Probabilistic Structural Mechanics

Definition

Deterministic - Algorithm, model, procedure, process, etc., whose resulting behavior is entirely determined by its initial state and inputs, and which is not random or stochastic. Processes or projects having only one outcome are said to be deterministic their outcome is 'pre-determined.' A deterministic algorithm, for example, if given the same input information will always produce the same output information.

Uncertainty - The estimated amount or percentage by which an observed or calculated value may differ from the true value.

Stochastic - means random. A stochastic process is one whose behavior is non-deterministic, in that a system's subsequent state is determined both by the process's predictable actions and by a random element. any kind of time development (be it deterministic or essentially probabilistic) which is analyzable in terms of probability deserves the name of stochastic process.

probability model of “choosing an object at random." - A ***probability model*** is a mathematical representation of a random phenomenon. It is defined by its ***sample space***, ***events*** within the sample space, and ***probabilities*** associated with each event.

Statistical - **Statistics** is the science of the collection, organization, and interpretation of data.
http://www.statsoft.com/textbook/basic-statistics/

procedures - a specified series of actions or operations [disambiguation needed] which have to be executed in the same manner in order to always obtain the same result under the same circumstances

failures - refers to the state or condition of not meeting a desirable or intended objective, and may be viewed as the opposite of success.

Reliability - The ability of a product to provide the desired and promised function to the customer or user.

Durability - The quality of equipment, structures, or goods of continuing to be useful after an extended period of time and usage

variation - When failures occur, it is not always the variation itself that is the problem but rather the uncertainty about the size of the load on the equipment

robust design - *Insensitivity to Variation* – eg misalignment bearing. robust design is an awareness of variation

complexity - Complexity is a natural property of every system. It is defined as a mix of interdependency and uncertainty. complexity is a fundamental and intrinsic property of all dynamical systems. eg ontonix

Limit state function - a function of random variables or previously defined functions

Limit state function can be defined to describe the relationship between the ability to load or resistance applied to the structure and it is the first step in structural reliability analysis.

Model X(w) - random is some function *X* of some hidden variable w or X(w).

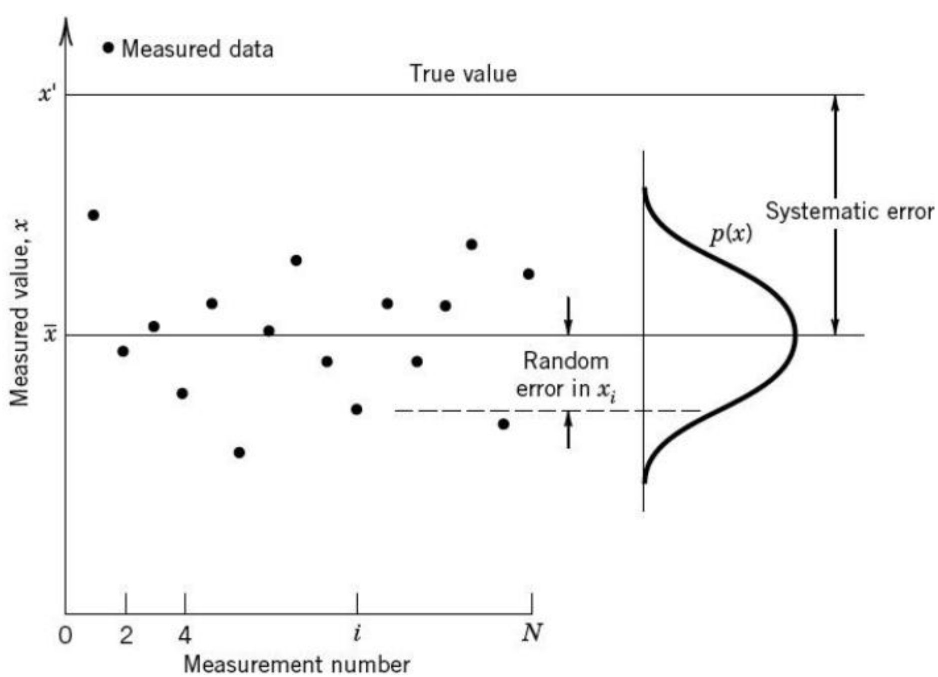
Data Analysis and Experimental Uncertainty

Introduction

related to human mistakes, such as misuse, unforeseen physical effects or unforeseen extreme events
lack of knowledge of identified physical behaviour, and variation in future – Bayesian perspective

1. Types of Uncertainty

two basic kinds of uncertainties, systematic and random uncertainties.



