3.2.1 Maximum Normal Stress

Internal force is the shear force, *V* has a magnitude equal to the load P and bending moment, M. Bending moments are then creating the normal stress on the cross section, while the shear force, *V* produces shear stress on the section. Maximum normal stress, S_{max} is a parameter that is commonly used in structural design. Maximum normal stress depends on the bending moment and section modulus, *s* and then:

$$S_{max} = \frac{|M_{max}|}{s} \tag{3.1}$$

and

$$s = \frac{I}{c} \tag{3.2}$$

where *c* is the maximum distance from the neutral surface and *I* is the moment of inertia of the cross-section. The bending moment equation for a uniform load beam structure having a support and a simple support is shown below:

$$M = -\frac{w}{8}(4x^2 - 5lx + l^2) \tag{3.3}$$

where *w*, *x*, dan *l* are the distributed loading, distance and length of the beam.

3.2.2 Reliability Assessment Approach Using Normal Probability

Reliability equation can be written as below:

$$P_r = 1 - P_f \tag{3.4}$$

where P_f is the probability of structural failure and P_r can be interpreted as a measure for the reliability.

Safety margin equation, Z is:

$$Z = S_y - S_{max} \tag{3.5}$$

The probability of a structural failure is when the Z value is less than or equal to zero and can be represented by the equation:

$$P_f = P(Z \leq 0) \tag{3.6}$$

where S_y and S_{max} are normal independent variables.
Thus, the mean value of yield strength and maximum stress are respectively m_{S_y} and $m_{S_{max}}$. While the standard deviation of the yield strength and maximum stress are respectively σ_{S_y} dan $\sigma_{S_{max}}$. The combination of the two independent variate will generate a new variate with mean and standard deviation different from the original. The equation of mean and standard deviation of the margin of safety is:

$$m_Z = m_{S_y} - m_{S_{max}} \tag{3.7}$$

$$\sigma_Z = \left(\sigma_{S_y} - \sigma_{S_{max}}\right)^{1/2} \tag{3.8}$$

The probability of structural failure can be identified with the function as below:

$$P_f = \Phi\left(\frac{0 - m_Z}{\sigma_Z}\right) \tag{3.9}$$

where Φ is the standard normal distribution function and m_Z and σ_Z are the mean and standard deviation of the safety margin.

3.3 STRUCTURAL ANALYSIS USING FUZZY THEORY

Procedures for structural reliability analysis using fuzzy sets theory begins with the fuzzification of uncertainty input and followed up with cutting-α, defuzzification and reliability assessment.

3.3.1 fuzzification of Uncertainty input

Fuzzification can be interpreted as a specification of the membership function $\mu(x)$ of an sets of uncertainty. The uncertainty of each parameter that is interpreted by a membership function that will bring value to the trend. Two parameters of the modulus of section, s and loading, *w* are used as fuzzy parameters.
Fuzzy normal stress is the result after the α-cut. It was found that, the normal stress depends on and bending moment and modulus of section, while the bending moment depends on the load.

Since both loading and bending moment is the dependent variable, then the bending moment is a fuzzy parameter. Triangular fuzzy numbers are used for understanding the function of all parameters of fuzzy membership. Upper limit value (s_u , w_u, M_u) and the lower limit (s_l , w_l, M_l) of these functions will be determined by expert opinion. While the middle value (s_t , w_t, M_t) is between the upper and lower limit. Then the two parameters are mapped to a decision (output) with the α-cut.

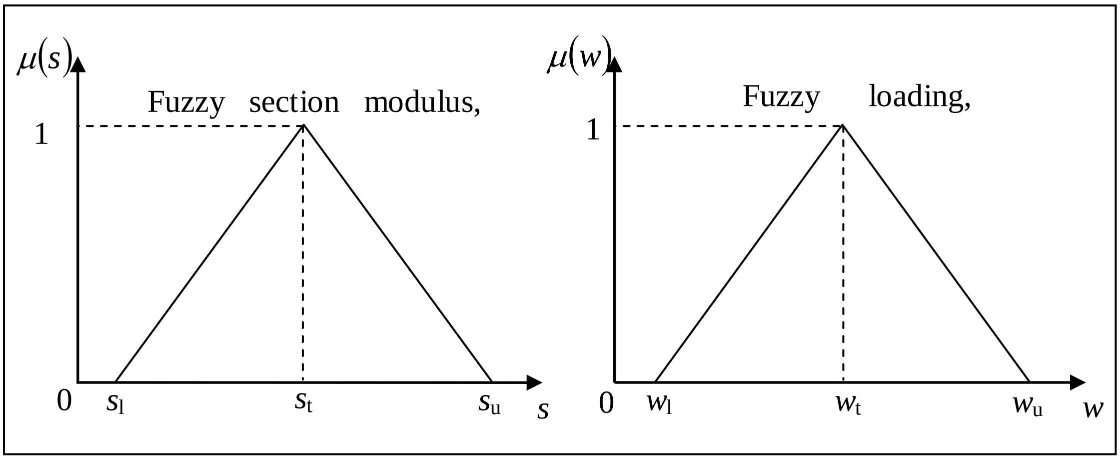


Figure 3.1 Membership functions with triangular fuzzy numbers for the modulus of section and loading

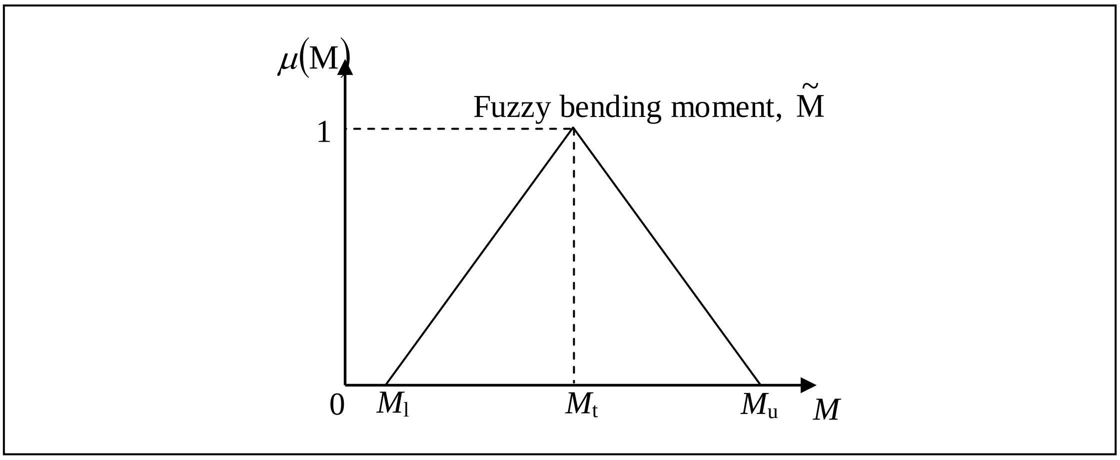


Figure 3.2 Membership functions with triangular fuzzy numbers of bending moment

Figure 3.1 and 3.2 shows that the tendency for the upper limit of the bending moment, M_l and the lower limit, M_u is zero, ie no tendency for the moment, whereas the trend with a value of 1 implies that the trend of the moment is getting a hundred per cent, and symbolized by the symbol M_t.

3.3.2 α-cut

The α-cut is one way of mapping that maps fuzzy input to fuzzy output with specific functions (Moller et al., 2000). The term mapping is specified here to mean logical relationship between two or more entities. Mapping of the input (modulus of section, *s* and the bending moment, *M*) to output (the maximum normal stress, S_{max}) performed after the fuzzification. In this process, all the fuzzy section modulus, $\tilde{s}$ and fuzzy bending moment, $\tilde{M}$ at each stage of the trend, α mapped by using equations 3.1. This mapping resulted in four significant values for each level of the trend, α at the normal stress space, S. The maximum and minimum values are selected from a combination of these results and used as the upper limit and lower limit for the output fuzzy.

