

## Bayesian Two-Sided Complete Group Chain Sampling Plan for Poisson Distribution with Gamma Prior

(Pelan Persampelan Bayesian Kumpulan Berantai Dua Sisi Lengkap untuk Taburan Poisson dengan Prior Gamma)

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### ABSTRACT

For statistical quality assurance based on the inspection of a random sample, acceptance sampling plan help to decide whether the lot should be accepted or rejected. Most traditional plans only focus on minimizing the consumer's risk, but producer's risk also should not be ignored in acceptance sampling plan. Therefore, this study focuses on reducing both producer's and consumer's risks through the quality region. This study proposes a Bayesian two-sided complete group chain sampling plan (BTSCGChSP) for the average probability of lot acceptance. The Poisson distribution with gamma as prior distribution is used to derive the average probability of lot acceptance. Next, R programming language is used to obtain the average number of defectives according to average probability of acceptance and pre-specified values of design parameters. For selected design parameters in BTSCGChSP, the acceptable quality level (AQL) associated with producer's risk and limiting quality level (LQL) associated with consumer's risk are considered to estimate quality regions. In this paper, four quality regions are measured: (i) probabilistic quality region (PQR), (ii) quality decision region (QDR), (iii) limiting quality region (LQR) and (iv) indifference quality region (IQR). Operating characteristic curves (OC) are used for performance comparison with existing Bayesian group chain sampling plan (BGChSP) for the same probability of lot acceptance and other design parameter values. Findings validate that BTSCGChSP provides more ideal OC curve than BGChSP for the same probability of acceptance. For quality regions with the same values of consumer's and producer's risks, then the BTSCGChSP region will contain fewer defectives than in the BGChSP region. Hence, the proposed plan is a better substitute for existing BGChSP.

Keywords: Consumer's risk; gamma distribution; producer's risk; quality region

### ABSTRAK

Bagi memastikan kualiti statistik berdasarkan pemeriksaan sampel secara rawak, pelan persampelan penerimaan dapat membantu memutuskan sama ada lot patut diterima atau ditolak. Kebanyakan pelan tradisional hanya menumpukan untuk meminimumkan risiko pengguna, tetapi risiko pengeluar juga tidak patut diabaikan dalam pelan persampelan penerimaan. Oleh itu, kajian ini memfokuskan pada pengurangan risiko pengeluar dan pengguna melalui wilayah kualiti. Kajian ini mencadangkan pelan persampelan Bayesian kumpulan berantai dua sisi lengkap (BTSCGChSP) untuk purata kebarangkalian penerimaan lot. Taburan Poisson dengan gamma sebagai taburan prior digunakan untuk mendapatkan purata kebarangkalian penerimaan lot. Seterusnya, bahasa pengaturcaraan R digunakan untuk mendapatkan purata bilangan kecacatan berdasarkan purata kebarangkalian penerimaan dan nilai reka bentuk parameter yang telah ditetapkan. Bagi reka bentuk parameter yang terpilih dalam BTSCGChSP, tahap kualiti yang boleh diterima (AQL) yang berkaitan dengan risiko pengeluar dan tahap kualiti terbatas (LQL) yang berkaitan dengan risiko pengguna dipertimbangkan untuk menilai wilayah berkualiti. Dalam makalah ini, empat wilayah kualiti dinilai: (i) wilayah kualiti kebarangkalian (PQR), (ii) wilayah kualiti keputusan (QDR), (iii) wilayah kualiti terbatas (LQR) dan (iv) wilayah kualiti indifferens (IQR). Keluk ciri operasi (OC) digunakan untuk perbandingan prestasi dengan pelan persampelan Bayesian kumpulan berantai (BGChSP) sedia ada dengan kebarangkalian penerimaan lot yang sama dan nilai reka bentuk parameter yang lain. Hasil kajian mengesahkan bahawa BTSCGChSP memberikan keluk OC yang lebih ideal berbanding BGChSP untuk kebarangkalian penerimaan yang sama. Sekiranya wilayah kualiti mempunyai risiko pengguna dan pengeluar yang sama, maka wilayah BTSCGChSP akan mengandungi bilangan kerosakan yang lebih sedikit berbanding wilayah BGChSP. Oleh itu, pelan yang dicadangkan adalah pengganti yang lebih baik berbanding BGChSP.

Kata kunci: Risiko pengeluar; risiko pengguna; taburan gamma; wilayah kualiti

## INTRODUCTION

In quality control statistics, a key tool that is used for inspection is acceptance sampling. There are numerous steps involved in the acceptance of a product (Montgomery 2020). The inspection of a product is based on defective or non-defective. One approach for deciding on a lot is to undertake a 100% inspection, while the other is to use acceptance sampling. As a result, the cost and time spent on inspection can be decreased because only a small number of samples are chosen for acceptance sampling (Dobbah et al. 2018; Teh et al. 2021). Several acceptance sampling plans have been established over decades. A single sampling plan (SSP) was introduced by Epstein (1954). To improve the probability of lot acceptance by considering the previous sample in SSP the chain sampling plan-1 (ChSP-1) was introduced by Dodge (1955). To minimize the average cost, an attribute single sampling plan was provided by Hald (1965).

Mughal et al. (2010) assessed the design parameters for the group acceptance sampling plan (GASP). In GASP, the inspection is completed in the form of groups by using several numbers of testers at a time. Where based on the available number of testers  $r$ , the sample size  $n$  is divided into different groups  $g$ . An economic reliability GASP by using a group sampling plan for a Pareto 2nd kind distribution was developed by Mughal et al. (2015a). Based on data theory to find the necessary design parameters, they used Poisson and weighted Poisson distributions. The proposed designs were found to require a shorter testing time. Later they developed a GChSP for a product lifetime following the Pareto distribution of the 2nd kind (Mughal et al. 2015b). The lot acceptance probability was obtained at several quality levels to satisfy the pre-assumed design parameters (Mughal et al. 2016).

A single sampling plan was modified by using the acceptable quality limit (AQL) and limiting quality level (LQL) in the Bayesian single sampling plan with the weighted Poisson distribution by Subbiah and Latha (2017) and Poisson distribution by Raju and Vidya (2017). Suresh and Veerakumasi (2007) used the gamma prior and proposed a Bayesian chain sampling plan for construction and performance measures. The Bayesian approach is based on conditional probability, which is based on current and past information. Bayesian sampling plans necessitate the explicit specification of the distribution from lot to lot, which is referred to as prior distribution. Chen et al. (2021a, 2021b), Hafeez and Aziz (2019), Hafeez et al. (2022a, 2022b), Prajapati et

al. (2020), and Wang and Park (2020), are a few recent studies that deal with the Bayesian approach.