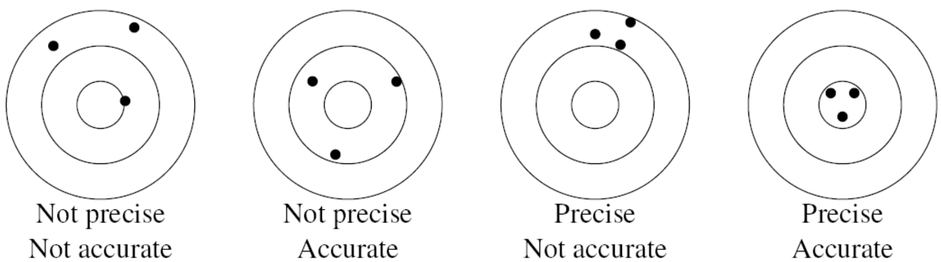
Systematic uncertainties are those due to faults in the measuring instrument or in the techniques used in the experiment. Eg kink steel tape, a pendulum with a clock that runs too fast, The stiffness of many springs depends on their temperature.

Random uncertainties are associated with unpredictable variations. Eg Electrical noise, The length of a table, Repeated measurements.

random uncertainties are much easier to deal with and to quantify.

If an experiment has low systematic uncertainty it is said to be accurate. If an experiment has low random uncertainty it is said to be precise.

When thinking about uncertainty, it is important to remember these associations, so they are worth repeating:
- Random uncertainty decreases the precision of an experiment.
- Systematic uncertainty decreases the accuracy of an experiment.



Review on statistics

The Mean, Standard Deviation, and Standard Deviation of the Mean

Random uncertainty is often associated with the concept of standard deviation.

Suppose ten students each measure the diameter of a steel ball with a micrometer caliper. For a variety of reasons we do not expect all the measurements to be identical.

The sources of error include:
- some students tighten the micrometer caliper more than others.
- the steel ball may not be perfectly round.
- some students may not exercise care to be sure they are measuring a \great diameter"|the ball is not centered between the jaws.
- the temperature of the steel ball may change with time as the ball is handled and hence its diameter may change slightly through thermal contraction or expansion.

- there may be varying amounts of corrosion on the steel ball.

Exercise 1
Which of the above sources of error contribute to systematic uncertainty? Which contribute to random uncertainty? Explain how you came up with your answers.

\What is the best value for the diameter of the steel ball?"

$$\overline{x} = \frac{x_1 + x_2 + \ldots + x_N}{N} = \frac{1}{N}\sum_{i=1}^{N} x_i.$$

Having obtained a mean or \best" value, x, it is important to have a way of stating quantitatively

how much the individual measurements are scattered about the mean.

For a precise experiment we expect all measurements to be quite close to the mean value.

The extent of scatter about the mean value gives us a measure of the precision of the experiment, and thus, a way to quantify the random uncertainty.

