MODIFIED MONTE CARLO WITH IMPORTANCE SAMPLING METHOD

Monte Carlo simulation methods apply a random sampling and modifications can be made of this method is by using variance reduction techniques (VRT). VRT objective is to reduce the variance due to Monte Carlo methods become more accurate with a variance approaching zero and the number of samples approaches infinity, which is not practical in the real situation (Chen, 2004). These techniques are the use of antithetic variate, variate control and sampling methods are different. In crack fatigue and reliability analysis of structures, other than random sampling, the types of sampling that has been used by researchers are:

a) importance sampling
b) Latin Hypercube sampling
c) adaptability of the radial-based importance sampling for determining the most likely point (MPP)

3.3.1 **I**mportance sampling

According Boessio et al. (2006), a modified Monte Carlo method with importance sampling can avoid a number of simulations, which is too much like the original Monte Carlo method. Importance sampling function by increasing concentrations of sample points in areas with a higher probability of failure. Distribution point penyampelan focused on important areas only, namely $f_w(X)$ the sample space is a random variable, X.

The equation is the probability of failure can be defined by:

$$P_f = \int_{g(X)\leq 0} f_X(X)dx = \int_X I[g(X)]f_X dX \tag{3.7}$$

$$P_f = \int_X I_w[g(X)]f_w dX = \frac{1}{ns}\sum_1^{ns} I_w[g(X)] \tag{3.8}$$

where $f_w(X)$ is a function of probability sampling, while I_w can be defined:

$$I_w = I[g(X)]\frac{f_X(X)}{f_w(X)} \tag{3.9}$$

Anderson (1999) states that it is important for the distribution are selected for sampling, or generating random numbers.

3.3.2 Latin Hypercube sampling

Latin Sampling Method Hypercube is one branch of the sampling layers are arranged in general. According to Choi et al. (2007), the distribution of each random variable can be divided into n intervals with equal probability. Any interval is not beyond themselves as they have the same probability, and has its own point of analysis. Thus, there are n-number of points of analysis are randomly mixed and each has probability 1 / n of the probability distribution. This will ensure that each of the input variable has a range of samples from all.

Implementation steps can be summarized as follows::

a) Distribution of the distribution for each of the n-interval is not beyond themselves.
b) Choose a random value for each variable in each interval.
c) The second step is repeated for all variables to completion.
d) Relate the value of *n* is found for x_i with a random value of $x_{j\neq 1}$.

Cumulative probability, P_m can be defined as:

$$P_m = \frac{1}{n}U_m + \left(\frac{m-1}{n}\right) \tag{3.10}$$

where U_m are random numbers from uniform distribution, and m is a value from 1 to n. The value of U_m located in each interval -m be used to obtain the probability of normal distribution of the, ξ ie:

$$\xi = \Phi^{-1}(P_m) \tag{3.11}$$

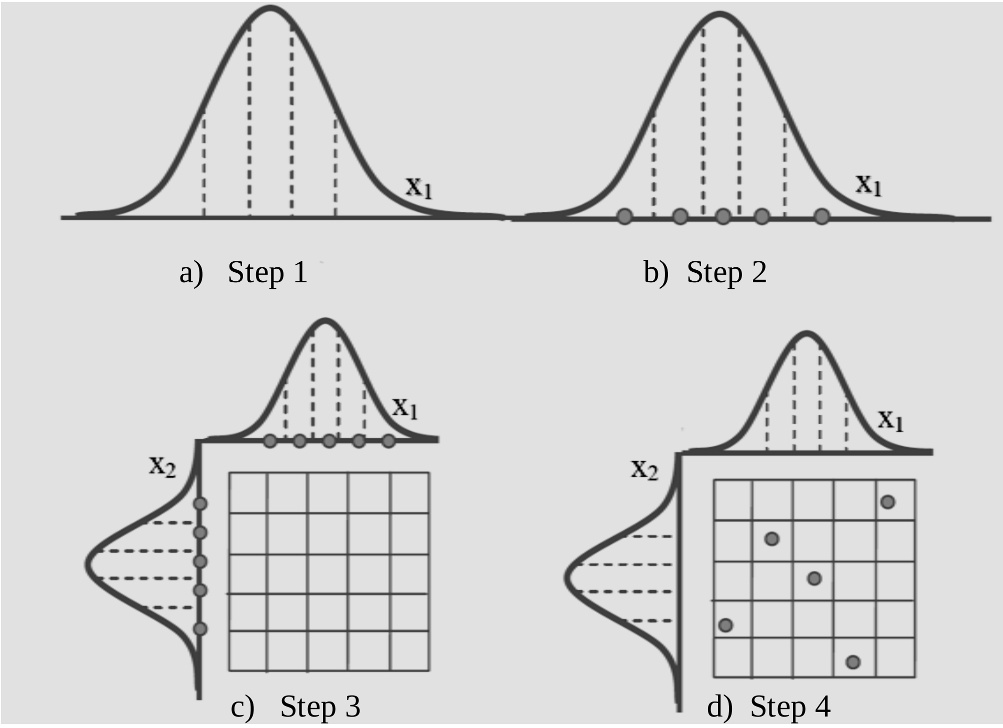

Figure 3.1: Basic steps for sampling the Latin Hypercube
(Source: Choi et al., 2007)

Orthogonal sampling is one of Latin Hypercube. Orthogonal layout is a fractional factorial matrix that ensures a uniform comparison between the level or relationship to any factor. This method is similar to the method of Latin origin Hypercube where the sample space is divided into spaces smaller with the same probability. All the sampling points chosen and sampled simultaneously with the same density. These techniques try to ensure that random numbers are representative of true randomness approach.

3.3.3 Importance Sampling-Based Radial Adaptability

According Grooteman (2008), based on Importance sampling technique was developed by the radial Harbitz that sphere-β ' ie to the n-dimensional sphere of domain sampling in the secure part. Sampling domain is restricted to the values that are beyond the sphere- β ' the joint probability density function. The value of β ' refers to the optimum radius of the sphere is the shortest distance to the limit in the most probable point (MPP). In general, the value of β ' is not known, but this method can save a lot of simulation due to the reduction of sampling is not needed in the safe area.

