Chapter 6

Dynamic

6.1 Introduction

In the last chapters, we have discussed the static analysis of structures. Static analysis holds when the loads are slowly applied. When the loads are suddenly applied, or when the loads are of a variable nature, the mass and acceleration effects come into the picture. If a solid body, such as an engineering structure, is deformed elastically and suddenly released, it tends to vibrate about its equilibrium position. This periodic motion is due to the restoring strain energy in the element and is called free vibration. The number of cycles per unit time is called the frequency. The maximum displacement from the equilibrium position is the amplitude.

6.2 Case Study/Engineering Application

Figure 6.1 and 6.2 below show a failed shaft of an air conditioning blower. The blower operating at 870 rpm. Observation from Figure 6.1 at the colour and surface texture shows brittle fracture and ductile fracture. The arrow in Figure 6.1 shows torque direction. The deformation in the red circle suggest that after britte fracture, it continues with ductile fracture. Early conclusion from observation suggest that the shaft failed because of fatigue and continues with plastic shear deformation due to the electric motor torque and rotational energy from the blower. To study the real factor contributing to the shaft failure, four analysis were carry out which are hardness test, ultrasonik analysis, static analysis and modal analysis.

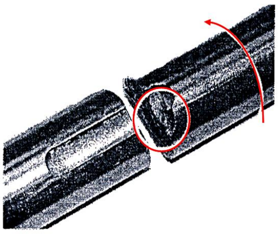


Figure 1:

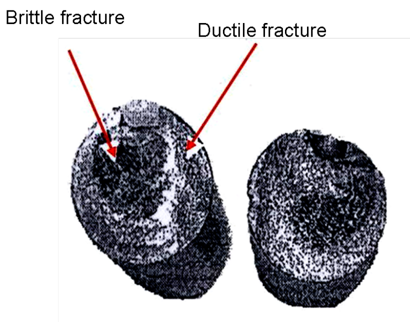


Figure 2:

6.2.1 Ultrasonic Analysis

Ultrasonic analysis was carried out to detect microcrack in the shaft. The measurement is done laterally along the shaft. Figure 6.3 below shows an example of the measurement result. All the result show that the first peak is 35 mm which prove that there are no microcrack present

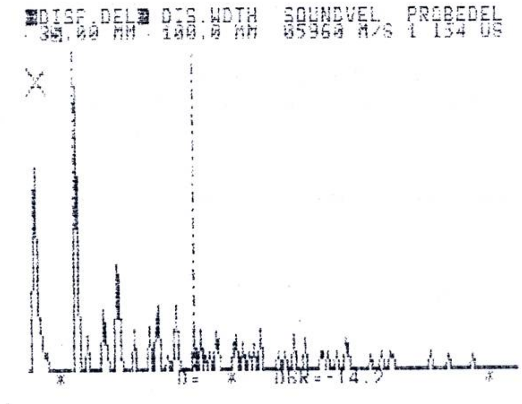


Figure 3:

6.2.2 Hardness Test

This test is carried out to compare the strength of the failed shaft with the strength of the new shaft. The average hardness of the failed shaft is 79.10 and the hardness range is between 71.50 and 83.50. However, for the new shaft, the hardness is between 87.60 and 91.70 with average value of 88.15. From the experiment, the hardness of the failed shaft is lower than the new shaft.

6.2.3 Static Analysis

The purpose of this analysis is to determine the shaft deflection due to the blower's weight.. The shaft's boundary condition is shown in Figure 6.4 below.

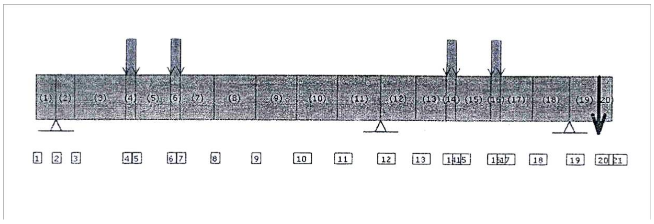


Figure 4:

The load due to the blower's weight at element 4, 6, 14 and 16 is -104.48 N while at element 20 is -10 N.