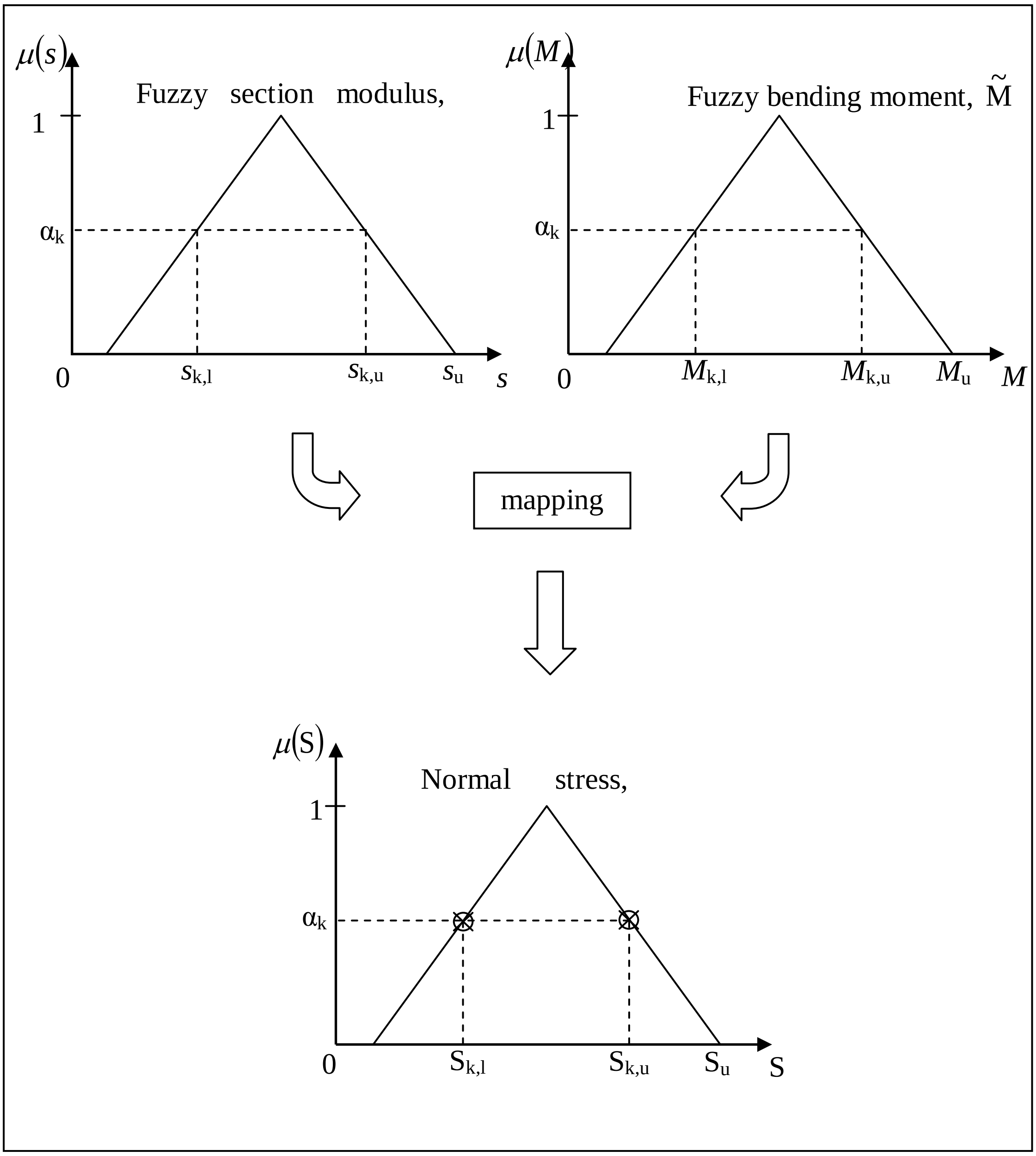


Figure 3.3 mapping of $\tilde{s}$ and $\tilde{M}$ at at all stages of the trend, α

3.3.3 Defuzzification of Normal stresses

Defuzzification of Normal stress is using the center of gravity or centroid. This technique determines the point at which it will distribute one area (area graph) into two parts which have the same value. Mathematically, the point is called center of gravity, COG (Negnevitsky, 2005). COG is expressed as equation 3.10.

$$\mathrm{COG} = \frac{\int_{S_l}^{S_u} S\mu_S(S)dS}{\int_{S_l}^{S_u} \mu_S(S)dS} \tag{3.10}$$

By applying the numerical solution methods, namely trapezoidal rule, the equation of the new COG would be formed as shown in equation 3.11.

$$\mathrm{COG} = \frac{\sum_{i=1}^{n-1} (S_o)\frac{1}{2}(\mu_i + \mu_{i+1})(S_{i+1} - S_i)}{\frac{h}{2}\left[(\mu_{S_l} + \mu_{S_u}) + 2\sum_{i=1}^{n-1}\mu_i\right]} \tag{3.11}$$

where

$$S_o = \frac{3(\mu_i + \mu_{i+1})S_i + (\mu_i + 2\mu_{i+1})h}{3(\mu_{i+1} + \mu_i)} \tag{3.12}$$

3.3.4 Fuzzy Structural reliability definition

The definition of fuzzy reliability of the structure begins with the determination of safety margins, Z with COG along with the real maximum normal stress, S_{max}. Given that the yield strength of the variable distribution is normal. So the probability is used in the calculation P_r with a mean value, m_{Sy} and standard deviation, σ_{S_y} is fixed. In this context, the probability of structural failure is the maximum normal stress exceeds the yield strength and can be represented by the following equation:

$$P_f = P(S_y \leq S_{max}) = \Phi\left(\frac{S_{max} - m_{S_y}}{\sigma_{S_y}}\right) \tag{3.13}$$

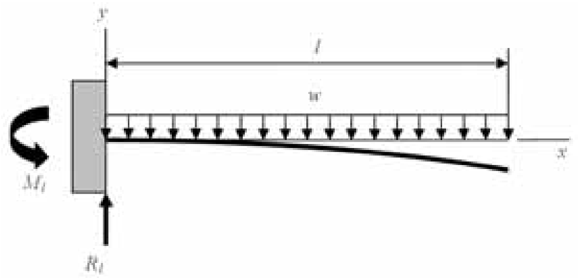
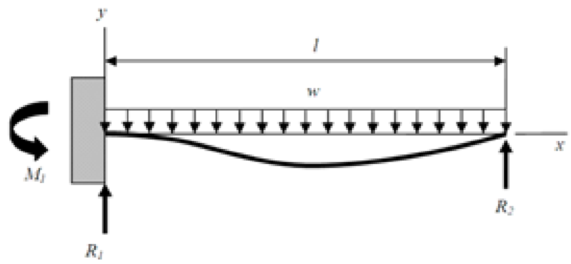
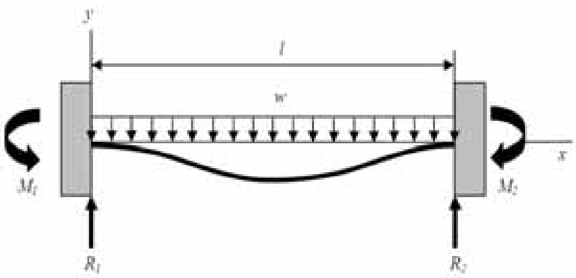
Then the reliability, P_r can be evaluated using equation 3.4.

Maximum normal stress:

$$\sigma_m = \frac{Mc}{I} = \frac{M}{s} \tag{3.19}$$

where s is the modulus of section.