Hafeez and Aziz (2019) worked on GChSP Mughal et al. (2015b)' plan and introduced the idea of Bayesian in GChSP. By considering the quality variation, they developed a plan called the Bayesian group chain sampling plan (BGChSP) for the binomial distribution by using beta as a prior distribution. For the average number of defectives, they consider Poisson distribution with gamma prior in BGChSP (Hafeez et al. 2022a). The average probability of acceptance was estimated for the average number of defectives based on only preceding lots.

Under the acceptance sampling, this study extends the idea of BGChSP for a sampling plan called BTSCGChSP. Where the decision about the current lot will consider preceding as well as succeeding lots. The BGChSP was limited to the preceding lots and provided less protection to the consumer and producer. The objective of this study was to consider preceding as well as succeeding lots with the current lot to provide more protection to the consumer and producer. From the addition of succeeding lots, the acceptance criteria will change and protect the current lot from both sides. If the BGChSP considers four preceding lots ( $i = 4$ ), then the same approach in the BTSCGChSP will consider four preceding and four succeeding lots ( $i = j = 4$ ). To design the BTSCGChSP, the indexed parameters are producer's risk ( $\alpha$ ), which is associated with acceptable quality level (AQL), and consumer's risk ( $\beta$ ), which is associated with limiting quality level (LQL). Other specified parameters are shape parameter ( $s$ ), number of groups ( $g$ ), number of testers ( $r$ ), number of preceding lots ( $i$ ) and number of succeeding lots ( $j$ ). Also, for prior distribution parameters, the numerical illustrations are provided for QDR, PQR, LQR, and IQR. This study is only limited to the average number of defective not for the average probability of acceptance. The results are obtained for some specific values of design parameters, it can be estimated for other values according to manufacturing requirement. This plan considers Poisson distribution with gamma as prior, further this work can be extended for other distributions with other priors.

## METHODS

The operating procedure for two-sided complete group chain sampling plan (TSCGChSP) is based on the following steps: i. Select an ideal number of  $g$  groups for each lot and assign  $r$  items to each group, which is the sample size required ( $n = g * r$ ). ii. Count the number of

defectives, which is the sum of current  $d$ , preceding  $d_i$  and succeeding  $d_j$  defectives. iii. If  $d = 0$  in the current sample, then accept the lot. iv. Reject the lot if more than one defective is found in the current sample, i.e., ( $d > 1$ ). v. If  $d = 1$  in the current sample but preceding  $i$  and

succeeding  $j$  samples have no defectives ( $d_i + d_j < 1$ ), then accept the lot. However, reject the lot if preceding  $i$  and succeeding  $j$  samples have one or more defective in total, i.e. ( $d_i + d_j \geq 1$ ). All these steps can be summarized in the flow chart presented in Figure 1.

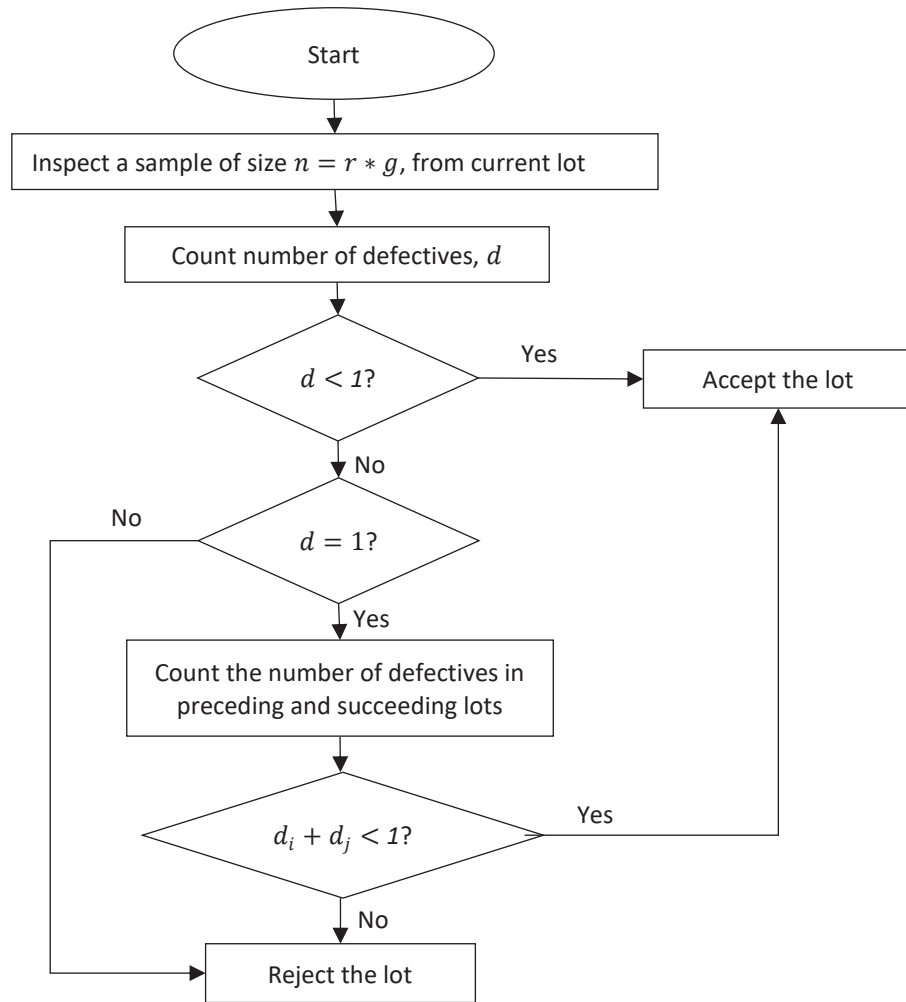


FIGURE 1. Operating procedure for TSCGChSP

For TSCGChSP, the above procedure can also be shown through a tree diagram for  $i = j = 1$  in Figure 2, where  $D$  denotes the defective and  $ND$  denotes non-defective products.