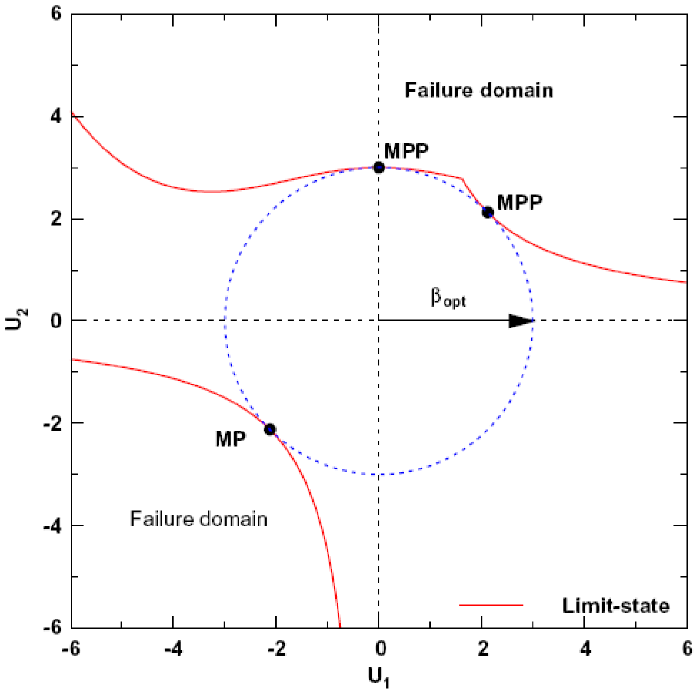

Figure 3.2 : Importance sampling based on berasaskan optimum radial

(Source: Grooteman, 2008)

This method also guarantees convergence to a solution with a much reduced number of sampling. However, this method considers the optimum value of β ' is known and in reality, this value is not known. So, Grooteman (2008) have suggested an Importance sampling method based on radial flexibility of finding the value β ' real first. This method starts by assuming an initial value β_0 high as this will ensure the probability of this value lies outside the true radius β. The value of β ' can be characterized by the equation 3.12:

$$\beta' = \sqrt{\chi_n^{-2}(1 - \frac{p_o}{p_{step}})} \quad (3.12)$$

where p_o adalah kebarangkalian domain sampel berada di luar sfera manakala χ_n is a chi distribution with n degrees of freedom as the number of stochastic variables and p_{step} is a probability sample is not in the β and $\beta_{optimum}$. Nilai p_{step} right is close to 1 to reduce the sample domain β' and $\beta_{optimum}$ while the value of 0.8 could be a suitable option.

3.4 MODIFIED MONTE CARLO SIMULATION (MMCS)

MCS method, introduced by Ulam and von Neumann in the 1940's era can be defined as a method to approach the expectations of the sample mean for the function of the simulated random variables (Anderson, 1999). This method is one of the methods used to conduct the reliability assessment of engineering structures and it is one of the most popular methods among researchers in the works or structural reliability assessment of engineering components. In this section, the steps to this method and a modified (MMCS) will be discussed in depth.

3.4.1 Performance Equation for Problem Statement

In the simulation, modeling is crucial in understanding the system or structure to identify the variables and coefficients that exist and to ignore factors that are not important. This means that the early stages of MCS is to identify the random variables involved in an engineering problem. Random variables that will lead to the construction or the issuance of the equation that connects all of them. According Grooteman (2007), if there is uncertainty whether the parameter or combination of parameters is random or not, sensitivity analysis can be conducted. If different parameter values are not shown diffusing it should be considered as a regulation. A sample of the great value of the random variable will be generated at random and according to the most appropriate statistical distributions in describing it. Subsequently, a calculation based on the performance of the equation may be carried out.

3.4.2 Modified Monte Carlo Method Implementation

According to Boessio et al. (2006), Monte Carlo method is simulating a large number of experiments that are generated in the form of artificial. This experiment is a sample of random variables, X and then the limit state equation will be evaluated. Relative frequency of cases of failure when $g(X) < 0$ and the number of samples.

Reliability analysis by Monte Carlo method can be summarized as:

a) Loop for $k=1$, initiated until it reaches the total number of simulations ns.
b) Random number in the vector u is distributed uniformly from 0 to 1 generated.
c) random numbers generated are based on probability distributions that characterize the engineering parameters involved.
d) Calculate the limit state function as in equation 3.1.
e) The calculation of equation 3.13:

$$I[g(X)] = \begin{cases} 1 \; if \; g(X) \leq 0 \\ 0 \; if \; g(X) > 0 \end{cases} \tag{3.14}$$

f) The calculation of the probability of failure run as equation 3.14 until the loop stops at $k = ns$,

$$P_f = \int_{g(X)\leq 0} f_X(X)dx = \int_X I[g(X)]f_X dX = \sum_1^{ns} I[g(X)] = \mu_{P_f} \tag{3.15}$$

g) The calculation of standard deviation and coefficient of variance of the next run:

$$\sigma_{P_f} \cong \left[\frac{(1 - P_f)P_f}{ns}\right]^{\frac{1}{2}} \tag{3.16}$$

(3.17)

$$COV(X) = \delta_X = \frac{\sigma_X}{\mu_X} \cong \left[\frac{(1-P_f)}{nsP_f}\right]^{\frac{1}{2}}$$

before the simulation is terminated and the results are displayed in the MATLAB workspace. Simulation results will dplotkan in the form of diagrams that are appropriate to the variables studied.