Table 3.1 Shear, moment and deflection of the beam structure statically determinate and indeterminate

	$R_1 = wl \qquad M_1 = \frac{wl^2}{2}$ $V = w(l-x) \qquad M = -\frac{w}{2}(l-x)^2$ $y = \frac{w\,x^2}{24E\,I}\left(4lx - x^2 - 6l^2\right)$ $y_{\max} = -\frac{w\,l^4}{8E\,I}$
	$R_1 = \frac{5wl}{8} \qquad R_2 = \frac{3wl}{8}$ $M_1 = \frac{wl^2}{8} \qquad V = \frac{5wl}{8} - wx$ $M = -\frac{w}{8}\left(4x^2 - 5lx + l^2\right)$ $y = \frac{w\,x^2}{48E\,I}(l-x)(2x-3l)$
	$R_1 = R_2 = \frac{wl}{2} \qquad M_1 = M_2 = \frac{wl^2}{12}$ $V = \frac{w}{2}(l-2x)$ $M = \frac{w}{12}\left(6lx - 6x^2 - l^2\right)$ $y = -\frac{w\,x^2}{24E\,I}(l-x)^2 \qquad y_{\max} = -\frac{w\,l^4}{384E\,I}$

3.4.3 Centroid

The equation can be written as:

$$\bar{x} = \frac{\int_A \tilde{x} dA}{\int_A dA} \tag{3.24}$$

3.5 ANALYSIS OF MAXIMUM NORMAL STRESS

The loading, w is 5.55 MN/m and moment of inersia, I is 6.58 x 10^{-5} m^4. Height of the beam structure is 0.161m and the structure is made of aluminum alloy 2024-T4 where the elastic modulus is 73.1 GPa. The statistical distribution of aluminum 2024-T4 where the mean value and standard deviation of the yield strength are 324 MPa and 32.4 MPa respectively.

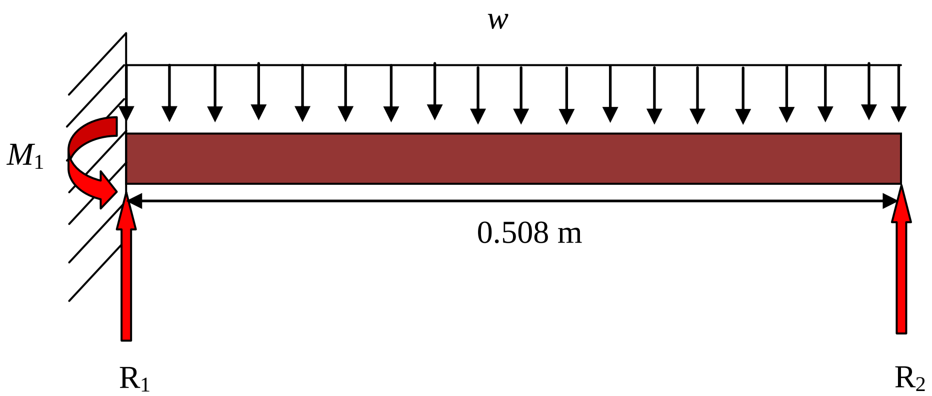


Figure 3.5 Free body diagram of the beam structure

3.5.1 Deterministic method

The deterministic method is the most common method to help engineers to solve problems relating to the loading on the beam structure. The stress values obtained using the method is 219 MPa. The stress value is then compared to the statistical distribution of aluminum 2024-T4 in which the mean and standard deviation of the yield strength is 324 MPa dan 32.4 MPa respectively.

3.5.2 Stochastic Methods

Stochastic methods are also derived from the deterministic method in which all common terms are also used in stochastic methods. The difference between the two methods are stochastic methods involving the distribution of data for input parameters, such as in this study were uniformly distributed load parameter, *w* and moment of inertia, *I*. The input parameters for uniformly distributed load and moment of inertia are in normal form with constant variance of 0.2. By using stochastic methods and taking into account the effects of error propagation in the beam of maximum stress is 366 MPa and the structural reliability is 0.9649.

3.5.3 Fuzzy methods

Normal stresses can be determined using equation 3.19 and bending beam structure can be determined using equation 3.3. Figure 3.6 shows the upper, middle and lower limit for each input parameter (uniformly distributed load and moment of inertia) and the fuzzy normal stress output. Figure 3.7 shows the location of COG in the fuzzy normal stress profile.

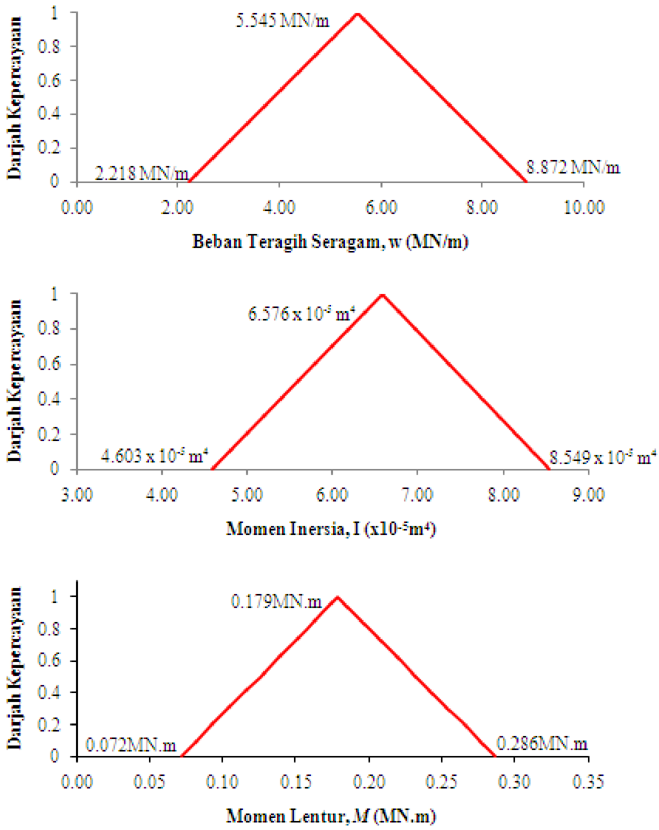


Figure 3.6 Figure membership functions for loading, *w*, section modulus, *s* and the bending moment, *M*

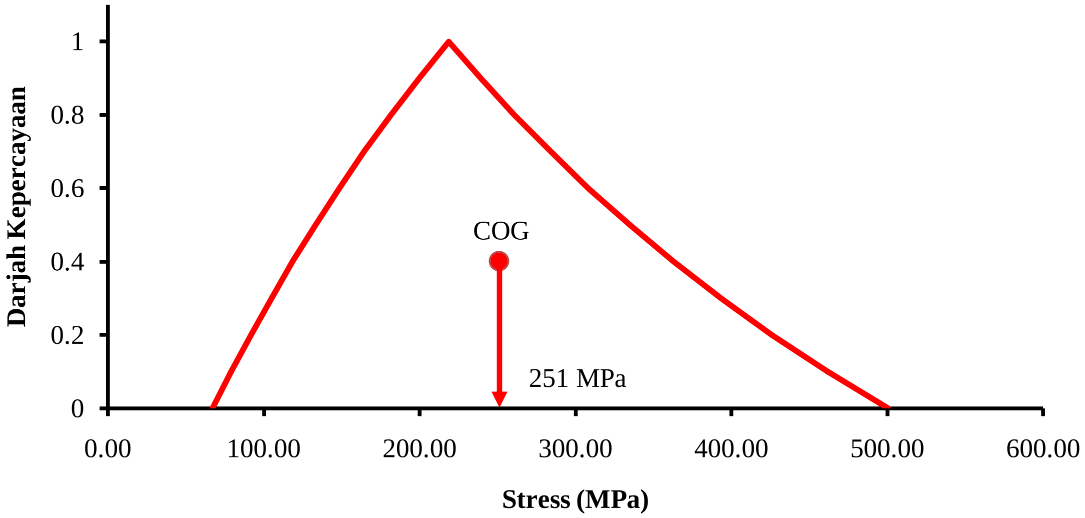


Figure 3.7 Fuzzy maximum normal stress diagram

Results showed that the maximum normal stress for the structural support beam is 251 MPa. The reliability of the structure is 0.9879 and the value obtained by comparing the value of maximum stress with the distribution of data in 2024-T4 aluminum material as described in section 3.5.

