

MODIFIED MONTE CARLO WITH LATIN HYPERCUBE METHOD

Latin hypercube sampling (LHS) was introduced by McKay, Conover and Beckman as a solution to increase the efficiency of computer simulations. This technique uses a stratified sampling on each input variable k . The distance of each input variable as a whole is divided into n intervals that are not connected with the same probability. At each input variable, an observation with a randomly drawn from each interval. The first n values of variables generated by this process is coupled with random without replacement value of the n variables. The combination of these n random without replacement coupled with the value of n input variables to form a triple third n . This process continues with each successive input variable so that nk tuple formed in all the input variables included in the vector. LHS can be summarized into four steps, and Figure 3.2 will give a clearer picture with variable coupling methods.

- 1) Divide the cumulative distribution for each variable on the probability of the interval N .
- 2) From each interval, a randomly selected value. At the i -th interval, the cumulative probability sample of this equation can be written to

$$Probability_i = \frac{u_i}{N} + \frac{(i-1)}{N} \quad (3.13)$$

where u_i are random numbers distributed uniformly in the range of numbers 0 to 1.

- 3) change the value of the probability sample x using the inverse cumulative distribution function F^{-1} .

$$x = F^{-1}(Probability) \quad (3.14)$$

- 4) N values of the variables selected at each value of x is coupled with other variables ns .

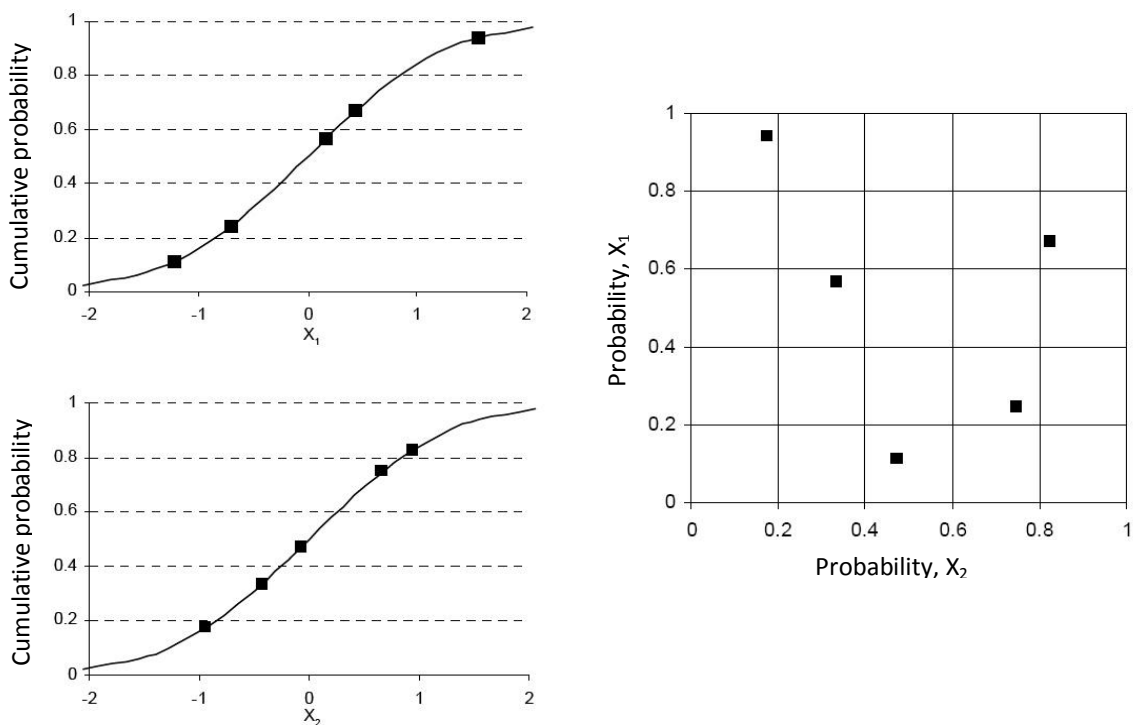


Figure 3.2 Example of LHS: Stratified random sampling for the variables X_1 dan X_2 at 5 intervals (left) and coupling random sample X_1 and X_2 form the Latin Hypercube (right)