From the tree diagram in Figure 2, it is clear that TSCGChSP contains five acceptance criteria (AC). The possible outcomes which comply with the acceptance criteria of chain sampling are  $\{(D, ND, D), (D, ND,$

$ND), (ND, D, ND), (ND, ND, D), (ND, ND, ND)\}$ . Therefore, the following outcomes  $\{(D, ND, D), (D, ND, ND), (ND, ND, D), (ND, ND, ND)\}$  concern with the current lot, hence their probability of lot acceptance is  $P_0$  and  $\{(ND, D, ND)\}$  has a probability of lot acceptance  $P_0 P_1 P_0$ .

$$L(p) = P_0 + P_0 P_1 P_0; \tag{1}$$

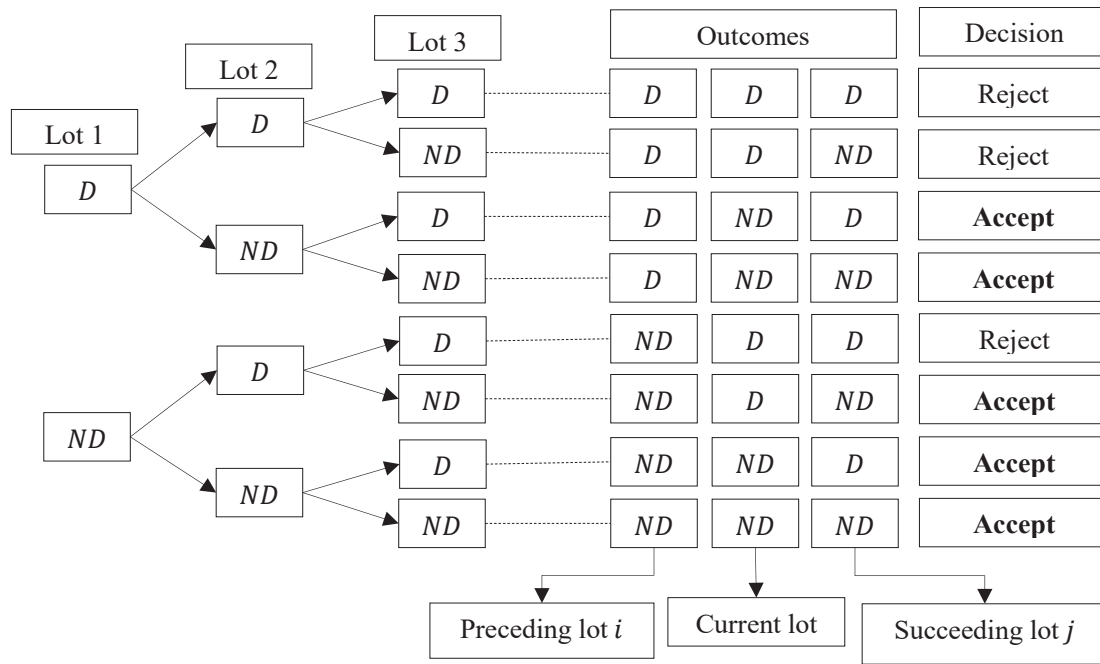


FIGURE 2. Tree diagram for the TSCGChSP

For TSCGChSP, the general expression of the probability of acceptance for  $i=j=1$  from equation (1) is:

$$L(p) = P_0 + (P_0)^i P_1 (P_0)^j. \tag{2}$$

$$L(p) = P_0 + P_1 (P_0)^{i+j}. \tag{3}$$

When developing the procedures,  $L(p)$  can be determined for the chain acceptance sampling plans, with the assumption that the underlying distribution for the plan is following either binomial or Poisson distribution (Latha & Arivazhagan 2015; Rosaiah & Kantam 2005; Suresh & Sangeetha 2011). For the average number of defectives, this paper considers Poisson distribution, such that the probability distribution function (PDF) is:

$$p(c) = \frac{\mu^c}{c!} e^{-\mu}. \tag{4}$$

For group chain sampling, replace mean  $\mu = np$  and  $n = r * g$  in equation (4) and solve for zero and one defective product. After solving equation (4) for  $c = 0$  and  $c = 1$ , the obtained probability of lot acceptance is:

$$P_0 = e^{-(r * g)p}; \tag{5}$$

$$P_1 = (r * g) p e^{-(r * g)p}. \tag{6}$$

After replacing equations (5) and (6) in equation (3), we get:

$$L(p) = e^{-r g p} + r g p e^{-r g p} e^{-r g p (i+j)}. \tag{7}$$

For the equal number of preceding and succeeding lots  $i = j$ , equation (7) can be written as:

$$L(p) = e^{-r g p} + r g p e^{-r g p (2i+1)}. \tag{8}$$

Let us consider gamma distribution as a suitable prior for the Poisson distribution, with PDF:

$$f(p) = \frac{t^s}{\Gamma(s)} p^{s-1} e^{-t p}, \tag{9}$$

where the shape parameter  $s > 0$ , and the rate parameter  $t > 0$  with mean  $\mu = \frac{s}{t}$  under the proposed sampling plan. For the average probability of lot acceptance, the general expression used in Bayesian is (Hafeez et al. 2022a, 2022b):

$$P = \int_0^{\infty} L(p) f(p) dp. \tag{10}$$

After replacing equations (8) and (9) in equation (10) and then from simplification, we get:

$$P = \frac{t^s}{\Gamma(s)} \left[ \frac{\Gamma(s)}{(r g + t)^s} + r g \frac{\Gamma(s + 1)}{(r g (2i + 1) + t)^{s+1}} \right]; \tag{11}$$

$$P = \left(\frac{t}{rg + t}\right)^s + \frac{rgst^s}{(rg(2i + 1) + t)^{s+1}}. \quad (12)$$

$$P = \frac{4}{(rg\mu + 2)^2} + \frac{8rg\mu}{(rg\mu(2i + 1) + 2)^3}; \quad (15)$$

Upon substituting mean  $\mu = s/t$  that gives  $t = s/\mu$  in equation (12) and simplifying:

$$P = \left(\frac{s}{rg\mu + s}\right)^s + \frac{rg\mu s^{s+1}}{(rg\mu(2i + 1) + s)^{s+1}}. \quad (13)$$

$$P = \frac{27}{(rg\mu + 3)^3} + \frac{81rg\mu}{(rg\mu(2i + 1) + 3)^4}. \quad (16)$$

Now simplifying equation (13), for  $s = 1, 2, 3$ , we get:

$$P = \frac{1}{(rg\mu + 1)} + \frac{rg\mu}{(rg\mu(2i + 1) + 1)^2}; \quad (14)$$

To estimate the quality regions for BTSCGChSP, Newton's approximation is used in equations (14)-(16), where  $\mu$  is used as a point of control by reducing  $P$ . For the specified values of shape parameter  $s = 1, 2, 3$ ; the number of testers  $r = 2, 3, 4$  and number of preceding and succeeding lots  $i = j = 1, 2, 3, 4$  the average number of defectives are represented in Table 1.