3.6 FUZZY FINITE ELEMENT METHODS (FFEM)

Figure 3.8 shows the flow chart for structural analysis. Mapping function is based on the finite element method.

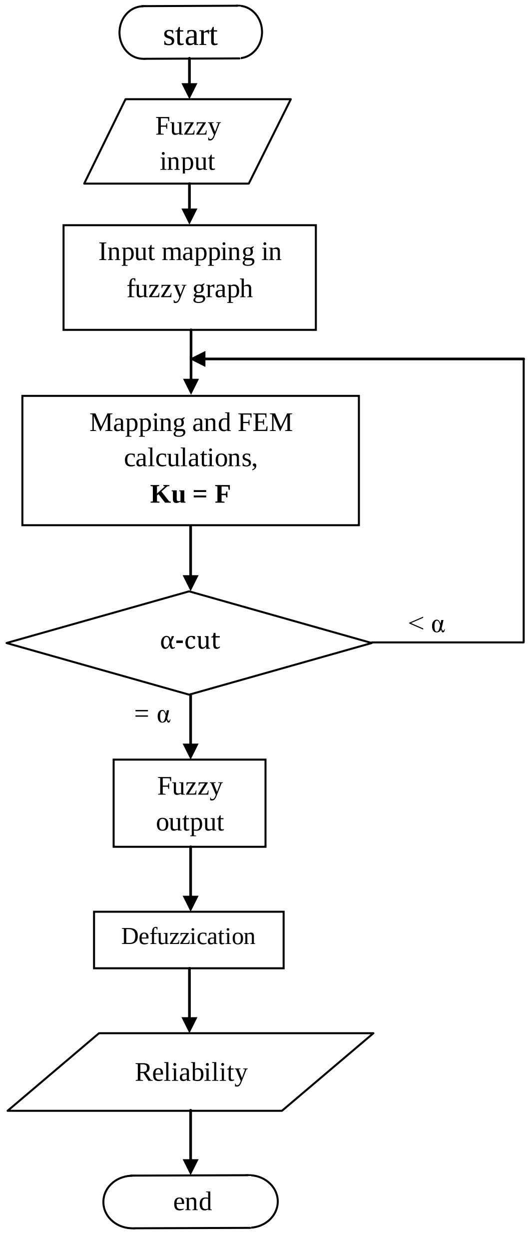


Figure 3.8 The flow chart of structural analysis

RESULTS AND DISCUSSION

Table 4.1 describe the physical properties involved in this reliability analysis.

Table 4.1 Mechanical Properties of Aluminum 2024-T4

Physical Properties	SI unit
Density	2780 kg/m^3
Ultimate strength (tension)	469 MPa
Yield Strength (Tension)	324 MPa
Elastic Modulus	73.1 GPa

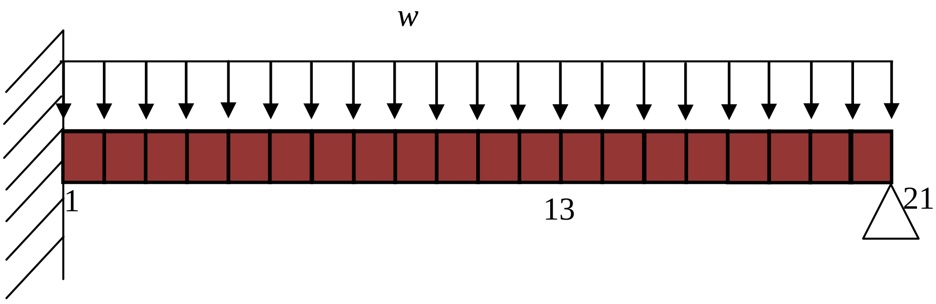


Figure 4.1 position of the nodes on the beam structure

Table 4.2 Types of entry

Method	**Input Type**	
	Uniform Distributed load	Moment of Inertia
Determinent	Average Value	Average Value
Stochastic	Normal distribution[a]	Normal distribution[b]
Fuzzy	Triangular Membership Function[c]	Triangular Membership Function[c]
FFEM	Triangular Membership Function[c]	Triangular Membership Function[c]

[a] COV = 0.2

[b] COV = 0.1

[c] width = 6σ normal distribution

4.3 FUZZY FINITE ELEMENT METHOD (FFEM)

4.3.1 Deflection

From the graph of deflection against the nodes, the highest possible deflection is located at node 13. Black lines represent the minimum and maximum, the green line is the highest confidence level. While the red line graph represents the deflection for the COG. COG value is not equal to the value at the highest peak. The value of the maximum deflection of the beam structure is equivalent to the value of the COG deflection components at node 13 is 0.5mm.