For the deterministic variables, $COV(X)$ are zero. The values of $COV(X)$ means that the smaller the smaller the uncertainty in the random variable. According to Grooteman (2008), with 95% confidence interval and COV_{P_f} the probability of failure, the relative error in the estimated probability of failure is:

$$E_{max}^{P_f} = 1.96COV_{P_f} \tag{3.18}$$

$$COV_{P_f} = \sqrt{\frac{1-P_f}{N_{sim}\,P_f}} \tag{3.19}$$

In reality, the error is less than 10% and it is acceptable for most engineering structures. COV led to the decline in value relative error reduction and increase in the number of simulations. Effects of decreasing COV of the simulation results will be discussed further in the next chapter.

3.4.3 Modifications to the Sample Generation

Generation of samples for each parameter is the most appropriate probability distribution characteristics. However, any distribution that is used has its own characteristics that have the statistics variables. Thus, the effects of statistical variables on the reliability of the structure of the simulation results are reviewed and discussed in the next chapter.

3.5 CONCLUSION

Thus, through the use of modification techniques, the number of samples required for the simulation and the simulation is expected to be reduced. Thus, the Monte Carlo method is modified to become more efficient and effective in analyzing a problem of fatigue in structural engineering. Importance sampling techniques are discussed in this chapter will be used to modify the basic Monte Carlo method, and subsequently applied to the structural reliability analysis will be discussed in subsequent chapters.

RESULTS AND DISCUSSION

Table 4.1 Statistical variables for specimens

Random Variables	Average Value	*COV*	Distribution Probability
Fracture toughness, K_{Ic}	44 MPa $\sqrt{m}$	0.30	Weibull
Crack Size, A	0.02 m	0.30	Lognormal
Tensile stress, σ_∞	100 MPa	0.30	Normal
Specimen Width, W	0.05 m	-	-

4.3 ANALYSIS OF PROBABILITY DISTRIBUTION

In the modified Monte Carlo method with importance sampling techniques, random numbers for random variables in each sample must be generated and characterized by appropriate probability distributions. However, for each type of probability distribution, there are statistical parameters that need to be set to control the properties of a distribution. In general, the statistical parameters are the form factor, the factor of location and scale factor. In this study, the effects on the reliability of statistical parameters studied before the values are selected for use in the simulation.

4.3.1 Lognormal distribution

For the lognormal probability distribution, the mean and standard deviation of the distribution representing the shape and scale factors respectively. The standard lognormal probability distribution having zero location factors and the scale factor 1 and zero form factor.

As discussed in chapter III, the initial crack size a more appropriate structure is characterized by a lognormal probability distribution as compared with the normal probability distribution. Both the Cross (2007) and Liu (2006) states that the size of the crack can not be characterized by a normal probability distribution for the negative value generated is not a physical meaning because the size of the crack is not possible to be less than zero.

Table 4.2 The value of statistical factors

Simulation	Mean	Standard Deviation	Location Factors	Type Distribution
1	1	1	0	Assumption
2	0	1	0	Standard
3	1	0.5	0	Assumption

Through simulations made, it is found that the standard lognormal probability distributions had a higher reliability value of about 80% compared with the other configuration at about 50 - 60%. According to Cross (2007), these factors should be determined statistically by the statistical analysis of experimental data because it depends on other variables such as geometric shapes and loading. Thus, it is sufficient to use the standard lognormal distribution for this study.

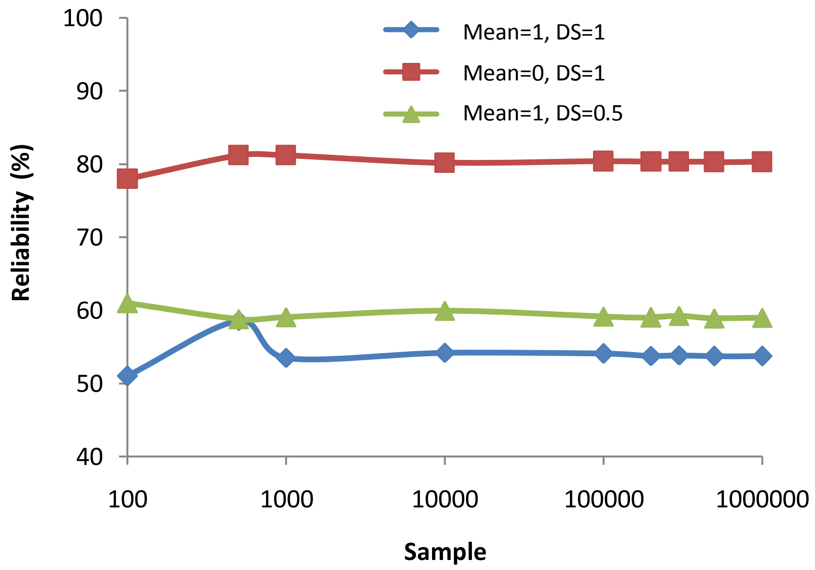

Figure 4.1 The reliability of the structure of a sampling Lognormal To Crack Size

4.3.2 Weibull distribution

For the Weibull probability distribution, statistical factors to be considered is the form factor, η and scale factor. In this study, which used the Weibull probability distribution is known as a 2-parameter Weibull distribution.

As already discussed, the Weibull distribution shape factor, η is used to characterize the behavior of engineering parameters that influence the rate review and the probability density distribution. Unknown parameters of stochastic fracture toughness of engineering and kerawakannya as characterized by the Weibull distribution. So in this study, the Monte Carlo method has been modified by the factor of different forms to see the impact on structural reliability.