Table 4.1 statistical variables for crack specimens

Random Variables	Average Value	COV^c	Probability Distribution
Fracture toughness, K_{Ic}	44 MPa \sqrt{m}	0.3 ^a	Normal ^b
Crack Size, a	0.01 m	0.3 ^a	Normal ^b
Tensile stress, σ_∞	100 MPa	0.3 ^a	Normal ^b
Specimen Width, w	0.05 m	-	-

Table 4.2 Geometry factor for crack specimens

Specimen Geometry	Geometry factor
Central crack tension (CCT)	$F = 1 + 0.128 \left(\frac{a}{w}\right) - 0.288 \left(\frac{a}{w}\right)^2 + 1.525 \left(\frac{a}{w}\right)^3$

Table 4.3 Reliability Result

Type of specimen	Reliability (%)	
	Simple Random Sampling Method	Latin Hypercube Sampling Method (LHS)
Central crack tension (CCT)	94.0402	94.0530

4.4 MODIFIED MONTE CARLO METHOD

Monte Carlo method is a popular method used for analyzing the reliability of the relevant stochastic factors. Therefore, in this part of the output direct Monte Carlo method (DMC) is referred to as the standard. With reference to Figure 4.1, both the sampling method yields fluctuations in the value of reliability, especially when the number of samples is less than 10,000. Use of common random sampling method produces a range of higher reliability of the method of LHS is close to 5% while the LHS approaches 1%. This item shows the LHS sampling can produce the output value is more stable if the contract or the number of samples is less. Reliability values are more stable after 10,000 samples to almost form a straight line parallel to the x-axis. At this point, it is clear that for the center crack problems based on variables such as Table 4.1, the reliability is 94%. This means that the probability of this plate to secure the charged is 94% successful.

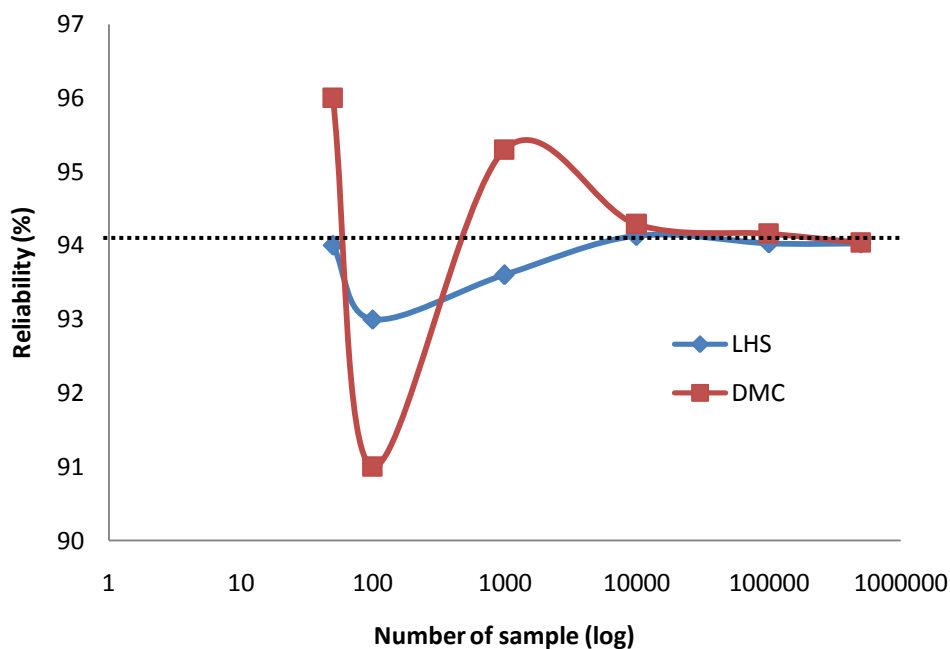


Figure 4.1 The effect of reliability on the number of samples

With reference to Figure 4.2, the constant variance function exponentially decreasing with the increasing number of samples. Variance values decrease and will eventually converging to zero values. Thus, it is evident that the effect of the randomness of the variables will be more focused on increasing the number of experimental samples. Both methods produce a sequence of events is almost the same graph with the graph of the negative logarithm function, it is possible that the LHS effective method to replace the existing DMC.