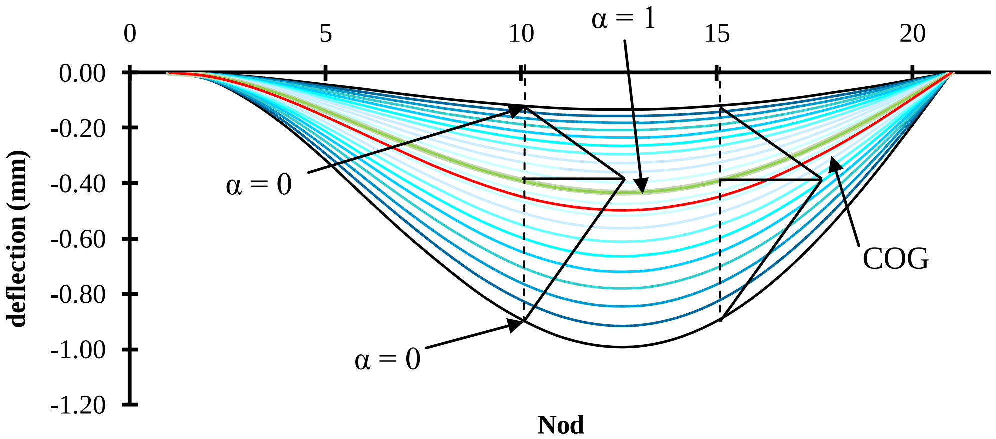


Figure 4.2 fuzzy deflection against beam length

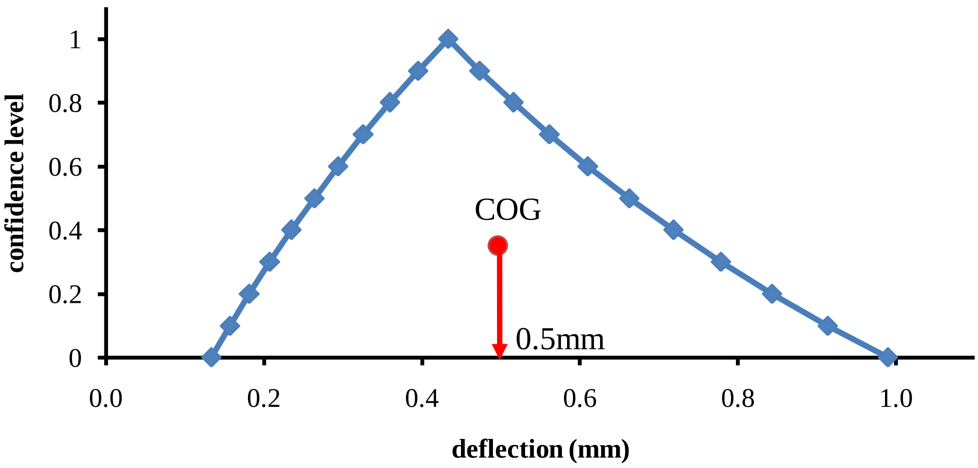


Figure 4.3 maximum deflection of the fuzzy

4.3.2 rotation

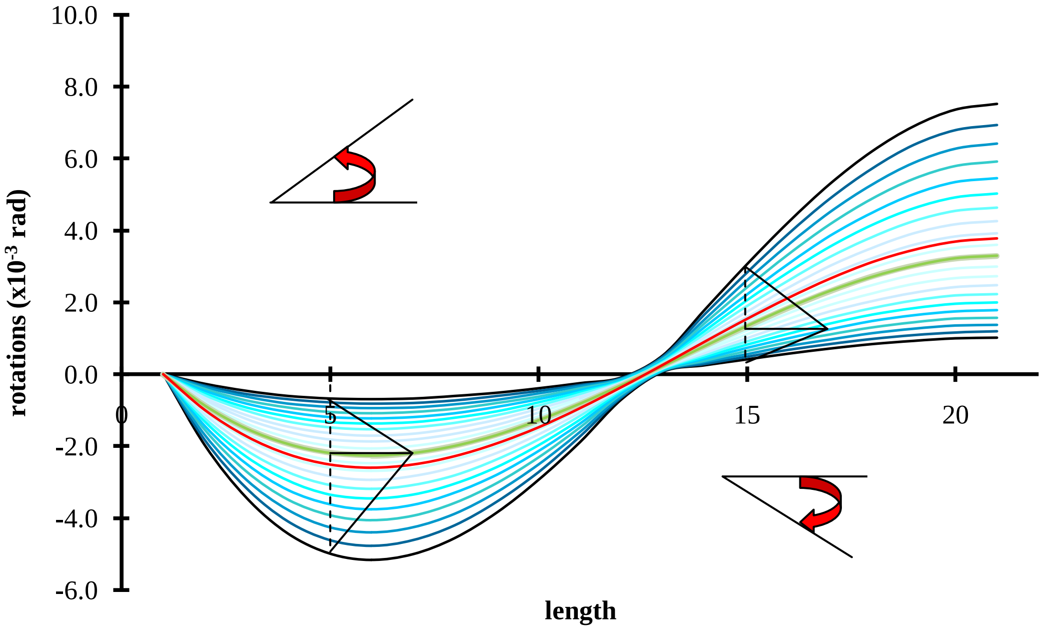


Figure 4.4 fuzzy rotation against beam length

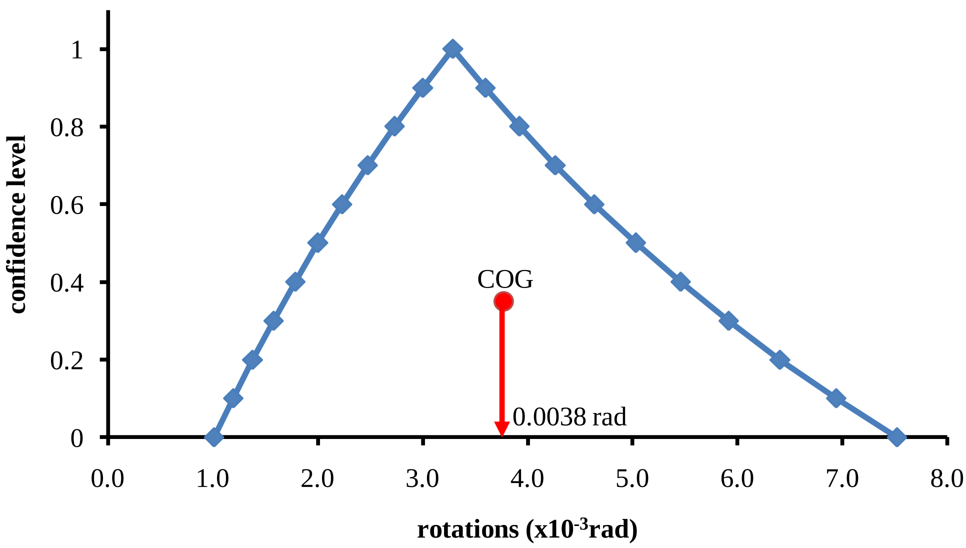


Figure 4.5 maximum fuzzy rotation

4.3.3 bending moment

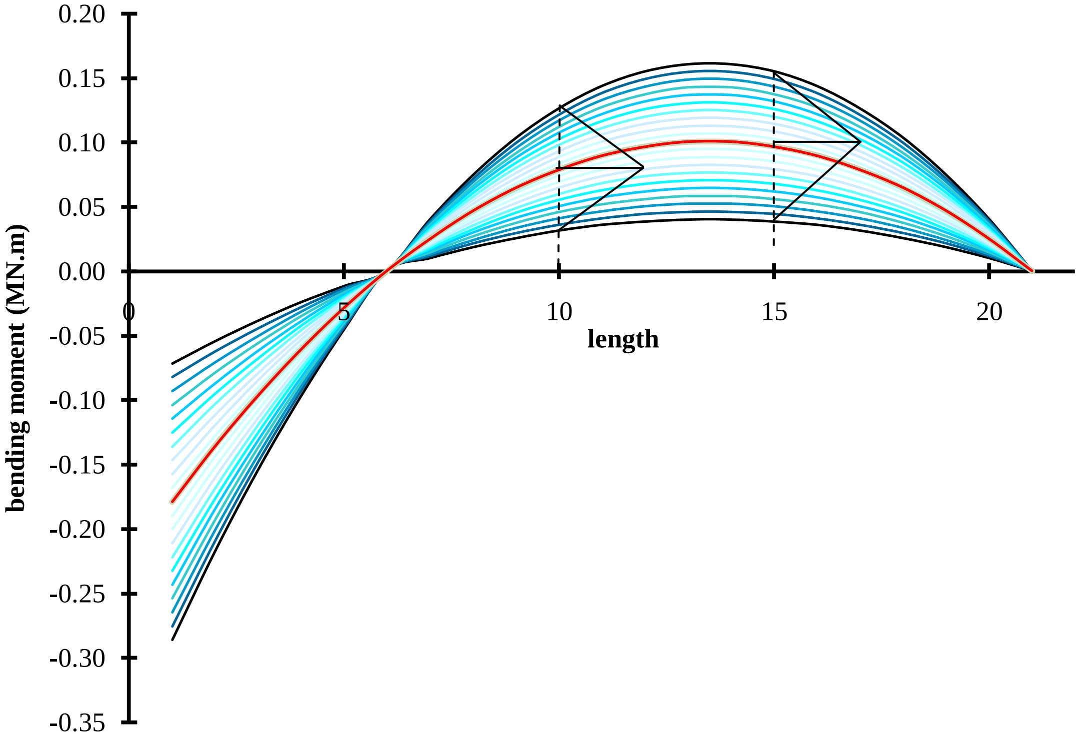