Input data for BEAM program:

```
 << BEAM ANALYSIS >>
Shaft Analysis of Air Conditioning Blower
          NN            NE           NM        NDIM           NEN          NDN
          17            16            1           1             2            2
          ND            NL         NMPC
           3             5            0
NODE#        X-COORD
           1             0
           2            70
           3           140
           4         344.7
           5         506.7
           6           650
           7           800
           8           950
           9          1100
          10          1260
          11          1390
          12        1519.8
          13        1681.9
          14          1820
          15          1955
          16          2060
          17        2109.8
         EL#            N1           N2        MAT#  Mom_Inertia
           1             1            2           1     7.37E+04
           2             2            3           1     7.37E+04
           3             3            4           1     7.37E+04
           4             4            5           1     7.37E+04
           5             5            6           1     7.37E+04
           6             6            7           1     7.37E+04
           7             7            8           1     7.37E+04
           8             8            9           1     7.37E+04
           9             9           10           1     7.37E+04
          10            10           11           1     7.37E+04
          11            11           12           1     7.37E+04
          12            12           13           1     7.37E+04
          13            13           14           1     7.37E+04
          14            14           15           1     7.37E+04
          15            15           16           1     7.37E+04
          16            16           17           1     7.37E+04
        DOF#        Displ.
           3             0
          19             0
          29             0
        DOF#          LOAD
           7       -104.48
           9       -104.48
          23       -104.48
          25       -104.48
          31           -10
        MAT#             E
           1        200000
B1            I             B2          J           B3
```

Result from BEAM program:

Results from BEAM Program

Shaft Analysis of Air Conditioning Blower

Node#	Displ.	Rotation
1	0.052587	-0.00075
2	-7.2E-12	-0.00075
3	-0.05212	-0.00073
4	-0.17802	-0.00044
5	-0.21918	-6.2E-05
6	-0.20595	0.000226
7	-0.15764	0.000396
8	-0.09393	0.000431
9	-0.03496	0.000333
10	-1.4E-11	7.91E-05
11	-0.00345	-0.0001
12	-0.01956	-0.00012
13	-0.02891	1.39E-05
14	-0.01881	0.000119
15	-4.2E-12	0.000147
16	0.015167	0.000143
17	0.022298	0.000143

DOF#	Reaction
3	120.9633
19	236.6904
29	70.26625

Figure 6.5 shows the deflection of the shaft.

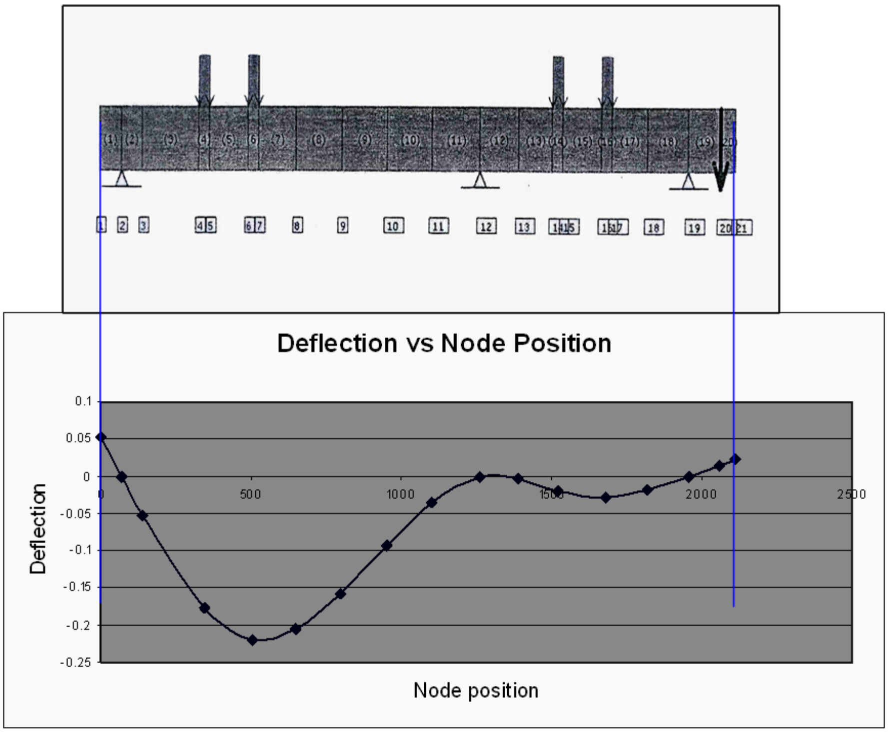


Figure 5:

6.2.4 Modal Analysis

The purpose of the modal analysis is to obtain the lowest eigenvalue and mode shape for the shaft with blower. This analysis will not consider unbalance condition.