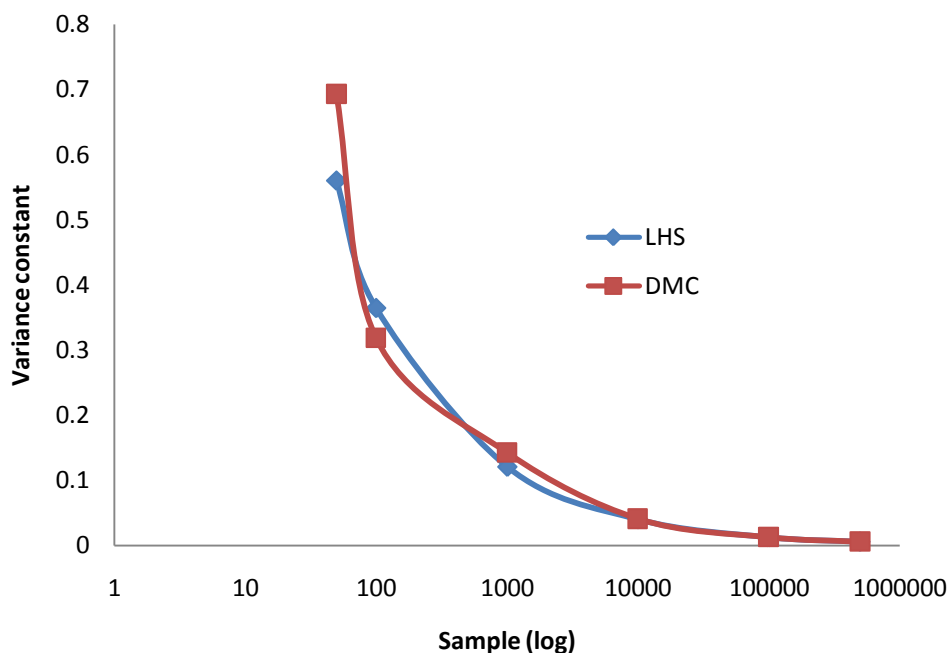


Figure 4.2 Effects of constant variance of the number of samples

Although the DMC method gives the exact value of the product, there is a problem in time efficiency for the simulation. With reference to Figure 4.3, simulation time difference between LHS and DMC methods for interval numbers to 100 000 samples are not significant. However, the interval after a number more than 100 000 samples, both methods show a considerable increase significantly. The slope of the line for the DMC method is higher than the LHS method. Time to operate when the number of simulated samples of 500 000 for the DMC method is 720 seconds (12 minutes) while the LHS method is 350 seconds (5.83 minutes). So reducing the time for the LHS method was 49.33%.