Figure 4.6 fuzzy bending moment against beam length

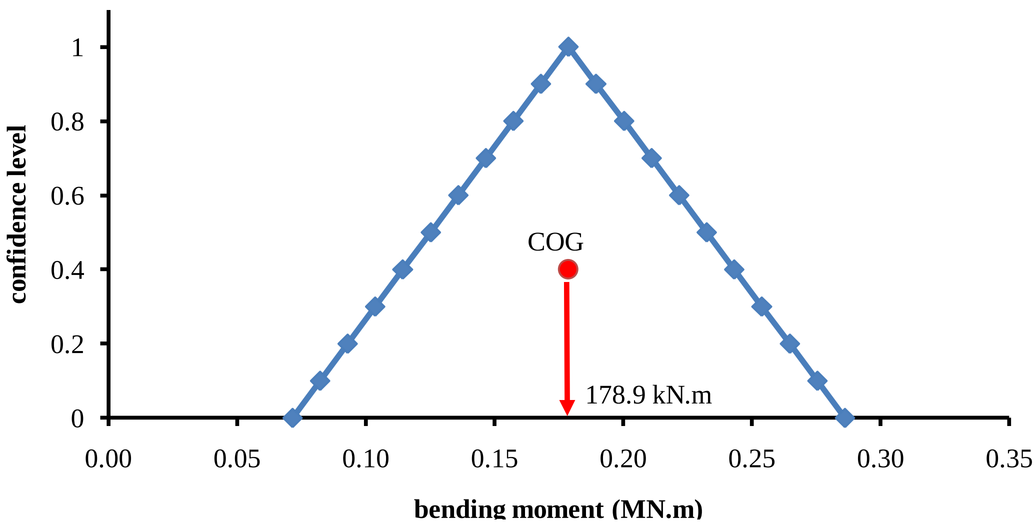


Figure 4.7 maximum fuzzy bending moment

4.3.4 shear force

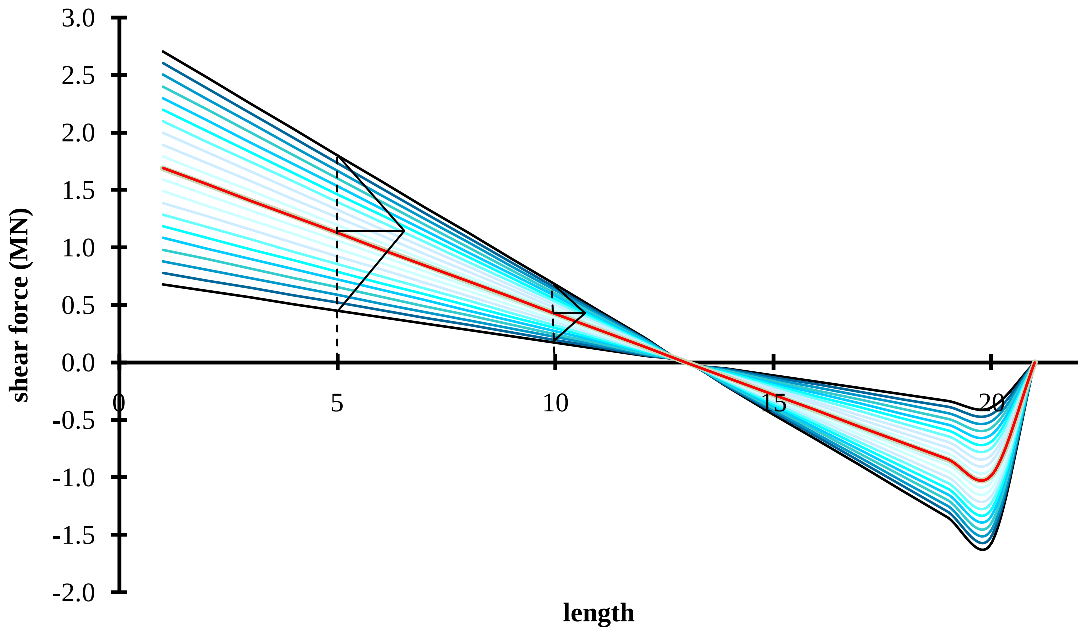


Figure 4.8 fuzzy shear force against beam length

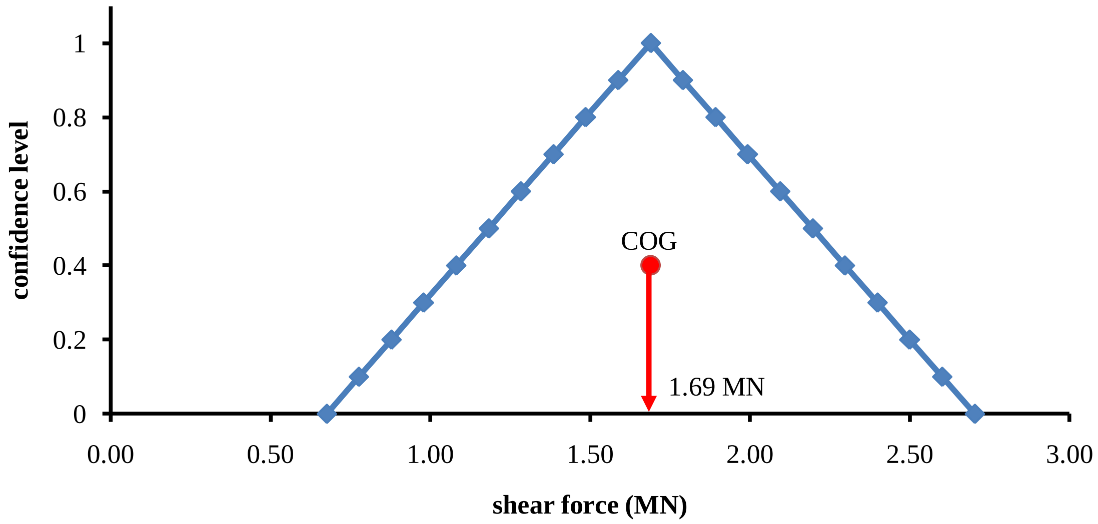


Figure 4.9 maximum fuzzy shear force

4.3.5 Bending Stress

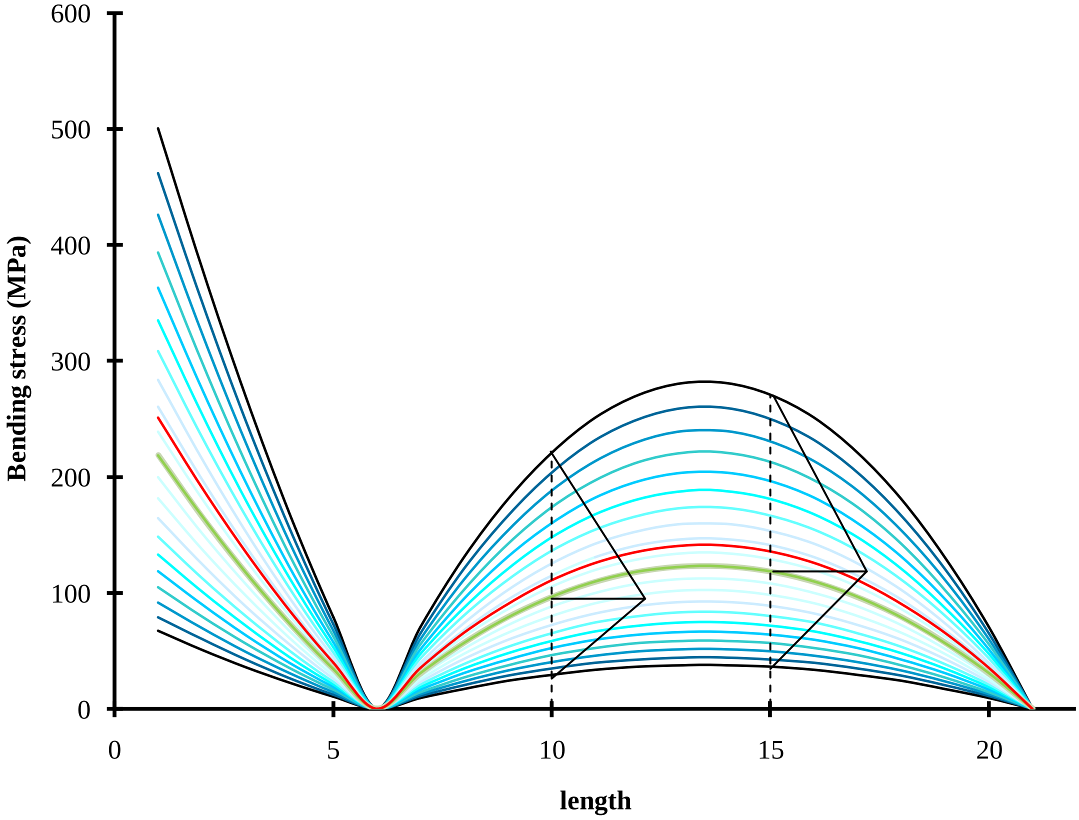


Figure 4.10 fuzzy bending stress against beam length