TABLE 1. Average number of defectives for BTSCGChSP for specified values of  $s, r, i, j$  and  $P$

| $s$ | $r$ | $i, j$ | 0.99   | 0.95   | 0.90   | 0.75   | 0.50   | 0.25   | 0.10   | 0.05    | 0.01    |
|-----|-----|--------|--------|--------|--------|--------|--------|--------|--------|---------|---------|
| 1   | 2   | 1      | 0.0253 | 0.067  | 0.1096 | 0.2465 | 0.6247 | 1.7383 | 5.0721 | 10.6277 | 55.0722 |
|     |     | 2      | 0.0193 | 0.0526 | 0.0881 | 0.208  | 0.5546 | 1.5931 | 4.7121 | 9.9118  | 51.5116 |
|     |     | 3      | 0.0163 | 0.0458 | 0.0785 | 0.1927 | 0.5308 | 1.5493 | 4.6097 | 9.7114  | 50.5276 |
|     |     | 4      | 0.0145 | 0.0418 | 0.0729 | 0.1847 | 0.5198 | 1.5305 | 4.5668 | 9.6284  | 50.1221 |
|     | 3   | 1      | 0.0168 | 0.0447 | 0.0731 | 0.1643 | 0.4164 | 1.1588 | 3.3814 | 7.0852  | 36.7148 |
|     |     | 2      | 0.0128 | 0.0351 | 0.0587 | 0.1387 | 0.3697 | 1.062  | 3.1414 | 6.6079  | 34.3411 |
|     |     | 3      | 0.0109 | 0.0305 | 0.0523 | 0.1285 | 0.3539 | 1.0329 | 3.0731 | 6.4743  | 33.6851 |
|     |     | 4      | 0.0097 | 0.0279 | 0.0486 | 0.1231 | 0.3465 | 1.0203 | 3.0445 | 6.4189  | 33.4147 |
|     | 4   | 1      | 0.0126 | 0.0335 | 0.0548 | 0.1233 | 0.3123 | 0.8691 | 2.5361 | 5.3139  | 27.5361 |
|     |     | 2      | 0.0096 | 0.0263 | 0.044  | 0.104  | 0.2773 | 0.7965 | 2.356  | 4.9559  | 25.7558 |
|     |     | 3      | 0.0082 | 0.0229 | 0.0392 | 0.0964 | 0.2654 | 0.7747 | 2.3048 | 4.8557  | 25.2638 |
|     |     | 4      | 0.0073 | 0.0209 | 0.0365 | 0.0924 | 0.2599 | 0.7653 | 2.2834 | 4.8142  | 25.061  |
| 2   | 2   | 1      | 0.0284 | 0.0722 | 0.1141 | 0.2354 | 0.5123 | 1.1180 | 2.3187 | 3.6742  | 9.4004  |
|     |     | 2      | 0.0215 | 0.0561 | 0.0905 | 0.1957 | 0.4526 | 1.0361 | 2.2032 | 3.5219  | 9.0919  |
|     |     | 3      | 0.0182 | 0.0485 | 0.0797 | 0.1795 | 0.4333 | 1.0154 | 2.1785 | 3.4912  | 9.0343  |
|     |     | 4      | 0.0161 | 0.0439 | 0.0734 | 0.1711 | 0.425  | 1.0079 | 2.1702 | 3.4814  | 9.0163  |
|     | 3   | 1      | 0.0189 | 0.0482 | 0.0761 | 0.1569 | 0.3415 | 0.7453 | 1.5458 | 2.4494  | 6.2669  |
|     |     | 2      | 0.0143 | 0.0374 | 0.0603 | 0.1305 | 0.3017 | 0.6907 | 1.4688 | 2.3479  | 6.0613  |
|     |     | 3      | 0.0121 | 0.0323 | 0.0531 | 0.1197 | 0.2889 | 0.6769 | 1.4523 | 2.3275  | 6.0228  |
|     |     | 4      | 0.0107 | 0.0293 | 0.0489 | 0.1141 | 0.2834 | 0.672  | 1.4468 | 2.3209  | 6.0109  |
|     | 4   | 1      | 0.0142 | 0.0361 | 0.0571 | 0.1177 | 0.2561 | 0.559  | 1.1594 | 1.8371  | 4.7002  |
|     |     | 2      | 0.0107 | 0.0281 | 0.0452 | 0.0978 | 0.2263 | 0.518  | 1.1016 | 1.7609  | 4.546   |
|     |     | 3      | 0.0091 | 0.0243 | 0.0398 | 0.0898 | 0.2166 | 0.5077 | 1.0892 | 1.7456  | 4.5171  |
|     |     | 4      | 0.0081 | 0.022  | 0.0367 | 0.0856 | 0.2125 | 0.504  | 1.0851 | 1.7407  | 4.5082  |

|   |   |   |        |        |        |        |        |        |        |        |        |
|---|---|---|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 3 | 2 | 1 | 0.0298 | 0.0746 | 0.1164 | 0.2325 | 0.4798 | 0.9698 | 1.8209 | 2.6659 | 5.5848 |
|   |   | 2 | 0.0225 | 0.0577 | 0.0917 | 0.1919 | 0.4226 | 0.9037 | 1.7492 | 2.5881 | 5.4809 |
|   |   | 3 | 0.019  | 0.0497 | 0.0804 | 0.1752 | 0.4047 | 0.8892 | 1.7371 | 2.5765 | 5.4675 |
|   |   | 4 | 0.0168 | 0.0448 | 0.0737 | 0.1666 | 0.3976 | 0.8847 | 1.7339 | 2.5736 | 5.4643 |
| 3 | 1 | 1 | 0.0198 | 0.0498 | 0.0776 | 0.155  | 0.3199 | 0.6465 | 1.2139 | 1.7773 | 3.7232 |
|   |   | 2 | 0.015  | 0.0385 | 0.0611 | 0.1279 | 0.2817 | 0.6024 | 1.1661 | 1.7254 | 3.654  |
|   |   | 3 | 0.0127 | 0.0331 | 0.0536 | 0.1168 | 0.2698 | 0.5928 | 1.1581 | 1.7177 | 3.645  |
|   |   | 4 | 0.0112 | 0.0299 | 0.0492 | 0.1111 | 0.2651 | 0.5898 | 1.1559 | 1.7157 | 3.6429 |
| 4 | 1 | 1 | 0.0149 | 0.0373 | 0.0582 | 0.1162 | 0.2399 | 0.4849 | 0.9104 | 1.3329 | 2.7924 |
|   |   | 2 | 0.0113 | 0.0289 | 0.0459 | 0.0959 | 0.2113 | 0.4518 | 0.8746 | 1.2941 | 2.7405 |
|   |   | 3 | 0.0095 | 0.0249 | 0.0402 | 0.0876 | 0.2024 | 0.4446 | 0.8686 | 1.2883 | 2.7338 |
|   |   | 4 | 0.0084 | 0.0224 | 0.0369 | 0.0833 | 0.1988 | 0.4423 | 0.8669 | 1.2868 | 2.7322 |