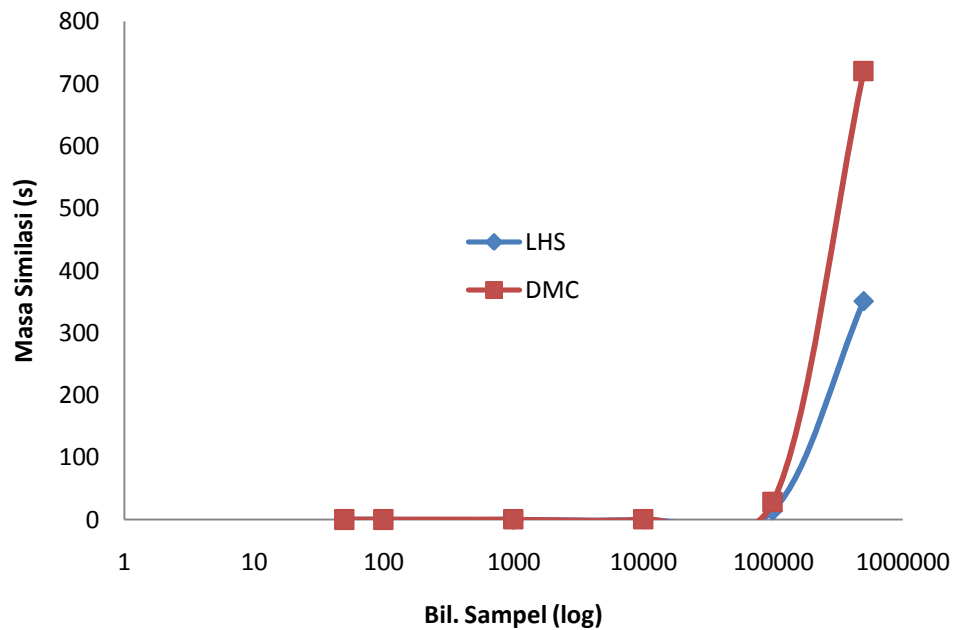


Figure 4.3 The effects of the number of samples

4.5 Sensitivity analysis of random variable

This will be discussed in the sensitivity analysis experiments performed on CCT specimens. Sensitivity analysis performed is the effect of loading and crack size and the changes that occur on the reliability of the different types of materials.

4.5.1 Loading

Referring to Figure 4.4, the figure is divided in two parts. Part A is the effect of stress to 225 MPa, while Part B is the effect of tensile force exceeding 225 MPa. It was found that the reliability of the A mode to follow a negative exponential graph, while Part B is the negative logarithm mode.

Found when the loading is less than 100 MPa, the value of the reliability change slightly, it can be concluded that the limit of resistance for this specimen are within about 100 MPa. Hence the reliability began to decline when the load began to approach the yield point.

In Part B, the LEFM approach was not applied again after loading beyond the yield point. Thus the appropriate approach is Elastic-Plastic Fracture Mechanics. This method is suitable to characterize the structure of that experience low cycle fatigue and high stress.

