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Grover's Algorithm Extensions – A Systematic Literature Review

Sambungan Algoritma Grover - Kajian Literatur Sistematik

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ABSTRACT

Quantum computing has emerged as a transformative field, offering superior computational capabilities over its classical counterparts in solving complex problems. Among the most well-known quantum algorithms is Grover's algorithm, which was introduced as a quantum search algorithm in an unstructured database. It achieves quadratic speedup, requiring only $O(\sqrt{N})$ queries compared to the classical $O(N)$, making it highly relevant for the information-processing. Despite its strengths, Grover's algorithm is widely recognized to have been limited, depending on its intended utilization, prompting numerous perceptive improvement extensions were born. The goal of this study was to identify established Grover's algorithm extensions through a systematic literature review. Indexed article items published between 2019 and 2025 were selected from different sources, based on specific defined keywords. The review classifies extensions into three primary aspects; time-complexity, optimization and quantum cost, including their bi-combinations and tri-combinations. Additionally, a conceptual framework is proposed to summarize these contributions and serve as guidance for future research. The findings highlight how the extensions address performance challenges and expand the applicability of Grover's algorithm across different domain applications. This review not only provides a clearer and establishes a foundation for future developments in quantum search algorithms and their broader utilization in quantum computing applications.

Keywords**:** Quantum algorithm, Quantum computing, Grover's algorithm, Quantum search.

ABSTRAK

Pengkomputeran kuantum telah muncul sebagai bidang yang transformatif, menawarkan keupayaan pengiraan yang lebih unggul berbanding kaedah klasik dalam menyelesaikan masalah yang kompleks. Elemen utama yang membolehkan keupayaan ini ialah algoritma kuantum. Antara algoritma kuantum yang paling terkenal ialah algoritma Grover, yang pada asalnya diperkenalkan sebagai algoritma carian kuantum dalam pangkalan data tidak berstruktur. Ia mencapai kelebihan kuadratik, memerlukan hanya O(√N) pertanyaan berbanding O(N) secara klasik, menjadikannya sangat relevan untuk meningkatkan kelajuan pemprosesan maklumat. Walaupun mempunyai kekuatan, algoritma Grover diakui mempunyai batasan tertentu bergantung pada penggunaannya, sekali gus mendorong pelbagai peluasan algoritma ini sepanjang kewujudannya. Kajian ini bertujuan mengenal pasti peluasan algoritma Grover yang telah dibangunkan melalui satu kajian literatur sistematik. Artikel terindeks yang diterbitkan antara tahun 2019 hingga 2025 telah dipilih daripada pelbagai sumber berdasarkan kata kunci tertentu. Kajian ini mengklasifikasikan peluasan tersebut kepada tiga aspek utama: kerumitan masa, pengoptimuman dan kos kuantum, termasuk kombinasi dua aspek (bi-combinations) dan tiga aspek (tri-combinations). Selain itu, satu rangka kerja konseptual turut dicadangkan untuk meringkaskan sumbangan tersebut dan menjadi panduan bagi penyelidikan masa depan. Dapatan kajian ini menekankan bagaimana peluasan algoritma Grover menangani cabaran prestasi serta mengembangkan kebolehgunaannya dalam pelbagai domain. Kajian ini bukan sahaja memberikan asas yang lebih jelas tetapi juga mewujudkan landasan untuk pembangunan masa hadapan algoritma carian kuantum dan penggunaannya yang lebih meluas dalam aplikasi pengkomputeran kuantum.

Kata kunci**:** Algoritma kuantum, Pengkomputeran kuantum, Algoritma Grover, Pencarian kuantum.

INTRODUCTION

Over recent decades, quantum computing has emerged as a transformative field, offering superior computational advantages over classical computing in resolving complex problems in various fields, including cybersecurity, finance, and healthcare (Díez-Valle et al., 2023; Kirsanov et al., 2023; Qu et al., 2023; Rao & Lakshmanan, 2023; Udvarnoki et al., 2023; Humayun et al., 2024). Unlike classical computing, which operates on binary bits (0s and 1s), quantum computing operates on quantum bit or qubit, utilizing the superposition of quantum states, which means the bits can be both (0s and 1s) at the same time. This capability enables quantum systems to process vast amounts of data simultaneously. Additionally, quantum entanglement strengthens the correlation between quantum states, further enhancing the capabilities of quantum computing (Zhahir et al., 2023).

The central key element that allows this transformative paradigm shift is the quantum algorithm, which enhances the computational efficiency founded on the heart of quantum mechanics, such as superposition and entanglement (Yu, 2021; Zhahir et al., 2024). Quantum algorithms have been developed to address various challenges across different fields, including searching, factorization, and optimization, demonstrating exponential or quadratic speedup compared to their classical counterparts. Among the most significant quantum algorithms are Shor's algorithm and Grover's algorithm. The former was notable for factoring large numbers, demonstrating exponential speedup and posing significant threats to cryptographic security (Shor, 1999; J. et al., 2022). Meanwhile, Grover's algorithm was notable for unstructured database searching and demonstrating quadratic speedup, positioning it as significantly relevant for data management and optimization (Grover, 1996; Jozsa, 1999).

Grover's algorithm, introduced by Lov Grover in the 1990s, is a centrepiece of the quantum search method. It employs amplitude amplification, enhancing the probability of finding the targeted items in an unstructured database. For an N -element database, Grover's algorithm is better able to achieve the desired results in $O(\sqrt{N})$ queries compared to its classical counterpart in $O(N)$ queries (Grover, 1996; Soni & Rasool, 2021). This gave it a significant advantage in terms of computational pace. The applications of Grover's algorithm extend beyond merely unstructured database searching to numerous areas such as machine learning, emphasizing its importance to a broader spectrum of quantum computation (Bowles et al., 2024; Gil-Fuster et al., 2024). Researchers have explored the possibilities over the years, modifying and extending Grover's algorithm to solve existing NP-hard problems.