CONSTRUCTION OF QUALITY REGIONS

*Probabilistic Quality Region (PQR)*

In PQR, the product is accepted with a maximum probability of 0.95 and minimum probability of 0.05, where 0.95 corresponds to AQL ( $1-\alpha$ ) and 0.05 corresponds to LQL ( $\beta$ ). In other words, PQR ( $R_1$ ) is

exactly the conventional setting of  $AQL = \mu_1$  and  $LQL = \mu_2$ . From the specified design parameters, this region considers  $\alpha = \beta = 0.05$  and PQR lies between two points  $\mu_1 \leq \mu \leq \mu_2$  as represented in Figure 3. Hence the range of PQR is  $R_1 = \mu_2 - \mu_1$  and the values of  $R_1$  can be estimated from Table 2.

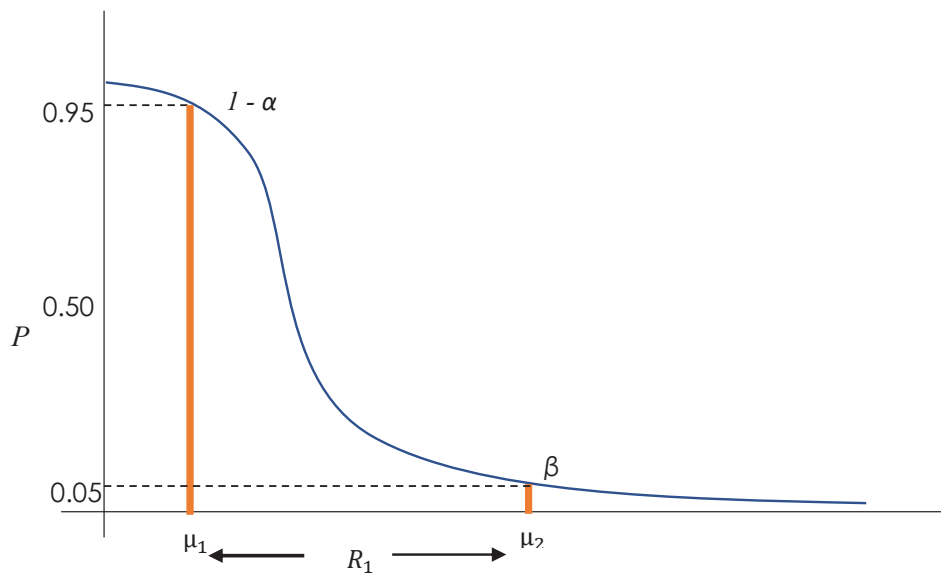


FIGURE 3. OC curve with pair of coordinates for PQR

*Quality Decision Region (QDR)*

In QDR, the product is accepted with a maximum and minimum probability 0.95 and 0.25, respectively. Where 0.95 corresponds to AQL ( $1-\alpha$ ) and 0.25 corresponds to LQL ( $\beta$ ). This explains that QDR ( $R_2$ ) is

exactly the conventional setting of  $AQL = \mu_1$  and  $LQL = \mu_\beta$ . This region considers consumer's risk  $\alpha = 0.05$  and producer's risk  $\beta = 0.25$ . QDR lies between  $\mu_1 \leq \mu \leq \mu_\beta$  and the range of QDR is  $R_2 = \mu_\beta - \mu_1$  can be estimated from Table 2.

*Limiting Quality Region (LQR)*

The product is accepted in LQR with a maximum probability of 0.75 and minimum probability of 0.05, where 0.75 relates to AQL ( $1-\alpha$ ) and 0.05 relates to LQL ( $\beta$ ). In other words, LQR ( $R_3$ ) is exactly the conventional setting of AQL =  $\mu_\alpha$  and LQL =  $\mu_2$ . From prespecified design parameters, this region considers  $\alpha = 0.25$  and  $\beta = 0.05$ . Thus, LQR lies between  $\mu_\alpha \leq \mu \leq \mu_2$  and the range of LQR is  $R_3 = \mu_2 - \mu_\alpha$  can be estimated from Table 2.

*Indifference Quality Region (IQR)*

The product is accepted in IQR with a maximum probability of 0.5 and a minimum probability of 0.05. These probabilities are corresponding to AQL ( $1-\alpha$ ) and LQL ( $\beta$ ). It is clear that, IQR ( $R_4$ ) is exactly the conventional setting of AQL =  $\mu_*$  and LQL =  $\mu_2$ . This region considers producer's risk  $\alpha = 0.5$  and consumer's risk  $\beta = 0.05$  from prespecified design parameters values. IQR lies between two points  $\mu_* \leq \mu \leq \mu_2$  and the range of IQR  $R_4 = \mu_2 - \mu_*$  can be estimated from Table 2.