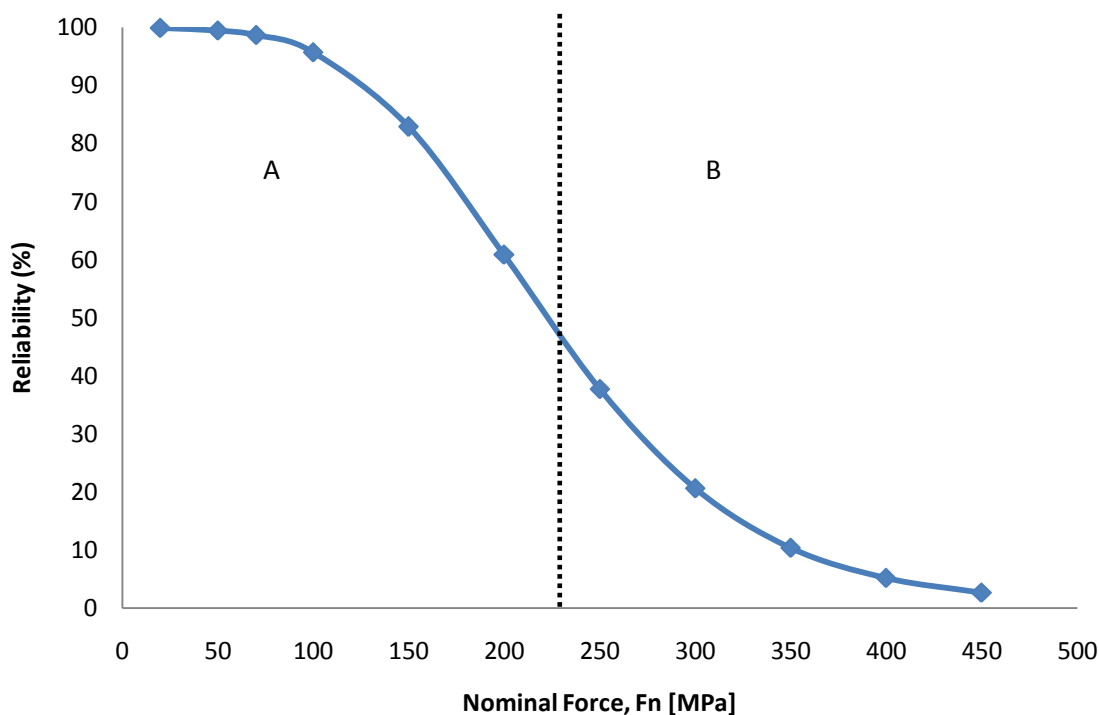


Figure 4.4 Reliability effects of the addition of force

4.5.2 Crack Size

This section discusses the size of the crack affects the reliability of CCT specimen. Crack size affects the reliability for the larger size will increase the intensity of stress cracks. Referring to Figure 4.5, found the reliability decreases exponentially with the size of the crack.

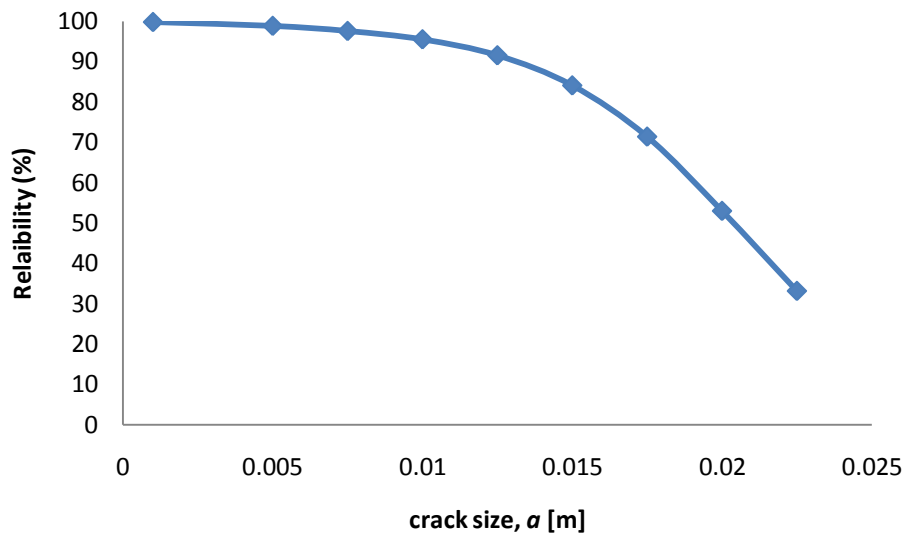


Figure 4.5 Effects of crack size on the reliability

4.5.3 Stress Intensity Factor

Figure 4.6 describes the effects of stress concentrations on the reliability. Different stress intensity value also means the use of different materials. Materials that have a lower stress intensity below $40 \text{ MPa}\sqrt{m}$ will experience a reduction in reliability when faced with a situation of stress of 100 MPa. The average reliability of a stress intensity above $40 \text{ MPa}\sqrt{m}$ is approaching 99% if the loading is maintained.

Thus, when an engineer wants to design a component or a structural engineering, material selection is important so that the same operating expenses.

