of the time to failure distribution is

$$t_{L-r} = (D-\emptyset)\alpha_L\left(\frac{r}{1-r}\right)^{\frac{1}{\beta_L}} \qquad (15)$$

To make the skew normal and log-logistic degradation models comparable, both the prior distributions for the scale and the shape parameters for each model are assumed to be the same as in the previous section where the values of the hyper parameters of the informative gamma prior for both models are equal, i.e., $a = b = 2$ and $d = s = 2$. Note that the location parameter, which is another parameter for the skew normal distribution, is assumed fixed and it is not estimated. In addition, for both models, the initial and the critical degradation values are also assumed the same as previous section. Then, it can be shown that the joint posterior distribution $\pi_L\ (\alpha_L, \beta_L; t_L)$ is proportional to

$$\frac{{\beta_L}^{d-1}\, e^{-\frac{\beta_L}{s}}}{\Gamma(d)\, d^s\, b}\left(\frac{\beta_L}{(D-\emptyset)\alpha_L}\right)^n \prod_{i=1}^{n}\left(\frac{t_{L_i}}{(D-\emptyset)\alpha_L}\right)^{\beta_L-1} \left(1+\left(\frac{t_{L_i}}{(D-\emptyset)\alpha_L}\right)^{\beta_L}\right)^{-2} \quad (16)$$

The results of the simulation study for comparison between the models are provided in Table 4.

The estimators are evaluated based on bias and MSE. From Table 4, we have found the following results: In most cases, the MSE of the estimated percentiles in the Bayesian approach of skew normal and log-logistic linear degradation model decreases as n increase. The MSE of the r^{th} percentile positions increases as r increases for all sample sizes except in the lower percentile position where n = 30 and 60 for skew normal linear degradation model. In most cases for all sample sizes, the B values of the estimated percentiles in both models increase as r increase. In most cases, for small sample size such as n = 30 and 60, for lower percentile position of time to failure distribution, the B values for skew normal linear degradation model are larger than the B values for log-logistic linear degradation model and vice versa for the case of larger percentile position.

Generally, the performance of the Bayesian approach of skew normal linear degradation model is found better than the performance of the Bayesian approach of log-logistic linear degradation model, particularly in the case of higher percentile position of time to failure distribution.

TABLE 4. Bias (B) and MSE of the percentiles for the skew normal and log-logistic degradation models for the simulated data for n = 30, 60 and 200

Parameter values	Skew normal degradation model						Log-logistic degradation model					
	n = 30		n = 60		n = 200		n = 30		n = 60		n = 200	
	B	MSE	B	MSE	B	MSE	B	MSE	B	MSE	B	MSE
$t_{0.05}$	0.787	17.805	0.466	7.825	0.613	2.900	0.027	2.420	1.226	3.194	0.672	0.806
$t_{0.2}$	0.679	6.845	1.080	4.090	0.668	1.347	0.656	6.099	1.659	6.295	1.594	3.346
$t_{0.5}$	2.686	17.146	1.997	7.333	2.246	5.943	2.093	21.337	1.760	12.834	3.372	13.649
$t_{0.75}$	4.790	48.523	3.048	17.183	3.779	16.386	4.380	84.856	1.172	38.467	6.043	45.353
$t_{0.9}$	6.927	99.588	4.252	33.804	5.375	33.108	8.412	395.311	0.851	177.867	10.739	159.160

APPLICATION OF REAL DEGRADATION DATA

In the second section, the GaAs laser degradation data is described and applied to investigate the adequacy of the skew normal, log-logistic and exponential degradation models based on probability plotting and the computed values of AIC of all models. In this section, the GaAs laser degradation data is applied to compare the performance of the weakly informative and informative prior of the skew normal degradation model. The comparison is made in terms of point estimate (PE) and standard error (SE) of the percentiles of the time to failure distribution. The results found are provided in Table (5).

TABLE 5. PE and SE for certain percentiles of the skew normal linear degradation model under informative and weakly informative prior based on the GaAs laser degradation data

parameters	Informative prior		Weakly informative prior	
	PE	SE	PE	SE
$t_{0.05}$	4.449	0.168	5.891	0.148
$t_{0.2}$	6.635	0.098	7.113	0.097
$t_{0.5}$	9.068	0.200	9.431	0.204
$t_{0.75}$	11.86	0.339	12.001	0.347
$t_{0.9}$	14.433	0.485	14.667	0.495

According to Table 5, referring to the estimated parameters and standard errors, when estimating the percentiles of the time to failure distribution under the application of laser degradation data, the performance of the skew normal degradation model based on informative prior is generally better than the performance of the skew normal degradation model based on weakly informative prior.