TABLE 2. For specified  $s, r, i$  and  $j$  values of QDR, PQR, LQR, IQR and operating ratios

| $s$ | $r$ | $i, j$ | $gR_1$  | $gR_2$ | $gR_3$  | $gR_4$  | $T$    | $T_1$  | $T_2$  |        |
|-----|-----|--------|---------|--------|---------|---------|--------|--------|--------|--------|
| 1   | 2   | 1      | 10.5607 | 1.6713 | 10.3812 | 10.0031 | 6.3191 | 1.0173 | 1.0557 |        |
|     |     | 2      | 9.8592  | 1.5404 | 9.7038  | 9.3572  | 6.4002 | 1.016  | 1.0536 |        |
|     |     | 3      | 9.6656  | 1.5035 | 9.5187  | 9.1807  | 6.4288 | 1.0154 | 1.0528 |        |
|     |     | 4      | 9.5865  | 1.4887 | 9.4437  | 9.1086  | 6.4396 | 1.0151 | 1.0525 |        |
|     | 3   | 1      | 7.0405  | 1.1142 | 6.9208  | 6.6688  | 6.319  | 1.0173 | 1.0557 |        |
|     |     | 2      | 6.5728  | 1.027  | 6.4692  | 6.2382  | 6.4001 | 1.016  | 1.0536 |        |
|     |     | 3      | 6.4438  | 1.0023 | 6.3458  | 6.1205  | 6.4289 | 1.0154 | 1.0528 |        |
|     |     | 4      | 6.391   | 0.9924 | 6.2958  | 6.0724  | 6.4398 | 1.0151 | 1.0525 |        |
|     | 4   | 1      | 5.2804  | 0.8356 | 5.1906  | 5.0016  | 6.3192 | 1.0173 | 1.0557 |        |
|     |     | 2      | 4.9296  | 0.7702 | 4.8519  | 4.6786  | 6.4002 | 1.016  | 1.0537 |        |
|     |     | 3      | 4.8328  | 0.7518 | 4.7594  | 4.5903  | 6.4287 | 1.0154 | 1.0528 |        |
|     |     | 4      | 4.7933  | 0.7443 | 4.7218  | 4.5543  | 6.4396 | 1.0151 | 1.0525 |        |
|     | 2   | 2      | 1       | 3.6019 | 1.0458  | 3.4388  | 3.1619 | 3.4442 | 1.0474 | 1.1392 |
|     |     |        | 2       | 3.4657 | 0.98    | 3.3262  | 3.0693 | 3.5366 | 1.042  | 1.1292 |
|     |     |        | 3       | 3.4427 | 0.9669  | 3.3117  | 3.058  | 3.5606 | 1.0396 | 1.1258 |
|     |     |        | 4       | 3.4375 | 0.964   | 3.3103  | 3.0564 | 3.5658 | 1.0384 | 1.1247 |
| 3   |     | 1      | 2.4013  | 0.6972 | 2.2925  | 2.1079  | 3.4442 | 1.0474 | 1.1392 |        |
|     |     | 2      | 2.3105  | 0.6533 | 2.2174  | 2.0462  | 3.5366 | 1.042  | 1.1292 |        |
|     |     | 3      | 2.2951  | 0.6446 | 2.2078  | 2.0386  | 3.5605 | 1.0396 | 1.1258 |        |
|     |     | 4      | 2.2917  | 0.6427 | 2.2069  | 2.0376  | 3.5657 | 1.0384 | 1.1247 |        |
| 4   |     | 1      | 1.801   | 0.5229 | 1.7194  | 1.5809  | 3.4442 | 1.0474 | 1.1392 |        |
|     |     | 2      | 1.7329  | 0.4899 | 1.6631  | 1.5346  | 3.5369 | 1.0419 | 1.1292 |        |
|     |     | 3      | 1.7214  | 0.4834 | 1.6559  | 1.529   | 3.5606 | 1.0396 | 1.1258 |        |
|     |     | 4      | 1.7187  | 0.482  | 1.6551  | 1.5282  | 3.5658 | 1.0384 | 1.1247 |        |
| 3   |     | 2      | 1       | 2.5912 | 0.8952  | 2.4334  | 2.1861 | 2.8947 | 1.0649 | 1.1853 |
|     |     |        | 2       | 2.5304 | 0.8459  | 2.3963  | 2.1655 | 2.9912 | 1.056  | 1.1685 |
|     |     |        | 3       | 2.5268 | 0.8395  | 2.4013  | 2.1718 | 3.0099 | 1.0523 | 1.1635 |
|     |     |        | 4       | 2.5287 | 0.8398  | 2.407   | 2.176  | 3.011  | 1.0506 | 1.1621 |
|     | 3   | 1      | 1.7275  | 0.5968 | 1.6223  | 1.4574  | 2.8948 | 1.0649 | 1.1853 |        |
|     |     | 2      | 1.6869  | 0.564  | 1.5975  | 1.4437  | 2.9912 | 1.056  | 1.1685 |        |
|     |     | 3      | 1.6845  | 0.5597 | 1.6008  | 1.4478  | 3.0099 | 1.0523 | 1.1635 |        |
|     |     | 4      | 1.6858  | 0.5599 | 1.6046  | 1.4506  | 3.011  | 1.0506 | 1.1621 |        |
|     | 4   | 1      | 1.2956  | 0.4476 | 1.2167  | 1.093   | 2.8947 | 1.0649 | 1.1853 |        |
|     |     | 2      | 1.2652  | 0.4229 | 1.1981  | 1.0828  | 2.9915 | 1.056  | 1.1685 |        |
|     |     | 3      | 1.2634  | 0.4197 | 1.2007  | 1.0859  | 3.0102 | 1.0522 | 1.1635 |        |
|     |     | 4      | 1.2643  | 0.4199 | 1.2035  | 1.088   | 3.011  | 1.0506 | 1.1621 |        |

SELECTION OF SAMPLING PLANS

For any given values of PQR ( $R_1$ ), QDR ( $R_2$ ), LQR ( $R_3$ ), IQR ( $R_4$ ), we can find the operating ratio  $T = \frac{R_1}{R_2}$ ,  $T_1 = \frac{R_1}{R_3}$  and  $T_2 = \frac{R_1}{R_4}$ . Find the value which is approximately equal to the required operating ratio under the column of  $T$ ,  $T_1$  and  $T_2$  in Table 2. From this operating ratio, the corresponding design parameters  $s$ ,  $r$ , and  $i$  can be determined from Table 2. By using these design parameters for BTSCGChSP, the values of quality regions; PQR ( $R_1$ ), QDR ( $R_2$ ), LQR ( $R_3$ ), IQR ( $R_4$ ) can be assessed from Table 2 and required AQL and LQL can be obtained from Table 1.

NUMERICAL EXAMPLES

*For Specified PQR and QDR*

Let a manufacturer demanded the same AQL;  $\mu_1 = 0.01$  for both PQR and QDR with an operating ratio of 3.45. From Table 2, the closest value to the specified ratio is  $T = 3.4442$  at  $s = 2$ ,  $i = 1$  and different values of  $r$ , the values of  $T$  that are approximately equal to the specified ratio are presented in Table 3. Here, the manufacturer has three options, but he needs to select one value that can reduce inspection cost and inspection time.