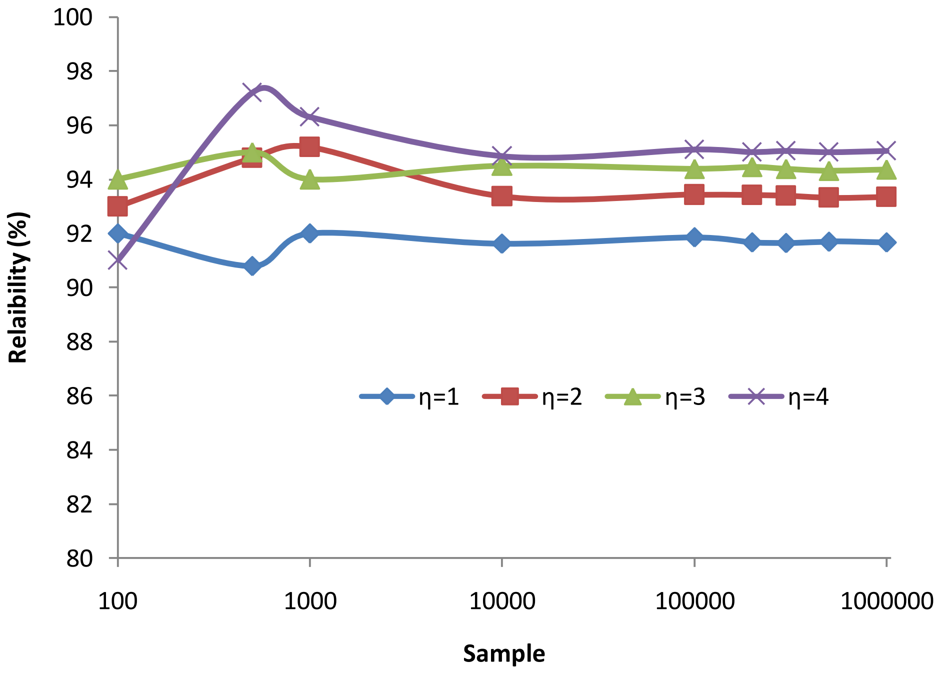

Figure 4.2 The reliability of the structure of a sample of Weibull For fracture toughness

It was found that, when η worth 4, the reliability is converging on the 300 000 samples with a value of 95.0523%, while the η value of 3, 2 and 1, the reliability is 94.3893%, 93.4027% and 91.6457%, respectively. This shows the higher value of η, the stochastic nature of the probabilistic parameters of fracture toughness has a higher value and leads to a higher structural reliability. For the fracture toughness parameter, η = 2 was chosen as a standard factor for the Weibull distribution for the subsequent simulations.

4.3.3 Normal Distribution

The standard normal probability distribution that is without any modification to the statistical factors, was used to characterize the stochastic nature of the tensile stresses imposed on the model structure studied.

4.3.4 The selection of distribution and distribution parameters

Selection of the appropriate statistical factors are important to characterize the randomness of engineering parameters to allow a more accurate simulation results and the factors summarized below.

Table 4.3 Statistical Variables Used For Individual Distribution Simulation

Distribution	Statistical parameters
Lognormal	Mean = 0 [a], Standard deviation = 1 [a]
Weibull	Location factors = 0 [a] , Scale factor = 1 [a] , geometry factor = 2 [b]
Normal	Mean = 0 [a] , Variance = 1 [a]