A widely accepted quantitative measure of scatter is the sample standard deviation, s.

$$s = \sqrt{\frac{\sum_{i=1}^{N}\left(x_i - \overline{x}\right)^2}{N-1}}.$$

the mean value of the measurements to have less random uncertainty than any one of the individual measurements.

$$\sigma_m = \sqrt{\frac{\sum_{i=1}^{N}(x_i - \overline{x})^2}{N(N-1)}} = \frac{s}{\sqrt{N}}.$$

9.80, 9.87, 9.89, 9.95, 9.91, 9.98, 9.92, 10.05, 9.97, 9.84

Stating Results with Uncertainty

two common ways to state the uncertainty of a result:

in terms of a σ, like the standard deviation of the mean σm, or in terms of a percent or fractional uncertainty, for which we reserve the symbol ε

$$\epsilon_x \equiv \frac{\sigma_x}{x} \, .$$

one typically uses the form $x \pm \sigma_x,$

eg if the mass of an object is found to be 9.2 g and the uncertainty in the mass is 0.3 g

$m = 9.2 \pm 0.3 \text{ g}$ or $m = (9.3 \pm 0.3) \times 10^{-3} \text{ kg}$

“The mass of the object is 9.2 grams with an uncertainty of 3 percent."

Comparing Quantities with Uncertainty

$10.0 \pm 0.1 \text{ g/cm}^3$ and $9.8 \pm 0.3 \text{ g/cm}^3$ agree

$9.81 \pm 0.02 \text{ g/cm}^3$ not agree

$$\left| \frac{9.8 - 10.0}{10.0} \right| \times 100 = 2\%$$

Significant Digits

no such thing as an “exact uncertainty"

$$9.5 \pm 0.3 \text{ g, or } 9.52 \pm 0.14 \text{ g, but not } 9.52 \pm 0.3 \text{ g.}$$

lose significant digits $9 \pm 2 \text{ g}$ or $m = 9.52 \pm 0.01 \text{ g.}$

Do not confuse round-off errors with uncertainty

Propagation of Uncertainty

The method of computing the uncertainty in a result which depends on several variables, each with its own uncertainty, is called propagation of uncertainty, or casually error propagation.

Functional Form	Formula	Uncertainty formula
Product or Quotient	$f = xy$ or $f = x/y$	$\epsilon_f = \sqrt{\epsilon_x^2 + \epsilon_y^2}$
Sum or Difference	$f = x + y$ or $f = x - y$	$\sigma_f = \sqrt{\sigma_x^2 + \sigma_y^2}$
Product of factors raised to powers	$f = x^m y^n$	$\epsilon_f = \sqrt{m^2\epsilon_x^2 + n^2\epsilon_y^2}$
Constant multipliers	$f = Kx$ (K =constant)	$\sigma_f = K\sigma_x$
Logarithmic functions	$f = \log_e(x)$	$\sigma_f = \epsilon_x$
	$f = \log_{10}(x)$	$\sigma_f = \log_{10}(e)\epsilon_x = 0.4343\epsilon_x$
Exponential functions	$f = e^x$	$\epsilon_f = \sigma_x$
	$f = 10^x$	$\epsilon_f = \log_e(10)\sigma_x = 2.303\sigma_x$

$$\sigma_f^2 = \sigma_x^2 \left(\frac{\partial f}{\partial x}\right)^2 + \sigma_y^2 \left(\frac{\partial f}{\partial y}\right)^2 + \sigma_z^2 \left(\frac{\partial f}{\partial z}\right)^2 + \dots ,$$

Least Squares Curve Fits/ method

chi-square

$$\chi^2 = \sum_i \frac{[y_i - f(x_i; a, b, \dots)]^2}{\sigma_i^2} , \qquad \chi_\nu^2 = \frac{\chi^2}{\nu} \equiv \frac{\chi^2}{N - n} .$$