As mentioned in the second section, the AIC found for fitted skew normal and log-logistic models are found close to each other. In order to compare the performance of those models, the point estimate (PE) and standard error (SE) of the percentiles of the time to failure distribution found based on skew normal and log-logistic distributions that are indicated in Equations (5) and (14), respectively, are computed. Also, deviance information criterion (DIC) is determined for selecting the best fitted model to the real data. Spiegelhalter, Best and Carlin (2002) define the DIC for the vector of parameters interest, say θ, as the following:

$$DIC = \bar{D}(\theta) + P_D$$

where D is the deviance which is defined by – 2log (l); l is the likelihood function; $\bar{D}$ is the posterior deviance mean which is given based on the values of the estimated parameters of the posterior distribution; and P_D is the effective number of parameters of the model which is defined as $\bar{D}(\theta)$ - $\bar{D}(\bar{\theta})$ where $\bar{D}(\bar{\theta})$ is the deviance which is found by finding the mean of each value of the estimated parameters in the posterior distribution. Results of the comparison involving the parameter estimates, standared error and DIC are provided in Table 6.

TABLE 6. PE and SE of the percentiles and DIC for the skew normal and log-logistic degradation models for laser degradation data

parameters	Interpolated values	Skew normal degradation model		Log-logistic degradation model	
		PE	SE	PE	SE
$t_{0.05}$	7.128	6.650	0.237	5.393	0.234
$t_{0.2}$	8.125	8.332	0.176	7.478	0.219
$t_{0.5}$	10.581	10.154	0.153	10.050	0.233
$t_{0.75}$	11.671	11.678	0.179	12.749	0.350
$t_{0.9}$	12.212	13.100	0.234	16.226	0.627
DIC		67.550		68.530	

Comparison of the skew normal and the log-logistic degradation model in Table 6 indicates the following results: In most cases, the standard error for the estimated percentiles of skew normal degradation model is smaller than the standard error for the estimated percentiles of log-logistic degradation model. The point estimate of the percentiles of skew normal degradation model is found closer to the observed value of the percentiles reported in the laser degradation dataset than those values found for the log-logistic degradation model. This finding can be proven by using the linear interpolation of the observed values. For example, consider the interpolated value of the 75th percentile of the time to failure values in Table 1, i.e., 11.671. Corresponding to the estimated values in the Table 6, we have for the skew normal degradation model while for the log-logistic degradation model , and the standard errors are 0.18 and 0.35, respectively. This indicates that the estimated value of the parameter for the skew normal degradation model is more precise and hence it is more appropriate to use this result as compared to that found based on log-logistic degradation model. The deviance information criterion for skew normal degradation model is slightly smaller than the deviance information criterion for log-logistic degradation model, indicating a better fitting for skew normal degradation model.

Conclusions

In this study, we present the Bayesian approach of skew normal linear degradation model based on the informative and weakly informative prior. From the simulation results, we conclude that the Bayesian approach which considers informative prior outperformed the Bayesian model found under weakly informative prior, especially in the case of small sample size while for large sample size both results are found to be close. In addition, comparison of the skew normal and log-logistic linear degradation models is presented in terms of point estimate, standard error, Akaike information criterion and deviance information criterion. It is found that the linear degradation model with the reciprocal of the degradation parameter as the slope in the degradation path equation where the degradation parameter follows the skew normal distribution presents as a better alternative model than the log-logistic linear degradation model for describing the degradation data and these results are found more apparent in the real data application of laser degradation data in terms of point estimate, standard error and deviance information criteria.

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References

Al-haj Ebrahem, M., Al-momani, N. & Eidous, O. 2021. Variable scale kernel density estimation for simple linear degradation model. *Electronic Journal of Applied Statistical Analysis* 14(2): 359-372. https://doi.org/10.1285/i20705948v14n2p359

Albert, J. 2008. *Bayesian Computation with R*. 3rd ed. New York: Springer.

Alhamidie, A.A., Kamarulzaman Ibrahim, Alodat, M.T. & Wan Zawiah Wan Zin. 2019. Bayesian inference for linear regression under alpha-skew-normal prior. *Sains Malaysiana* 48(1): 227-235.

Azzalini, A. 1985. A class of distributions which includes the normal ones. *Scandinavian Journal of Statistics* 12(2): 171-178.

Dakhn, L.N.B., Ebrahem M.A-H. & Eidous, O. 2017. Semi-parametric method to estimate the time-to-failure distribution and its percentiles for simple linear degradation model. *Journal of Modern Applied Statistical Methods Article* 16(2): 322-346. https://doi.org/10.22237/jmasm/1509495420

Bayes, C.L. & Branco, M.D. 2007. Bayesian inference for the skewness parameter of the scalar skew-normal distribution. *Brazilian Journal of Probability and Statistics* 21: 141-163.