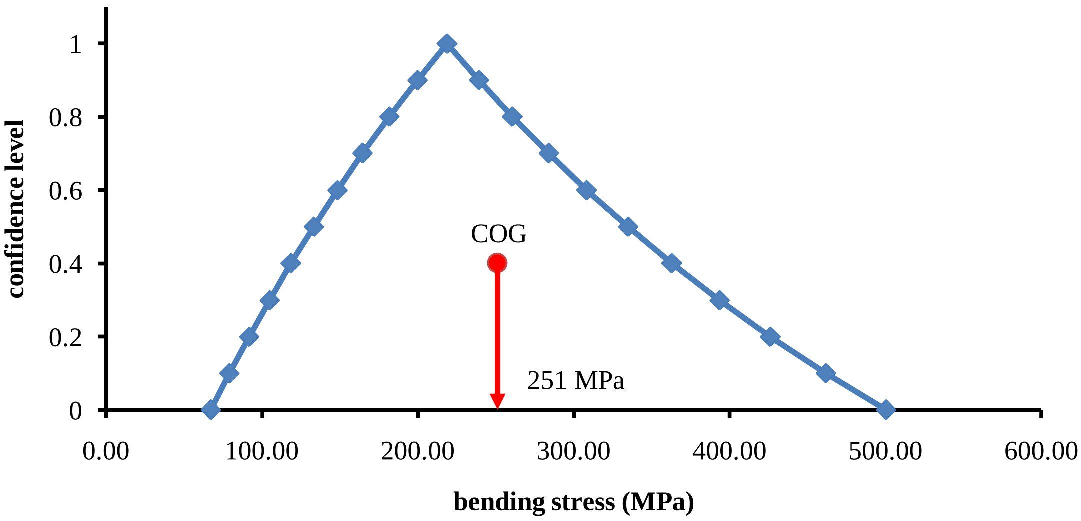


Figure 4.11 maximum fuzzy bending stress

4.4.1 node number

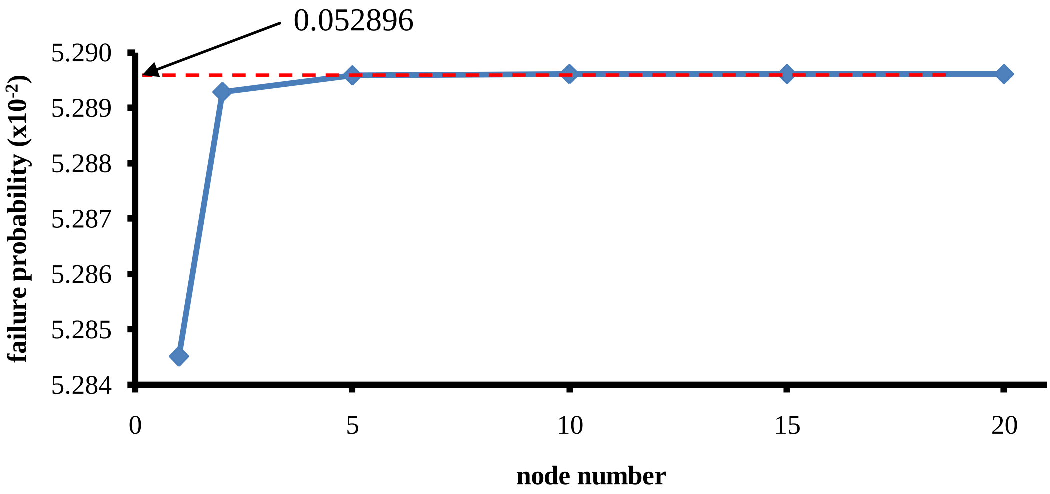


Figure 4.12 failure probability against node number.

4.4.2 α-cut

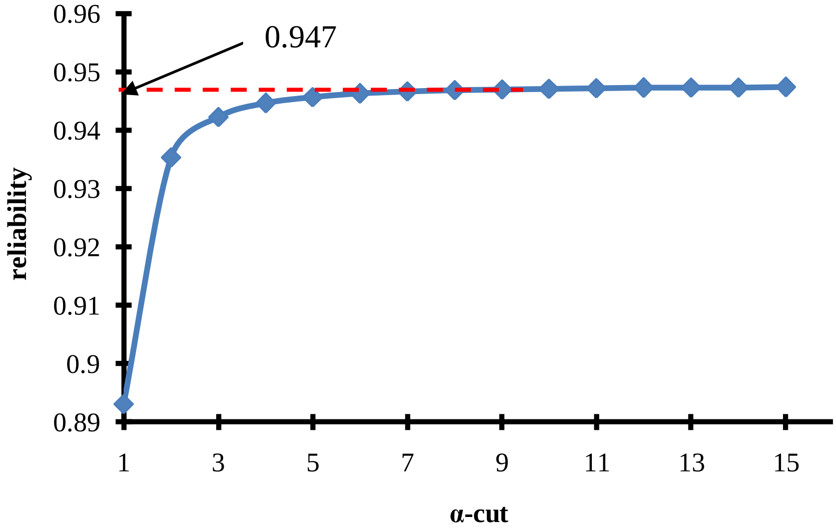


Figure 4.13 reliability against α-cut

4.4.3 Pemalar Varian

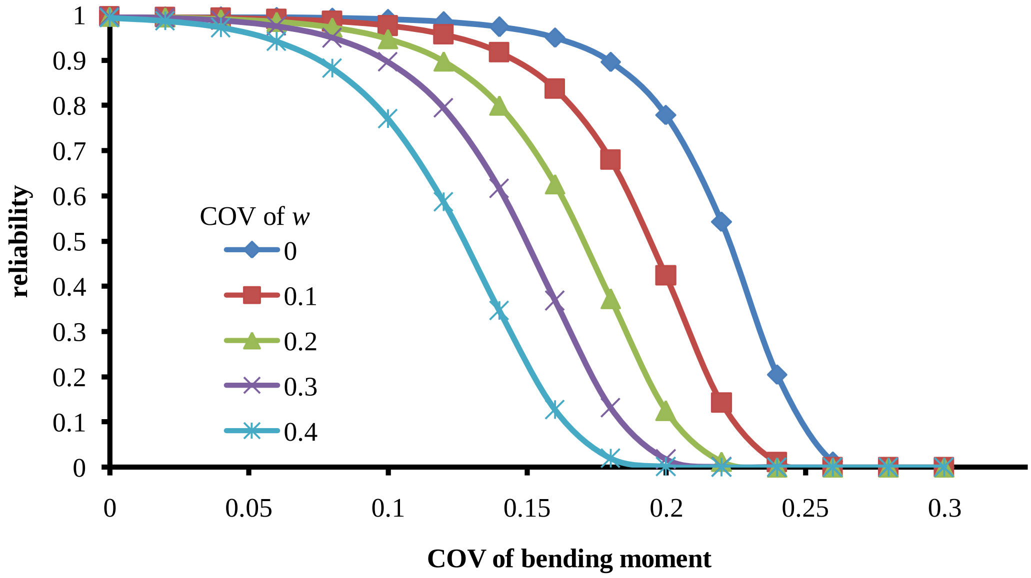


Figure 4.14 Relationship between the reliability of COV of moment of inertia and COV of distributed load