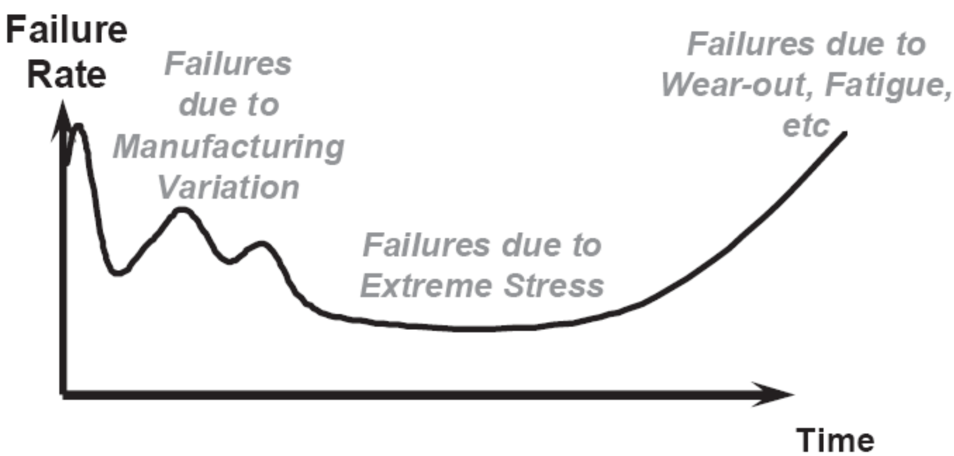
The method of least squares is one that produces the \best" prediction of the output based on the input

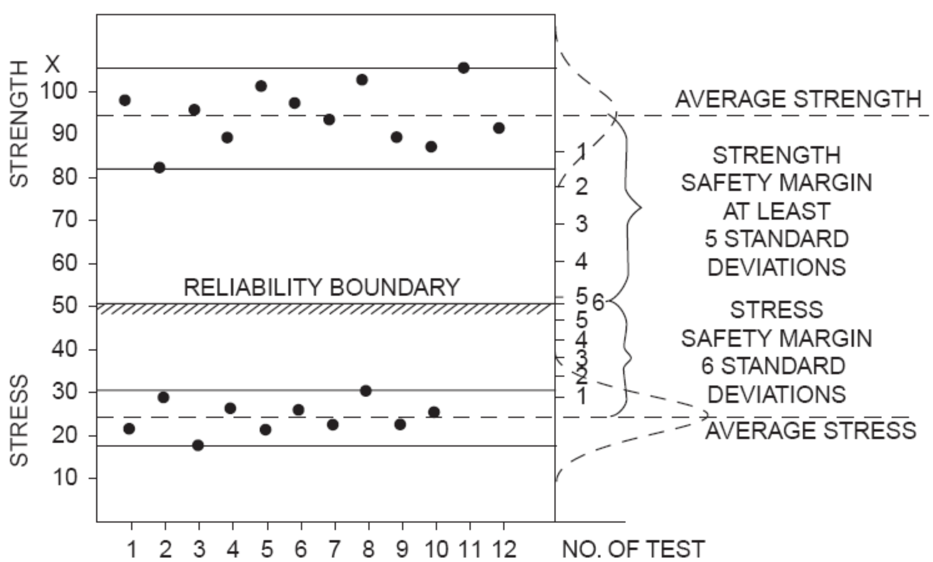
Incorporation of engineering experience

It is obvious that using only one to four specimens, as it is common in geotechnical engineering, a purely statistically determined characteristic value tends to be very low