Bryson, M.C. 1974. Heavy-tailed distributions: Properties and tests. *Technometrics* 16(1): 61-68. https://doi.org/10.1080/00401706.1974.10489150

Chen, X., Sun, X., Ding, X. & Tang, J. 2019. The inverse gaussian process with a skew-normal distribution as a degradation model. *Communications in Statistics - Simulation and Computation*. https://doi.org/10.1080/03610918.2018.1527351

Coro, G. 2017. *Gibbs sampling with Gibbs Sampling with JAGS : Behind the Scenes*. https://www.researchgate.net/publication/313905185

Ebrahem, M.A-h., Alodat, M.T. & Amani Arman. 2009. Estimating the time-to-failure distribution of a linear degradation model using a Bayesian approach. *Applied Mathematical Sciences* 3(1): 27-42.

Ebrahem, M.A-h., Eidous, O. & Kmail, G. 2009. Estimating percentiles of time-to-failure distribution obtained from a linear degradation model using kernel density method. *Communications in Statistics - Simulation and Computation* 38(9): 1811-1822. https://doi.org/10.1080/03610910903145130

Eidous, O., Ebrahem, M.A-h. & Dakhn, L.N.B. 2017. Estimating the time-to-failure distribution and its percentiles for simple linear degradation model using double kernel method. *Journal of Probability and Statistical Science* 15(1): 121-134.

Giraldo Gómez, N. 2005. An example of a heavy tailed distribution. *Matematicas* 13(1): 43-50.

Guure, C.B. & Noor Akma Ibrahim. 2014. Approximate Bayesian estimates of Weibull parameters with Lindley's method. *Sains Malaysiana* 43(9): 1433-1437.

Hamada, M. 2005. Using degradation data to assess reliability. *Quality Engineering* 17(4): 615-620. https://doi.org/10.1080/08982110500225489

Jin, Z. 2016. Semiparametric accelerated failure time model for the analysis of right censored data. *Communications for Statistical Applications and Methods* 23(6): 467-478. https://doi.org/10.5351/csam.2016.23.6.467

Meeker, W.Q., Escobar, L.A. & Lu, C.J. 1999. Accelerated degradation tests: Modeling and analysis. *Statistics Preprints* 2: 1-24.

Meeker, W.Q. & Escobar, L.A. 1998. *Statistical Method for Reliability Data*. New York: John Wiley and Sons, Inc.

Oliveira, R.P.B., Loschi, R.H. & Freitas, M.A. 2018. Skew-heavy-tailed degradation models: An application to train wheel degradation. *IEEE Transactions on Reliability* 67(1): 129-141. https://doi.org/10.1109/TR.2017.2765485

Pan, D., Liu, J-B. & Yang, W. 2018. A new result on lifetime estimation based on Skew-Wiener degradation model. *Statistics and Probability Letters* 138: 157-164. https://doi.org/10.1016/j.spl.2018.03.009

Plummer, M. 2003. JAGS : A program for analysis of Bayesian graphical models using Gibbs sampling. *Distributed Statistical Computing*. https://www.r-project.org/nosvn/conferences/DSC-2003/Drafts/Plummer.pdf

Puggard, W., Niwitpong, S-a. & Niwitpong, S. 2022. Comparison analysis on the coefficients of variation of two independent Birnbaum-Saunders distributions by constructing confidence intervals for the ratio of coefficients of variation. *Sains Malaysiana* 51(7): 2265-2281.

Rawashdeh, A., Ebrahem, M.A-h. & Momani, A. 2018. A Bayesian approach to estimate the failure time distribution of a log-logistic degradation model. *METRON* 76: 155-176. https://doi.org/10.1007/s40300-018-0141-7

Robins, J. & Tsiatis, A.A. 1992. Semiparametric estimation of an accelerated failuare time model with time-dependent covariates. *Biometrika* 79(2): 311-319.

Shafiq, M., Alamgir & Atif, M. 2016. On the estimation of three parameters lognormal distribution based on fuzzy life time data. *Sains Malaysiana* 45(11): 1773-1777.

Spiegelhalter, D.J., Best, N.G., Carlin, B.P. & Van Der Linde, A. 2002. Bayesian measures of model complexity and fit. *Biostatistics* 64(4): 583-616.

Thangjai, W., Niwitpong, S-A. & Niwitpong, S. 2021. A Bayesian approach for estimation of coefficients of variation of normal distributions. *Sains Malaysiana* 50(1): 261-278. https://doi.org/10.17576/jsm-2021-5001-25

Tsai, C-c. & Lin, C-t. 2015. Lifetime inference for highly reliable products based on skew-normal accelerated destructive degradation test model. *IEEE Transactions on Reliability* 64(4): 1340-1355.

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