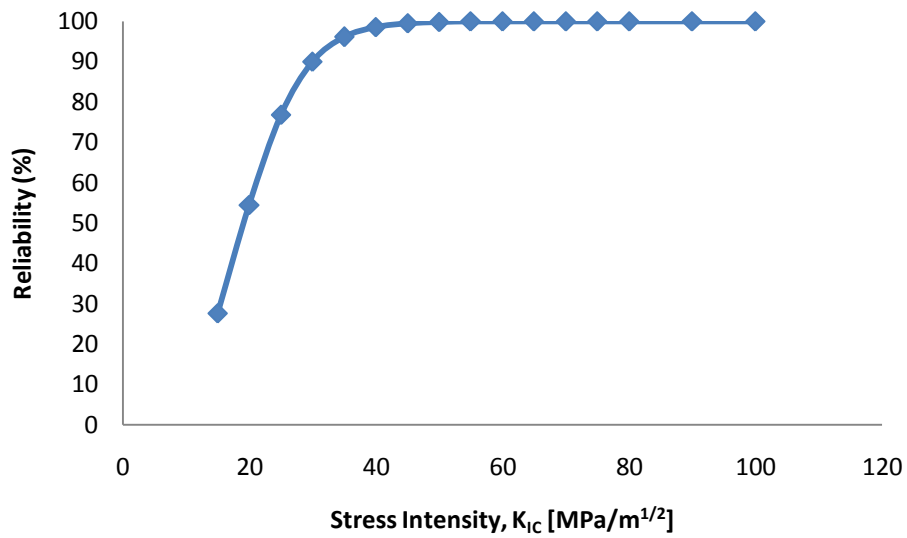


Figure 4.6 Effect of stress intensity on the reliability

4.6 ENGINEERING APPLICATION

4.6.1 Problem statement

Figure 4.7 refers to the initial crack in this case is caused by two full penetration welds on the flange below the base of the box girder bridge (Chung 2004). Randomness of the uncertainty prevailing in the variable loading, crack size and mechanical properties of materials used. Therefore, the reliability analysis is applied to the structure.

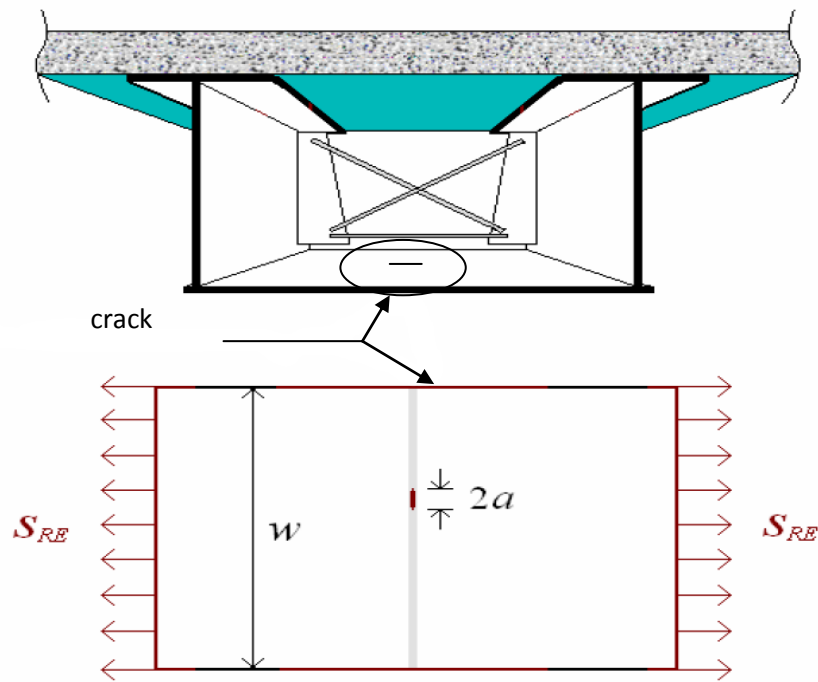


Figure 4.7 Modeling of Box Bridge to CCT specimens

Referring to equation 3.1 the reliability and fatigue reliability index can be obtained. Based on the Chung (2004), equation 3.1 can be adjusted to factor into the equation 4.2 can be included in the operating cycle.

$$g(x) = \int_{a_0}^{a_c} \frac{da}{[f(a/w)\sqrt{\pi a}]^m} - C \cdot S_{RE}^m \cdot (365 \cdot ADTT \cdot C_s \cdot Y) \quad (4.2)$$

where a_c is critical crack size, a_0 is initial crack size, $f(a/w)$ is geometry factor, C and m are fatigue propagation parameters, C_s is the number of stress cycles per truck passage, $ADTT$ is the average daily truck traffic, S_{RE} is the spectral range of stress for the flange under the bridge, and Y is the length of service in years.