TABLE 3. Located values for a specified operating ratio of PQR and QDR, at  $s = 2$  and  $i = 1$

| $T$    | $r$ | $g\mu_1$ | $g = g\mu_1/\mu_1$ |
|--------|-----|----------|--------------------|
| 3.4442 | 2   | 0.0722   | 8                  |
| 3.4442 | 3   | 0.0482   | 5                  |
| 3.4442 | 4   | 0.0361   | 4                  |

From Table 3, the minimum sample size that takes the short time for inspection is at  $r = 4$  and  $g = 4$ . Hence from Table 2, for this operating ratio, the range of PQR and QDR are  $gR_1 = 1.8010$  and  $gR_2 = 0.5229$ , respectively. Hence, with minimum sample size for specified operating ratio, the required design parameters for BTSCGChSP are  $s = 2$ ,  $g = 4$ ,  $r = 4$  and  $i = 1$ , with  $\mu_1 = 0.0090$ ,  $\mu_\beta = 0.1398$ , and  $\mu_2 = 0.4593$ , from Table 1. The range of PQR and QDR for the average proportion of defectives are  $R_1 = 0.4503$  and  $R_2 = 0.1308$ , respectively.

*For Specified PQR and LQR*

Suppose a manufacturer needed the same LQL;  $\mu_2 = 0.95$  for both PQR and LQR with an operating ratio of 1.05. From Table 2, for  $s = 3$ ,  $i = 4$  and different values of  $r$ , the values of  $T_1$  that is approximately equal to the specified ratio are presented in Table 4. Here the manufacturer has three options, but he needs to select one value that can reduce inspection cost and inspection time.

TABLE 4. Located values for a specified operating ratio of PQR and LQR, at  $s = 3$  and  $i = 4$

| $T_1$  | $r$ | $g\mu_2$ | $g = g\mu_2/\mu_2$ |
|--------|-----|----------|--------------------|
| 1.0506 | 2   | 2.5736   | 3                  |
| 1.0506 | 3   | 1.7157   | 2                  |
| 1.0506 | 4   | 1.2858   | 2                  |

From Table 4, the minimum sample size that takes the short time for inspection is at  $r = 3$  and  $g = 2$ . Hence from Table 2, for this operating ratio, the range of PQR

and LQR is  $gR_1 = 1.6858$  and  $gR_3 = 1.6046$ , respectively. Hence, with minimum sample size for specified operating ratio, the required design parameters for BTSCGChSP



are  $s = 3, g = 2, r = 3$  and  $i = 4$ , with  $\mu_1 = 0.01495, \mu_a = 0.05555$ , and  $\mu_2 = 0.8579$ , from Table 1 and the range of PQR and LQR for the average proportion of defectives are  $R_1 = 0.8429$  and  $R_3 = 0.8023$ , respectively.

*For Specified PQR and IQR*

Let Suppose a manufacturer required the same LQL;  $\mu_2 = 0.95$  for both PQR and IQR with an operating ratio of 1.15. From Table 2, for  $s = 2, i = 1$  and different values of  $r$ , the values of  $T_2$  that are close to the specified ratio are presented in Table 5.

TABLE 5. Located values for a specified operating ratio of PQR and IQR, at  $s = 2$  and  $i = 1$

| $T_2$  | $r$ | $g\mu_2$ | $g = g\mu_2/\mu_2$ |
|--------|-----|----------|--------------------|
| 1.1392 | 2   | 3.6742   | 4                  |
| 1.1392 | 3   | 2.4494   | 3                  |
| 1.1392 | 4   | 1.8371   | 2                  |

From Table 5, the minimum sample size that takes the short time for inspection is at  $r = 4$  and  $g = 2$ . Hence from Table 2, for this operating ratio, the range of PQR and IQR are  $gR_1 = 1.801$  and  $gR_4 = 1.5809$ , respectively. Hence, for specified operating ratio, the required design parameters for BTSCGChSP are  $s = 2, g = 2, r = 4$  and  $i = 1$ , with  $\mu_1 = 0.0181, \mu_a = 0.1281$ , and  $\mu_2 = 0.9187$ , from

Table 1 and the range of PQR and IQR for the average proportion of defectives are  $R_1 = 0.9005$  and  $R_4 = 0.7905$ , respectively.

GRAPHICAL REPRESENTATION

Consider shape parameter  $s = 2$ , preceding and succeeding lots  $i = j = 3$ , then, for the different number of testers  $r = 2, 3, 4$ , the OC curves are displayed in Figure 4.

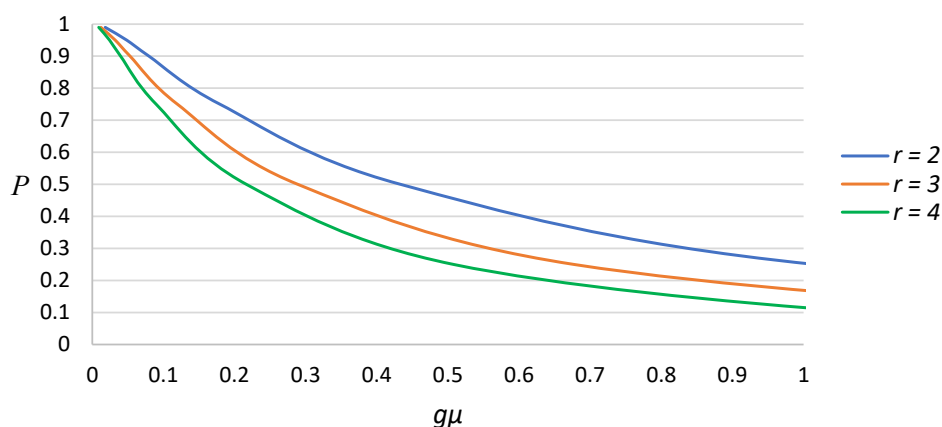


FIGURE 4. OC curves for  $r = 2, 3, 4$

When the number of testers  $r = 3$ , preceding and succeeding lots  $i = j = 3$  are considered, then OC curves for different values of shape parameters  $s = 1, 2, 3$  are shown in Figure 5.

From Figures 4 and 5, we can conclude that the ideal OC curve can be achieved by increasing the value of the shape parameter and the number of testers.