4.4 CENTRAL CRACK TENSION (CCT)

$$K_I = \sigma\sqrt{\pi a}\, f(a/w)$$

$$f(a/w) = 1 + 0.128(a/w) - 0.288(a/w)^2 + 1.525(a/w)^3 \quad (4.2)$$

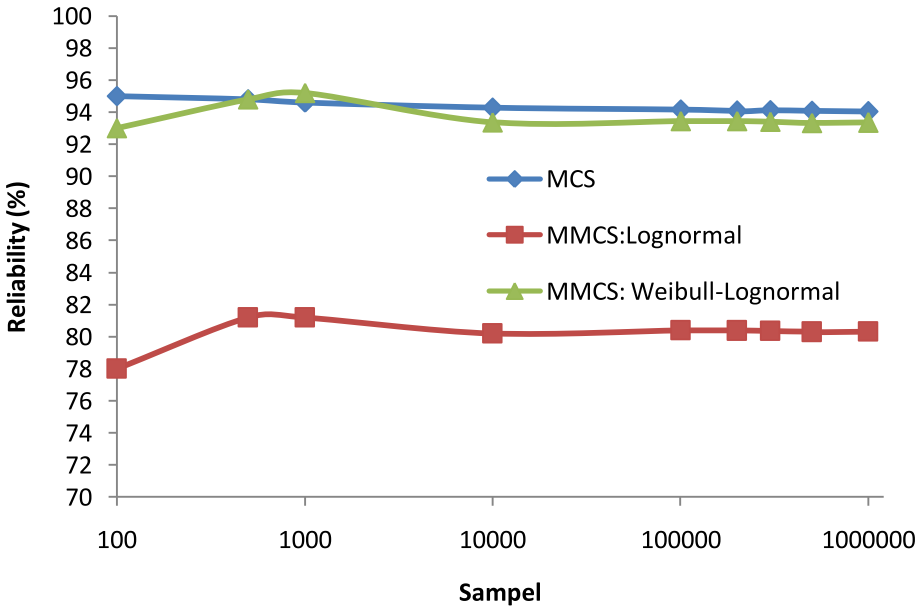

Figure 4.3 The reliability of the structure of a central crack tension

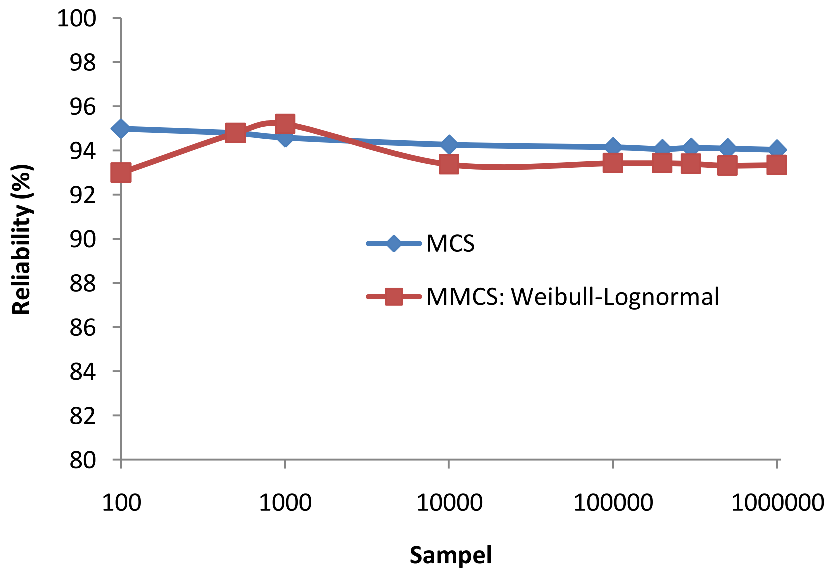

Figure 4.4 First simulation: The reliability of the structure of a CCT

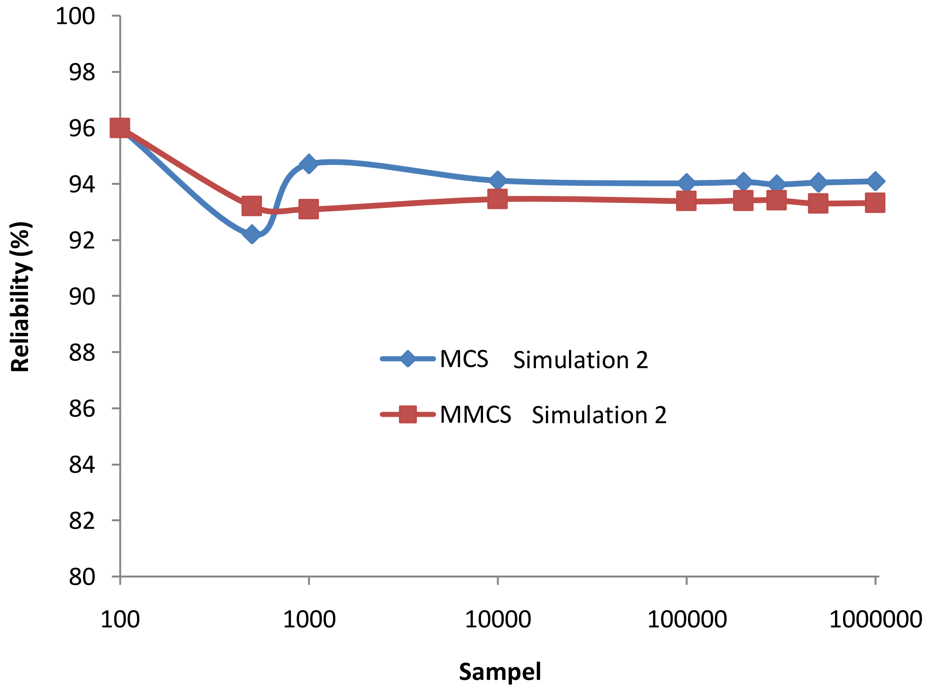

Figure 4.5 Second simulation: The reliability of the structure of a CCT

Table 4.4 Converged Value Comparison Between MCS and MMCS

Simulation	MCS (%)	MMCS (%)	difference between the methods (%)
1	94.162	93.370	0.8411
2	94.022	93.460	0.5977
Difference between simulation (%)	0.1487	-0.0964	-