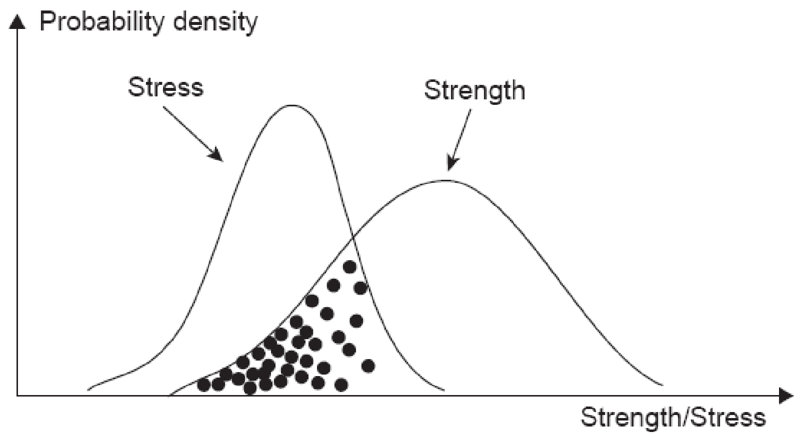
Probabilistic Analysis

Reliability and Variation



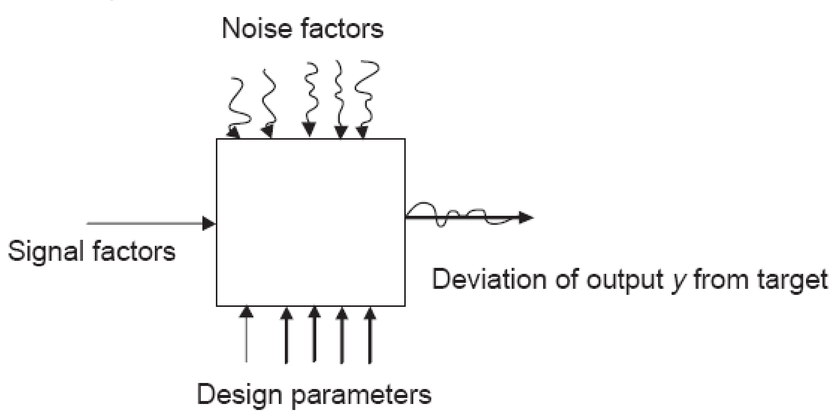


Failure Mode Avoidance



load and capacity

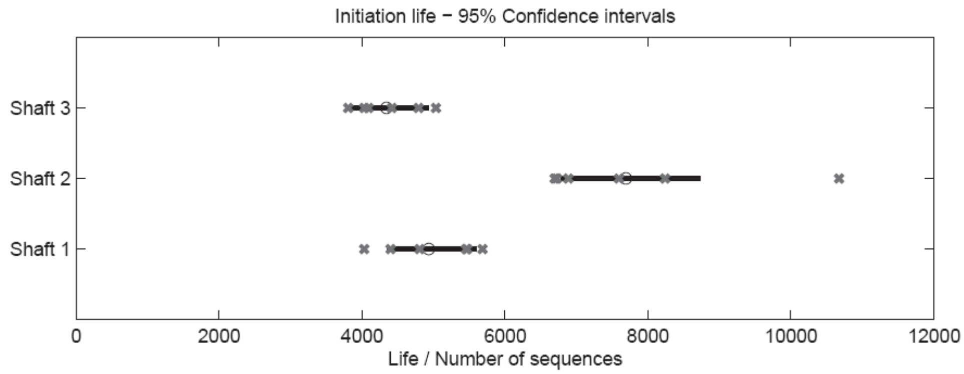
Robust Design



The P-diagram (P stands for product/process)

Estimation of Prediction Uncertainty

1.Scatter or physical uncertainty, which is uncertainty identified with the inherent random nature of the phenomenon, e.g. the variation in strength between different components.



2. *Statistical uncertainty*, which is uncertainty due to statistical estimators of physical model parameters based on available data, e.g. estimation of parameters in the Coffin–Manson model for life based on fatigue tests.

3. *Model uncertainty*, which is uncertainty associated with the use of one (or more) simplified relationship to represent the 'real' relationship or phenomenon of interest, e.g. a finite element model used for calculating stresses is only a model for the 'real' stress state.

	Logarithmic life ln(N)		
Type of scatter and uncertainty	Scatter	Uncertainty	Total
Strength scatter			0.38
Material, within shaft	0.15		
Material, between shaft	0.29		
Geometry	0.20		
Statistical uncertainty			0.07
LCF curve		0.07	
Model uncertainty			0.84
LCF curve		0.05	
Mean stress model		0.30	
Multi- to uniaxial		0.20	
Plasticity		0.72	
Stress analysis		0.24	
Temperature		0	
Load scatter and uncertainty			0.50
Service load, scatter	0.40		
Service load, uncertainty		0.30	
Total	0.55	0.90	1.05

Ang, Ng & Tan