4.5 ANALYSIS OF FFEM

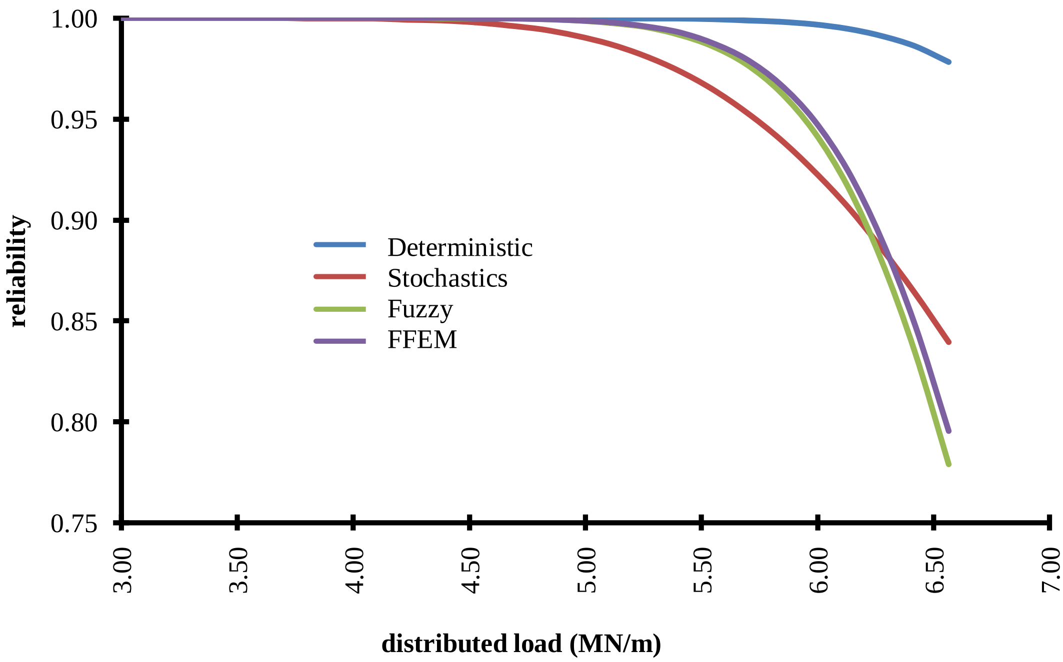
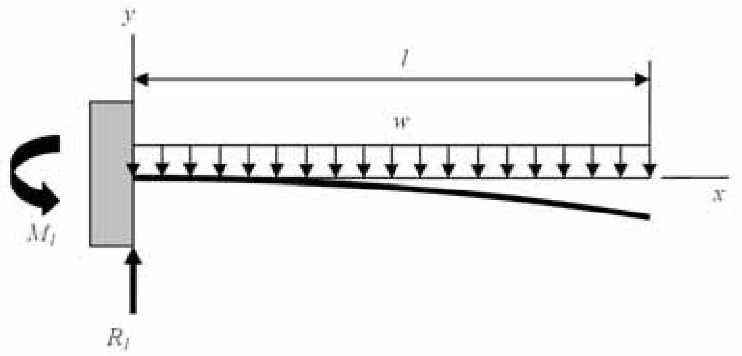


Figure 4.15 Comparison of analytical methods in structural reliability analysis

4.6 OTHER TYPE OF BEAMS



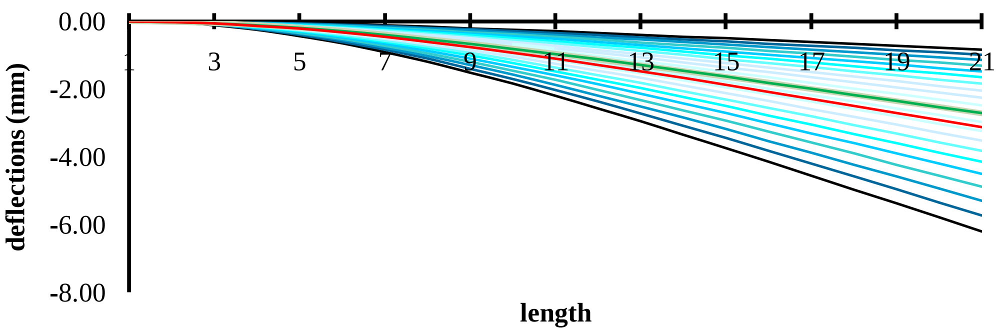


Figure 4.16 fuzzy deflections

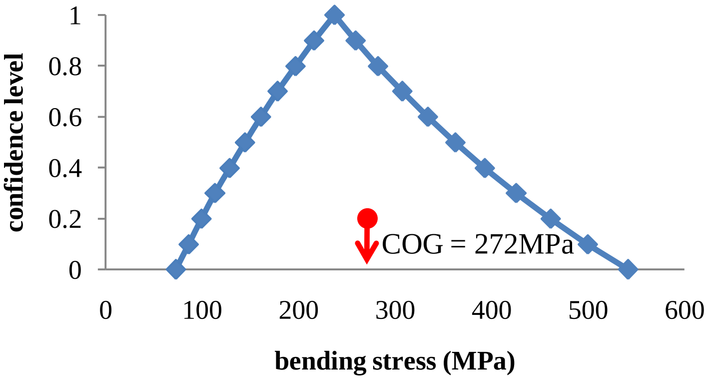


Figure 4.17 membership function of bending stress

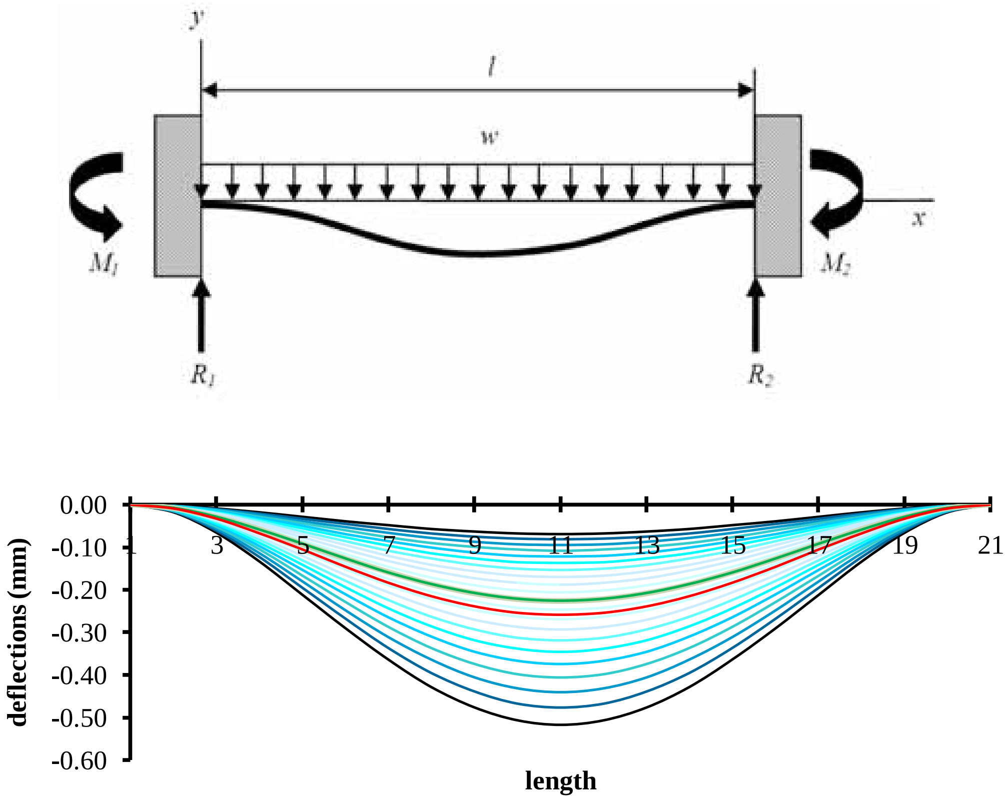


Figure 4.18 fuzzy deflections

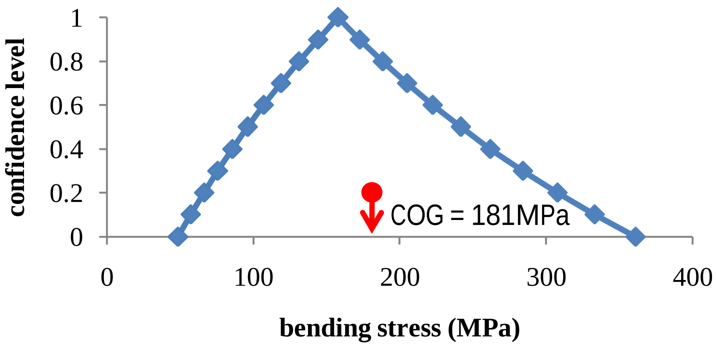


Figure 4.19 membership function of bending stress