\[ v = \frac{RT}{p} = \frac{0.082054(400)}{2.5} = 13.12864 \]

\[ f(v) = \left( p + \frac{a}{v^2} \right)(v - b) - RT \]

\[ f(v) = \left( 2.5 + \frac{12.02}{v^2} \right)(v - 0.08407) - 0.082054(400) \]

\[ v_{i+1} = v_i - \frac{\left( p + \frac{a}{v_i^2} \right)(v_i - b) - RT}{\left( p + \frac{a}{v_i^2} \right) - (v_i - b) \frac{2a}{v_i^3}} \]

\[ = 12.84073 \]
Fig P8.2

\[
\ln \frac{1 + R(1 - X_{Af})}{R(1 - X_{Af})} = \frac{R + 1}{R[1 + R(1 - X_{Af})]}
\]

\[
f(R) = \ln \frac{1 + R(1 - X_{Af})}{R(1 - X_{Af})} - \frac{R + 1}{R[1 + R(1 - X_{Af})]} = 0
\]

\[X_{Af} = 0.95\]
function \( f = V(x) \)
\[
f = 20 \cdot (\text{sing}(x, 0, 1) - \text{sing}(x, 5, 1)) - 15 \cdot \text{sing}(x, 8, 0) - 57;
\]

The singularity function can be set up as

function \( s = \text{sing}(x, a, n) \)
if \( x > a \)
    \( s = (x - a)^n \);
else
    \( s = 0 \);
end

```matlab
>> x=fzero(@V,5)
x =
   2.8500
```
function \( f = \text{Mx}(x) \)
\[
\begin{align*}
  f &= -10 \times (\text{sing}(x,0,2) - \text{sing}(x,5,2)) + 15 \times \text{sing}(x,8,1) \\
  &\quad + 150 \times \text{sing}(x,7,0) + 57 \times x;
\end{align*}
\]

In addition, the singularity function can be set up as

function \( s = \text{sing}(x, a, n) \)

\[
\begin{align*}
  \text{if } x > a \\
  s &= (x - a)^n; \\
\text{else} \\
  s &= 0;
\end{align*}
\]

\[
>>
\begin{align*}
  x &= \text{fzero}(\text{@Mx}, 5) \\
  x &= 5.8140
\end{align*}
\]
function $f = \texttt{duydx}(x)$
\[ f = -\frac{10}{3}(\text{sing}(x, 0, 3) - \text{sing}(x, 5, 3)) + 7.5\text{sing}(x, 8, 2) + 150\text{sing}(x, 7, 1) + \frac{57}{2}x^2 - 238.25; \]

In addition, the singularity function can be set up as

function $s = \text{sing}(x, a, n)$
if $x > a$
    \[ s = (x - a)^n; \]
else
    \[ s = 0; \]
end

$$x = 3.9$$

>> $x = \text{fzero}(@\texttt{duydx}, 5)$

$x = 3.9357$
Fig P8.36

\[ f(\theta_0) = (\tan \theta_0) x - \frac{g}{2v_0^2 \cos^2 \theta_0} x^2 + y_0 - y \]

\[ f(\theta_0) = 0 = 35 \tan \left( \frac{\pi \theta_0}{180} \right) - \frac{15.0215625}{\cos^2 \left( \frac{\pi \theta_0}{180} \right)} + 1 \]

\[ \theta = 27 \]
Newton’s 2\textsuperscript{nd} law of Motion

- States that “the time rate change of momentum of a body is equal to the resulting force acting on it.”
- The model is formulated as
  \[ F = m a \]
  - \( F \): net force acting on the body (N)
  - \( m \): mass of the object (kg)
  - \( a \): its acceleration (m/s\(^2\))
• Formulation of Newton’s 2\textsuperscript{nd} law has several characteristics that are typical of mathematical models of the physical world:
  – It describes a natural process or system in mathematical terms
  – It represents an idealization and simplification of reality
  – Finally, it yields reproducible results, consequently, can be used for predictive purposes.
• Some mathematical models of physical phenomena may be much more complex.

• Complex models may not be solved exactly or require more sophisticated mathematical techniques than simple algebra for their solution.
  
  - Example, modeling of a falling parachutist:

\[
\frac{dv}{dt} = \frac{F}{m}
\]
\[
\frac{dv}{dt} = \frac{F}{m}
\]

\[
F = F_D + F_U
\]

\[
F_D = mg
\]

\[
F_U = -cv
\]

\[
\frac{dv}{dt} = \frac{mg - cv}{m}
\]

\[
\frac{dv}{dt} = g - \frac{c}{m}v
\]
• This is a differential equation and is written in terms of the differential rate of change $\frac{dv}{dt}$ of the variable that we are interested in predicting.

• If the parachutist is initially at rest ($v = 0$ at $t = 0$), using calculus

\[
\frac{dv}{dt} = g - \frac{c}{m}v
\]

\[
\frac{v(t_{i+1}) - v(t_i)}{t_{i+1} - t_i} = g - \frac{c}{m}v
\]

\[
v(t) = \frac{gm}{c} \left( 1 - e^{-\left(\frac{c}{m}\right)t} \right)
\]