4.4.1 Probability of Failure

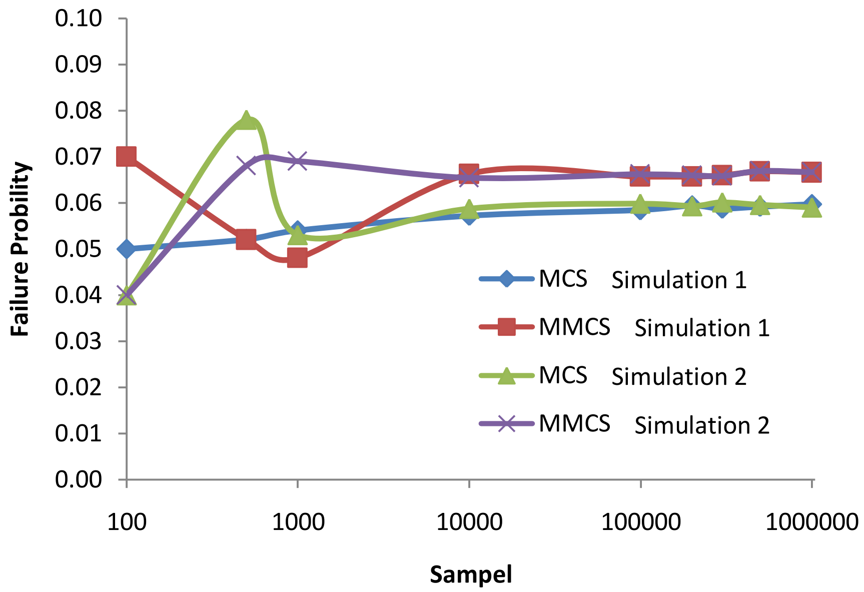

Figure 4.6 Structural Failure Probability Model CCT

4.4.2 Kecekapan Kaedah Pengubahsuaian Penyelakuan Monte Carlo

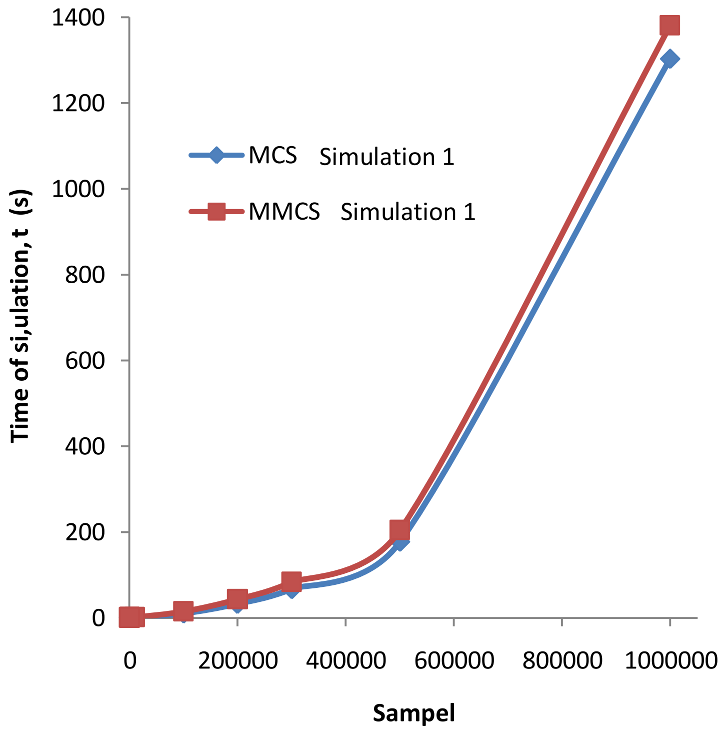

Figure 4.7 number of samples on Simulation

Table 4.5 Time Needed To Make Every sample with MMCS

Samples	Time (s)	Time to generate each sample (s)
1 000 000	1381.2860	0.00138
500 000	203.7105	0.000406
200 000	42.6096	0.000213
100 000	14.5166	0.000145
10 000	1.0930	0.0001093
1000	0.2023	0.0002023