For the geometrical factors Chung (2004) has been modeled as a central crack as equation 4.3

$$f(a/w) = \frac{1 - 0.5(a/w) + 0.370(a/w)^2 - 0.044(a/w)^3}{\sqrt{1 - (a/w)}} \quad (4.3)$$

where a is the crack size and w is half the specimen width W . Summary information related to the values of random variables in this study are shown in Table 4.4

Table 4.4 Description of the relevant variables in the Bridge Case Box

Variable	Type Distribution	Average	COV
a_0	Lognormal	0.020 in	0.500
a_c	constant	2.000 in	-
C	Lognormal	2.05×10^{-10}	0.630
m	Normal	3.000	0.100
S_{RE}	Normal	9.85 ksi	0.300
C_s	constant	1.000	-
$ADTT$	constant	300.000	-
W	constant	42.000 in	-

source Chung (2004)

According to (Chung, 2004) if the initial crack size, a_0 when compared with the relatively wide bottom flange is small (a_0/W) $\approx 3.3 \times 10^{-4}$, the geometry factor $f(a/w)$ can be simplify as the stress intensity factors for cracks.

After a simulation run based on the equation of 4.2, according to Cheung and Li (2003) the probability of failure that can be used to find the reliability index based on equation 4.4.

$$\beta = -F^{-1}(P_f) \quad (4.4)$$

where $F^{-1}()$ is the inverse of the cumulative probability distribution function of normal.

4.6.2 Results

Figure 4.8 describes the effects of operating life of reliability index. Engineering structures are usually designed to achieve the operation for 100 years. Reliability

index of the bridge was found to decrease with a logarithmic function. This has coincided that reliability will decrease the longer the period of operation.

The probability of failure is agreed to by (Chung 2004) is 0005. If translated in the form of reliability index is 2.5758 which is a horizontal red line in figure 4.8. Crosses between the red line to the line against the lives of the reliability index is the optimum frequency of the inspections of the box girder bridge is approximately 10 years. A more accurate examination period can be obtained from equation 4.5, which is the fitting curve against the lives of the reliability index.

$$\beta = -0.85 \ln Y + 4.552 \quad (4.5)$$

after the modified equation 4.5 will be the equation 4.6

$$Y = 211.73e^{-\beta/0.85} \quad (4.6)$$

hence, the optimum inspection period is 10.22 years. So, to check the appropriate box girder bridge after construction is completed after 122 months of operation.

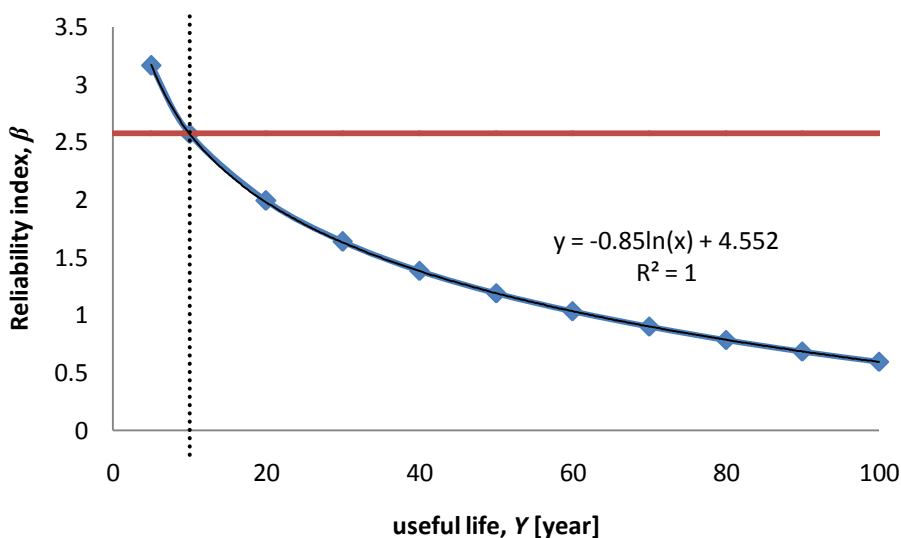


Figure 4.8 Effects of useful life of structural reliability index