Input data for BeamKM program:

```
PROGRAM  BeamKM  << for Stiffness and Mass Matrices
Shaft Analysis of Air Conditioning Blower
      NN            NE           NM     NDIM           NEN          NDN
      17            16            1        1             2            2
      ND            NL         NMPC
       3             0            0
NODE#       X-COORD
       1             0
       2            70
       3           140
       4         344.7
       5         506.7
       6           650
       7           800
       8           950
       9          1100
      10          1260
      11          1390
      12        1519.8
      13        1681.9
      14          1820
      15          1955
      16          2060
      17        2109.8
    ELEM#           N1           N2     MAT#        Mom_In         Area
       1             1            2        1      7.37E+04      167.416
       2             2            3        1      7.37E+04      167.416
       3             3            4        1      7.37E+04      167.416
       4             4            5        1      7.37E+04      167.416
       5             5            6        1      7.37E+04      167.416
       6             6            7        1      7.37E+04      167.416
       7             7            8        1      7.37E+04      167.416
       8             8            9        1      7.37E+04      167.416
       9             9           10        1      7.37E+04      167.416
      10            10           11        1      7.37E+04      167.416
      11            11           12        1      7.37E+04      167.416
      12            12           13        1      7.37E+04      167.416
      13            13           14        1      7.37E+04      167.416
      14            14           15        1      7.37E+04      167.416
      15            15           16        1      7.37E+04      167.416
      16            16           17        1      7.37E+04      167.416
     DOF#       Displ.
       3             0
      19             0
      29             0
     MAT#            E          Rho   << Properties E and MassDensity
       1      2.00E+05     8.00E-07
SPRING SUPPORTS                       <DOF#=0 Exits this
     DOF#  Spring Const
       0
LUMPED MASSES                         <DOF#=0 Exits this
     DOF#  Lumped Mass
       7    1.0860E+00
       9    1.0860E+00
      23    1.0860E+00
      25    1.0860E+00
      31    1.0390E-01
```

Result from BeamKM program as input for Jacobi program:

Results from Program BeamKM
Shaft Analysis of Air Conditioning Blower

NumDOF	BandWidth
34	4

Banded Stiffness Matrix

515685.1	18048980	-515685	18048980
8.42E+08	-1.8E+07	4.21E+08	0
1.75E+13	0	-515685	18048980
1.68E+09	-1.8E+07	4.21E+08	0
536306.9	-1.6E+07	-20621.7	2110635
1.13E+09	-2110635	1.44E+08	0
62225.6	1259279	-41603.9	3369913
6.52E+08	-3369913	1.82E+08	0
101713	936903.2	-60109.1	4306816
7.75E+08	-4306816	2.06E+08	0
112518	-376150	-52408.9	3930667
8.05E+08	-3930667	1.97E+08	0
104817.8	0	-52408.9	3930667
7.86E+08	-3930667	1.97E+08	0
104817.8	0	-52408.9	3930667
7.86E+08	-3930667	1.97E+08	0
95592.48	-475979	-43183.6	3454688
7.62E+08	-3454688	1.84E+08	0
1.75E+13	1778449	-80509.8	5233136
8.22E+08	-5233136	2.27E+08	0
161392.3	16139.19	-80882.5	5249275
9.08E+08	-5249275	2.27E+08	0
122409.4	-1883519	-41526.9	3365757
8.18E+08	-3365757	1.82E+08	0
108684.9	1271503	-67158	4637259
7.91E+08	-4637259	2.13E+08	0
139049.5	215415.6	-71891.5	4852675
8.64E+08	-4852675	2.18E+08	0
1.75E+13	3169094	-152796	8021769
9.98E+08	-8021769	2.81E+08	0
1584953	27638946	-1432157	35660715
1.75E+09	-3.6E+07	5.92E+08	0
1432157	-3.6E+07	0	0
1.18E+09	0	0	0