4.5 SENSITIVITY ANALYSIS OF RANDOM VARIABLES

4.5.1 loading conditions

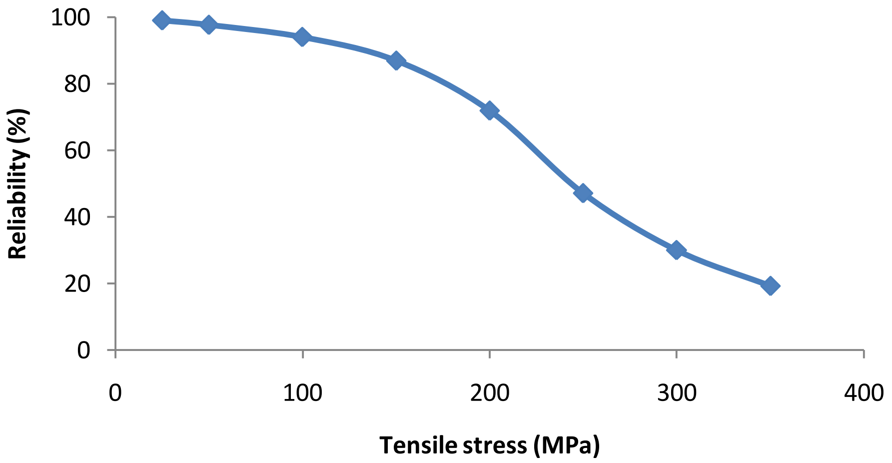

Figure 4.8 Tension Stress Effect on Reliability

4.5.2 Initial crack size

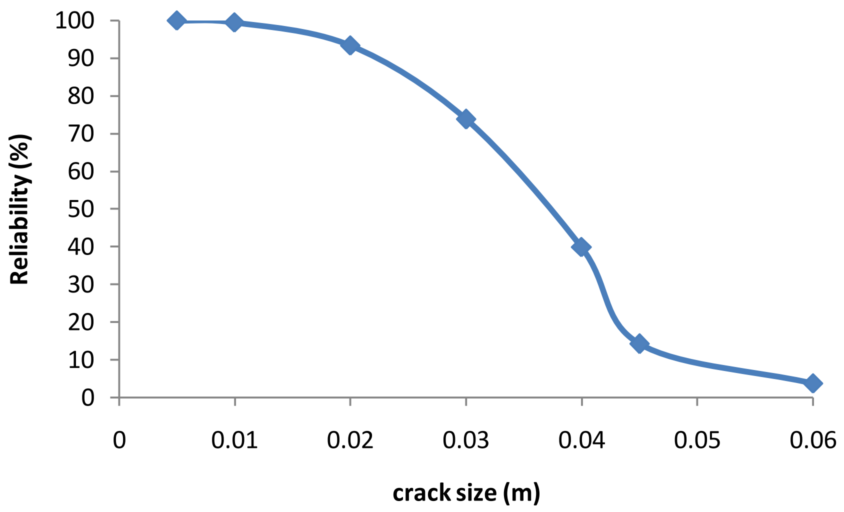

Figure 4.9 Effect of Crack Size on Reliability

4.5.3 Fracture toughness

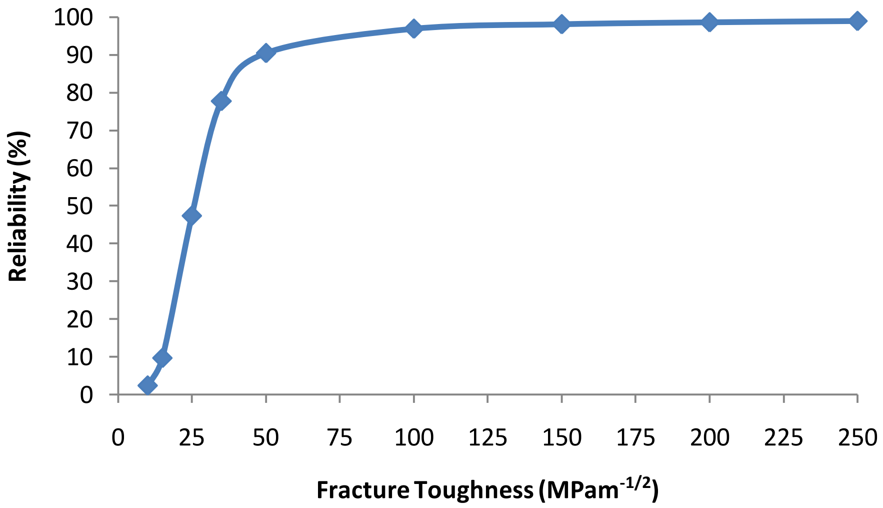

Figure 4.10 The effect on reliability of material fracture toughness

4.5.4 Varian constant probability of failure

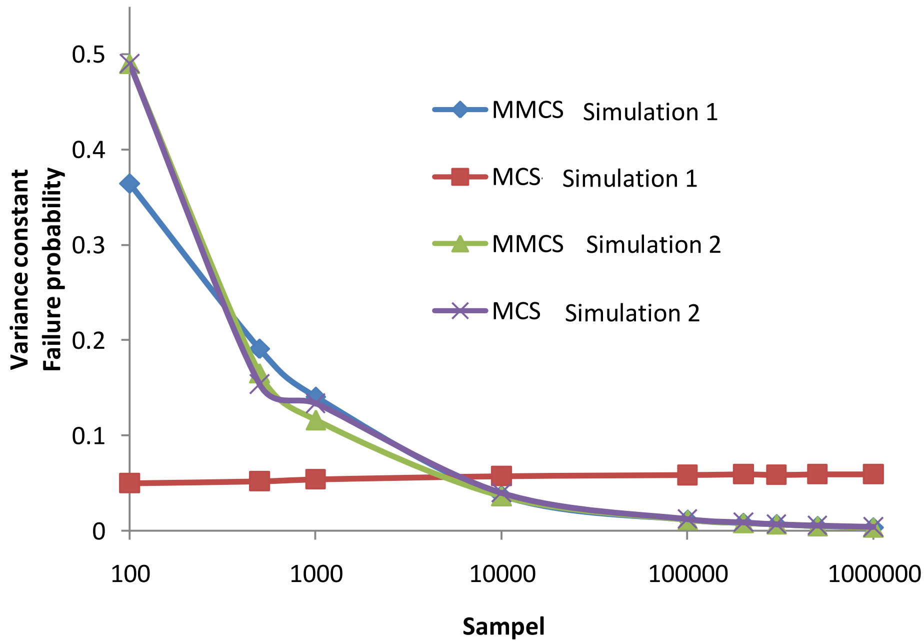

Figure 4.11 Effects of constant probability of failure on Simulation Variance

4.7 ENGINEERING APPLICATION

4.7.1 Model simulations on Different Materials

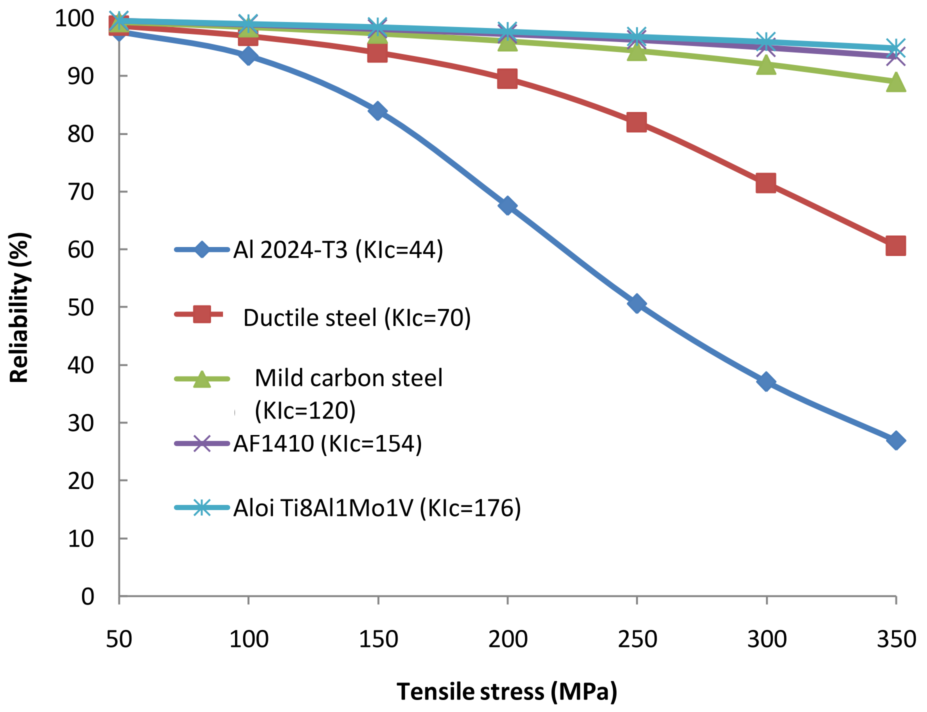

Figure 4.13 Simulation Comparison of Different Materials

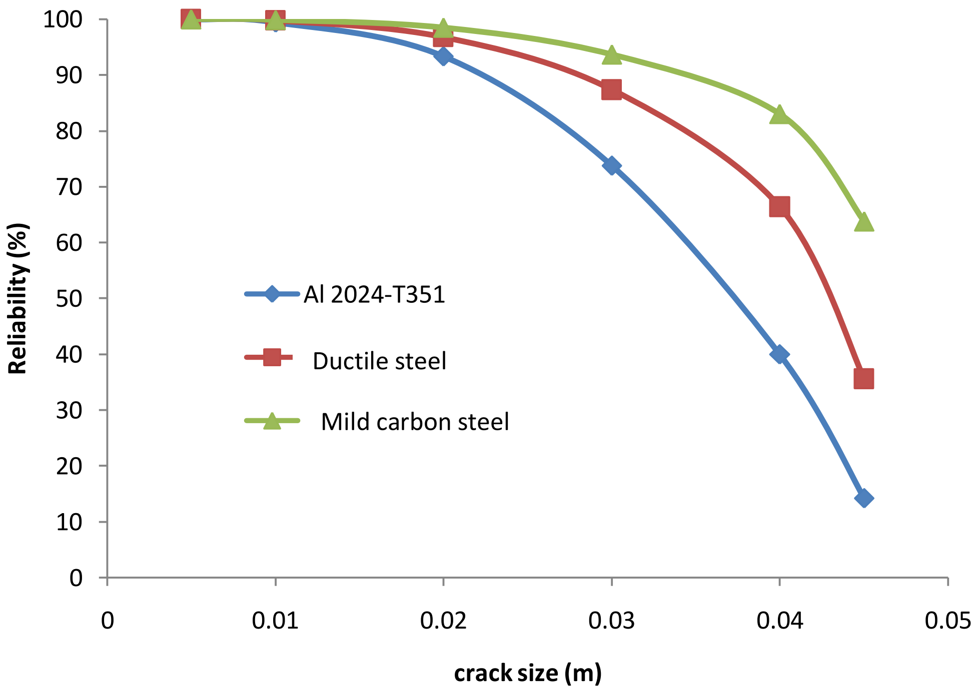

Figure 4.14 Simulation Comparison of Different Materials