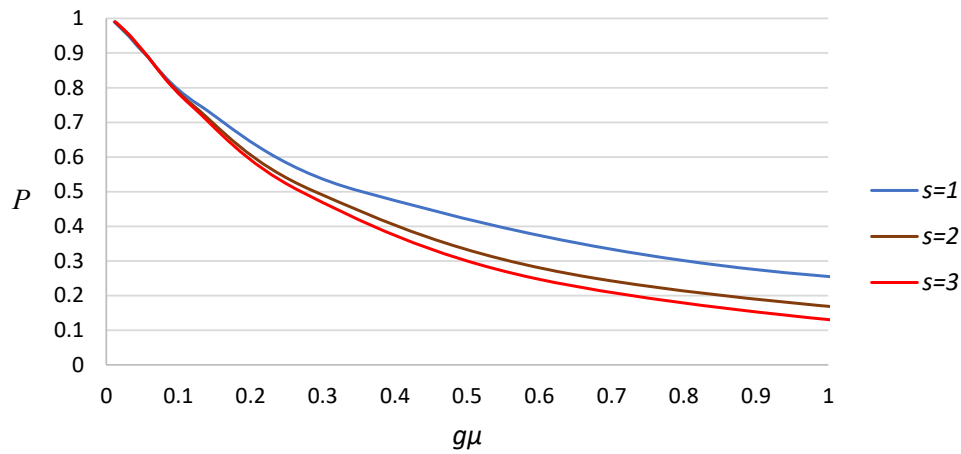


FIGURE 5. OC curves for  $s = 1, 2, 3$

For comparison purposes for the same values of design parameters, BTSCGChSP is compared with the existing BGChSP given by Hafeez et al. (2022a). For the

specified design parameters,  $s = 2, r = 3$  and  $i = j = 2$ , the average number of defectives is plotted against the average probability of acceptance in Figure 6.

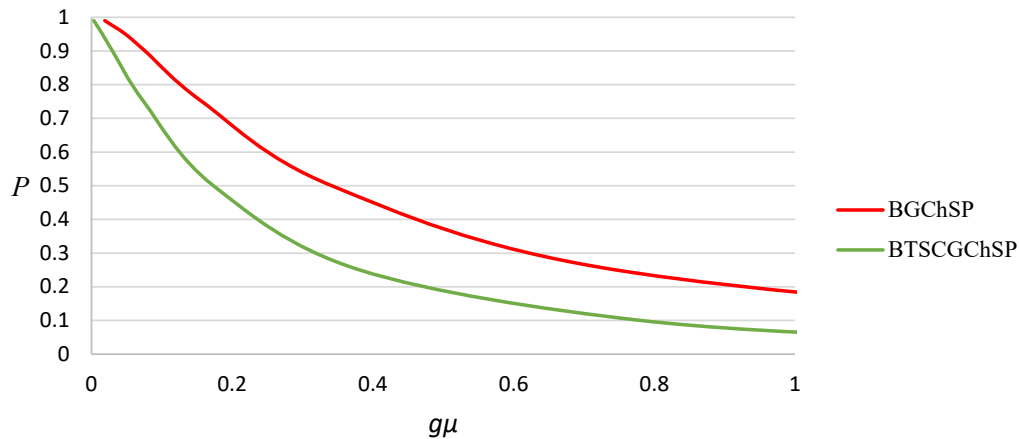


FIGURE 6. OC curves for BGChSP and BTSCGChSP

From Figure 6, it can be concluded that the BTSCGChSP OC curve is more ideal than the existing BGChSP. For both plans, if the values of all design parameters are the same, BTSCGChSP gives a smaller number of defectives than BGChSP.

APPLICATION ON REAL DATA SET

For the application of the proposed plan, a data set is taken from Walpole et al. (2007), where large steel plates are being manufactured. Every hour a sample size of 50 is collected, and the number of defectives is noted in each

sample. From manufacturing lots 20 samples are selected in which number of defectives are found; 4, 2, 1, 3, 0, 1, 2, 2, 3, 1, 4, 5, 3, 2, 2, 4, 3, 2, 1, and 3. Based on the Bayesian information criterion (BIC), the number of defectives follow a gamma distribution with an estimated value of BIC is 279.6998 and mean square error (MSE) is 0.8361. By maximum likelihood estimates, the shape parameters of the distribution  $s = 3.0016$  and  $t = 1.1428$ , are estimated. Suppose the design parameters, preceding and succeeding lots  $i = j = 2$  and the available number of testers  $r = 5$ . Hence the sample size  $n = 50$  is divided into  $g = 10$  groups, i.e.,  $n = r * g = 5 * 10 = 50$ .

When the experimenter set up the BTSCGChSP plan according to the above-mentioned specifications, for PQR consider consumer's risk 0.05 and producer's risk 0.05. Now use equation (16) because  $s = 3$  and follow the same procedure as for Table 1. Then, the estimated values for AQL ( $\mu_1 = 0.00234$ ), LQL ( $\mu_2 = 0.10352$ ), and the range of PQR is  $R_1 = \mu_2 - \mu_1 = 0.10119$ .

Now, for QDR, the approximate value for AQL is  $\mu_1 = 0.00234$ , LQL is  $\mu_\beta = 0.03614$  and the range of IQR is  $R_2 = 0.03380$ . Based on PQR and QDR, the operating ratio  $T = \frac{R_1}{R_2} = 2.993$ .

Similarly, for LQR, the estimated values for AQL is  $\mu_\alpha = 0.00765$ , LQL is  $\mu_2 = 0.10352$  and the range of IQR is  $R_3 = 0.09587$ . Based on PQR and LQR, the operating ratio  $T_1 = \frac{R_1}{R_3} = 1.056$ .

Likewise, for IQR, the estimated values for AQL is  $\mu_* = 0.01689$ , LQL is  $\mu_2 = 0.10352$  and the range of IQR is  $R_4 = 0.08664$ . Based on PQR and IQR, the operating ratio  $T_2 = \frac{R_1}{R_4} = 1.168$ .

#### CONCLUSION

The presented work in this paper is limited to BTSCGChSP and four quality regions are estimated for the specified producer's and consumer's risks. This plan gives protection to both producer and consumer. The proposed plan considers preceding and succeeding lots at the same time. Many electronic components can be evaluated by using the proposed plan, such as transport electronics systems, global positioning systems, wireless systems, and computer-supported and integrated manufacturing systems. Many other quality and reliability characteristics with other distributions can be explored for this plan in the future.

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