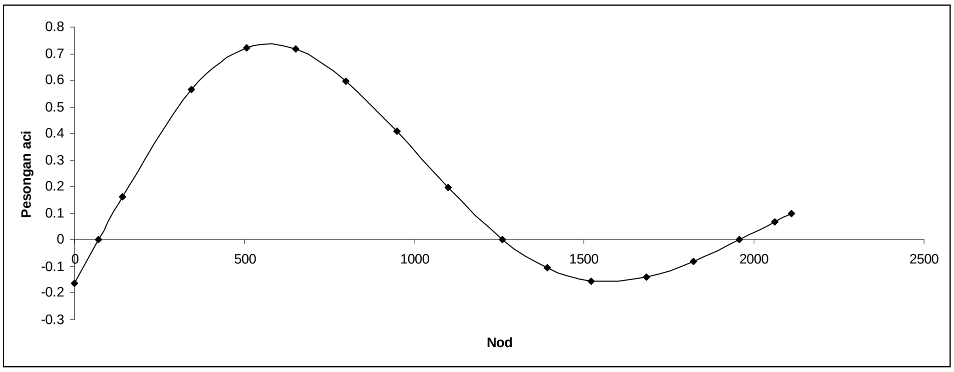
Banded Mass Matrix

0.003482	0.034376	0.001205	-0.02031
0.437513	0.020313	-0.32813	0
0.006964	0	0.001205	-0.02031
0.875025	0.020313	-0.32813	0
0.013665	0.259588	0.003525	-0.17371
11.37833	0.173706	-8.20562	0
1.104242	-0.10985	0.00279	-0.1088
16.36384	0.108795	-4.06727	0
1.101188	-0.04005	0.002468	-0.08513
9.176509	0.085128	-2.81512	0
0.014591	0.013786	0.002583	-0.09327
8.058457	0.093274	-3.22873	0
0.014924	0	0.002583	-0.09327
8.60994	0.093274	-3.22873	0
0.014924	0	0.002583	-0.09327
8.60994	0.093274	-3.22873	0
0.015421	0.021748	0.002755	-0.10613
9.529609	0.106125	-3.91848	0
0.014426	-0.06103	0.002239	-0.07006
8.027015	0.070059	-2.10178	0
0.012924	-0.00036	0.002235	-0.06984
5.591838	0.069844	-2.0921	0
1.100521	0.066145	0.002791	-0.10893
8.222533	0.10893	-4.0748	0
1.100934	-0.05055	0.002378	-0.07906
8.792592	0.079062	-2.51964	0
0.013586	-0.00594	0.002325	-0.07555
6.497844	0.075552	-2.35374	0
0.011939	-0.05051	0.001808	-0.0457
4.614928	0.045704	-1.10745	0
0.111601	-0.05995	0.000858	-0.01028
1.634142	0.010281	-0.11815	0
0.002477	-0.0174	0	0
0.157538	0	0	0