4.7.3 Bridge Case Study Box Galang

Limit state function equation can be given

$$g(X) = \int_{a_o}^{a_c} \frac{da}{\left[f(a/w)\sqrt{\pi a}\right]^m} - C.S_{RE}^m.(365.ADTT.C_s.Y) \tag{4.3}$$

the geometric factor, $f(a/w)$ can be defined as

$$f(a/w) = \frac{1 - 0.5(a/w) + 0.37(a/w)^2 - 0.044(a/w)^3}{\sqrt{1-(a/w)}} \tag{4.4}$$

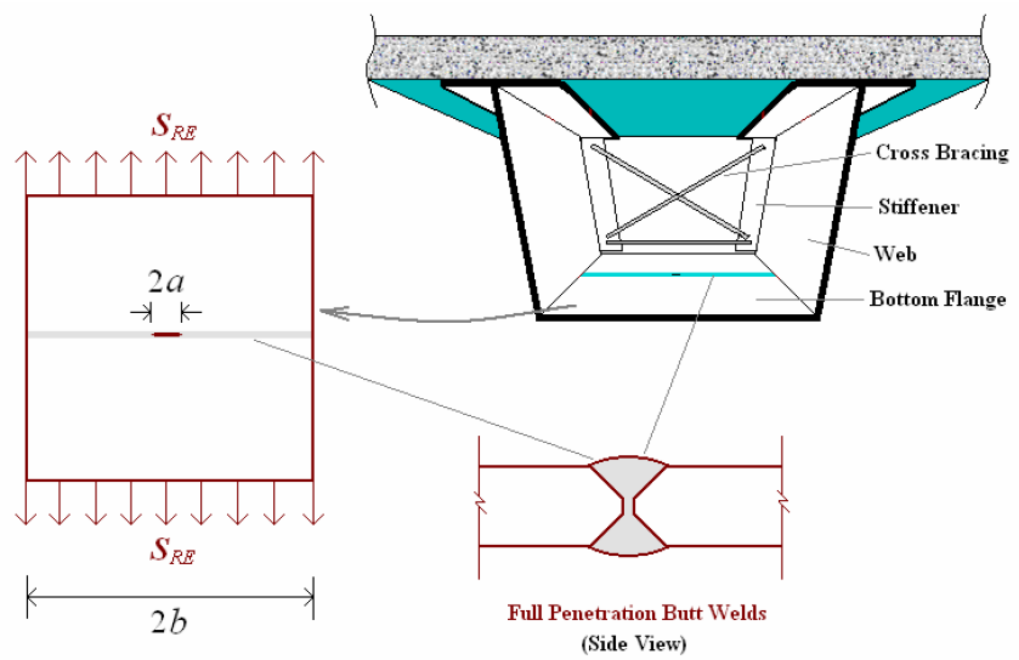

Figure 4.15 Bridge Box

(Source: Chung, 2004)

Table 4.6 Input Data For Variable-Variable Involved

Variable	Type Distribution	Average	COV
a_0	Lognormal	0.020 in	0.500

a_c	constant	2.000 in	-
C	Lognormal	$2.05\text{x}10^{-10}$	0.630
m	Normal	3.000	0.100
S_{RE}	Normal	9.85 ksi	0.300
C_s	constant	1.000	-
$ADTT$	constant	300.000	-
W	constant	42.000 in	-

4.7.4 Results

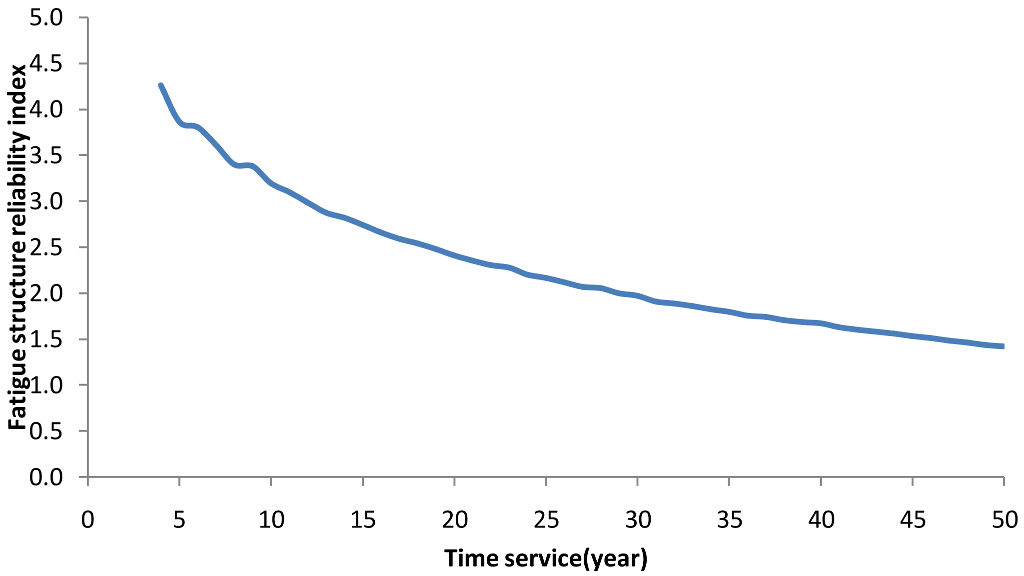

Figure 4.16 Decrease in fatigue reliability index structure with Time Service