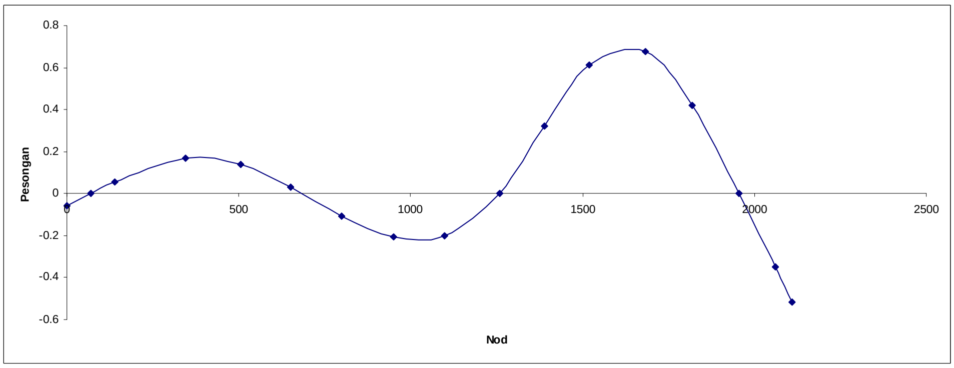
Result from Jacobi program:

```
Eigenval#           1
Eigenval =   366.5143
Omega  =     19.14456
Freq Hz =    3.046954
Eigenvector
   -0.16309   0.00233    2.64E-11   0.00233    0.161824   0.002276   0.563598   0.001496   0.721842
   0.000429   0.716905   -0.00045   0.598357   -0.00108   0.409454   -0.00139   0.197481   -0.00138
   1.37E-11   -0.00102   -0.10461   -0.00059   -0.15324   -0.00016   -0.14092   0.000294   -0.08103
   0.000545   -2.7E-11   0.000628   0.066061    0.00063   0.097412    0.00063

Eigenval#           2
Eigenval =    1600.85
Omega  =     40.01062
Freq Hz =    6.367893
Eigenvector
   -0.05797   0.000828   -1.1E-11   0.000828   0.056982   0.000786   0.168438   0.000185   0.139445
   -0.00053   0.031295   -0.00091   -0.10825   -0.00087   -0.20783   -0.00038   -0.20026   0.000555
   1.17E-10    0.00203   0.319904   0.002624   0.612707   0.001622   0.679033   -0.00089   0.418116
   -0.00269   5.56E-11   -0.00332   -0.34999   -0.00334   -0.51642   -0.00334

Eigenval#           3
Eigenval =   17255.06
Omega  =     131.3585
Freq Hz =    20.90638
Eigenvector
   0.423551   -0.00605   -2.2E-10   -0.00605    -0.4072   -0.00536   -0.70482    0.00431   0.507595
   0.007826   1.314842   0.003469   1.519153    -0.0006   1.195938   -0.00343   0.587301   -0.00431
   1.29E-10   -0.00255    -0.1743   -0.00024   -0.09081   0.001411   0.157775   0.000966   0.163214
   -0.00074   5.16E-12   -0.00153    -0.1695   -0.00166   -0.25211   -0.00166

Eigenval#           4
Eigenval =   29671.79
Omega  =      172.255
Freq Hz =    27.41526
Eigenvector
   -0.10462   0.001495    5.7E-11   0.001494   0.100058   0.001303   0.147864   -0.00134    -0.1487
    -0.0012   -0.13161    0.00119   0.132914   0.002051   0.399416   0.001237   0.432094   -0.00101
   -4.3E-10   -0.00456   -0.63321   -0.00387   -0.67136   0.004534   0.552453   0.006002   0.739562
   -0.00271   -2.1E-11   -0.00761    -0.8811   -0.00879   -1.31894   -0.00879

Eigenval#           5
Eigenval =   108201.7
Omega  =     328.9402
Freq Hz =     52.3525
Eigenvector
   -0.07459   0.001066   7.73E-11   0.001063   0.070154   0.000885   0.058248    -0.0014   -0.08988
   0.001391   0.479695   0.005514   1.240099   0.003751   1.433157   -0.00139   0.878198   -0.00546
   2.36E-10   -0.00434   -0.30757   -0.00036   -0.09809   0.003519   0.259511   -0.00266   -0.53321
   -0.00475   2.91E-09   0.016356   2.594527   0.028967   4.039838   0.029041

Eigenval#           6
Eigenval =   124861.1
Omega  =     353.3568
Freq Hz =    56.23853
Eigenvector
   -0.21205   0.003032   1.71E-10   0.003022   0.198968   0.002499   0.151077   -0.00409     -0.242
   0.004731   1.544544   0.016835   3.776775   0.010234   4.127267   -0.00593    2.28248    -0.0165
    1.5E-09   -0.00791   -0.18665   0.002868   0.169522   0.000392   -0.13969   -0.00078   0.064048
   0.002091   -1.1E-09   -0.00459   -0.78957   -0.00902   -1.23952   -0.00904

Eigenval#           7
Eigenval =   895802.7
Omega  =     946.4685
Freq Hz =    150.6353
Eigenvector
   0.444365   -0.00639   -3.8E-10   -0.00623    -0.3899   -0.00447   -0.03749   0.008195   0.107189
   -0.01746   -3.73728   -0.02152   -3.31365   0.027931   2.354895   0.035124   4.224118    -0.0133
   4.63E-09   -0.02315   -1.00038   0.004669   0.045319   0.005829   -0.01644   -0.00242   -0.11535
   0.000626     -8E-11   0.000399   -0.02279   -0.00053   -0.04969   -0.00054
```

Mode shape 1: 182.82 rpm



Mode shape 2: 382.07 rpm



Mode shape 3: 1254.4 rpm