This review explores the fundamental principles of Grover's algorithm and its mathematical formulation. This study followed a strict set of guidelines, systematically reviewing identified article items concerning Grover's algorithm extensions and proposing a conceptual framework of these extensions. The paper is organized as follows: the research methodology is detailed in Section 2. Section 3 covers the results and discussion. Finally, Section 4 concludes the study.

LITERATURE REVIEW

In this section, the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) is discussed as it was the publication standard used. This study followed a strict sequential strategy, including 1) the PRISMA review protocol, 2) research question formulation, 3) a systematic searching strategy, 4) quality appraisal, and 5) data extraction and analysis.

A. PRISMA review protocol

This systematic literature review utilized PRISMA as the review protocol (Page et al., 2021). The protocol served as a guiding principle for conducting the systematic literature review by formulating research questions; identifying the inclusion and exclusion criteria for the systematic searching strategy; the quality appraisal of selected items by the field expert; and the data extraction and analysis of the selected items over the specified time period. The scope of the study was Grover's algorithm extensions. This study specifically focused on recent, application-oriented extensions of Grover's algorithm emphasizing time-complexity, optimization and quantum cost. Theoretical generalizations such as continuous-time, Hamiltonian, and quantum walk were intentionally excluded.

B. Research question formulation

In the preliminary phase, the study developed research questions, which pillared the systematic literature review. The following research questions were formulated to conform to the research objectives of extensively reviewing Grover's algorithm extensions through a systematic literature review and presenting a proposed conceptual framework of Grover's algorithm extensions: 1) What are the established extensions of Grover's algorithm?; 2) What are the established extensions of Grover's algorithm applications?; and 3) How can a conceptual framework on Grover's algorithm extensions be proposed for future work?

C. *Systematic searching strategy*

The study employed several sequential steps in the systematic searching strategy, including identification, screening, and eligibility.

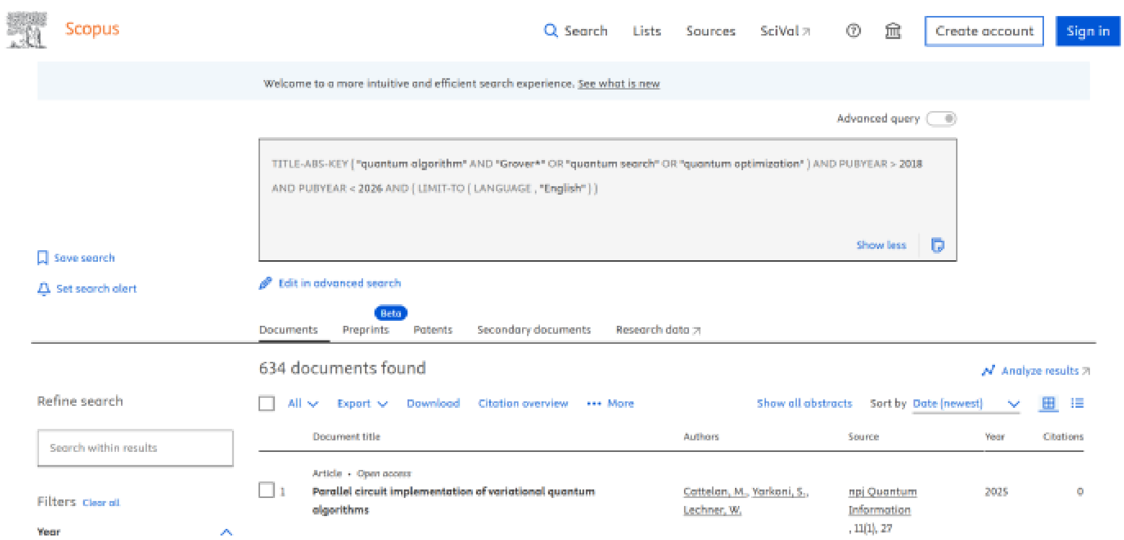
I. *Identification*

Identification is a vital process as it determines relevant article items for a systematic literature review. The article items were drawn primarily from the multidisciplinary search engines of Scopus and Web of Science (WOS). An advanced search was conducted using the field tags “TITLE-ABS-KEY” (title, abstract, and keywords) within the Scopus search engine. In WOS, the field tag “TS” (topic) was used. Specified keywords were employed in the preliminary phase, including “quantum algorithm,” “Grover’s algorithm,” and “quantum search,” along with the usage of the Boolean operators “AND” and “OR”. The specified keywords were transformed into search strings searching for related article items. These search strings are listed in Table 1.

TABLE 1. Systematic Searching Search Strings

Database	Search String
Scopus	TITLE-ABS-KEY (("quantum algorithm" AND "Grover*") OR "quantum search")
WOS	TS = (("quantum algorithm" AND "Grover*") OR "quantum search")

The keywords used for the systematic searching were 1) “quantum algorithm”; 2) “Grover*”, in which the asterisk (*) represented zero or more characters, thus covering terms such as “Grover*’s algorithm*” and “Grover *search algorithm*”; and 3) “quantum search”. These keywords were determined as being able to extract relevant article items related to the topic. The search process was conducted between February and March of 2025. The systematic searching identified a total of 1,317 potentially related article items from both search engines, while a total of 183 article items were considered for further analysis. The search results from Scopus and WOS are displayed in Figure 1.



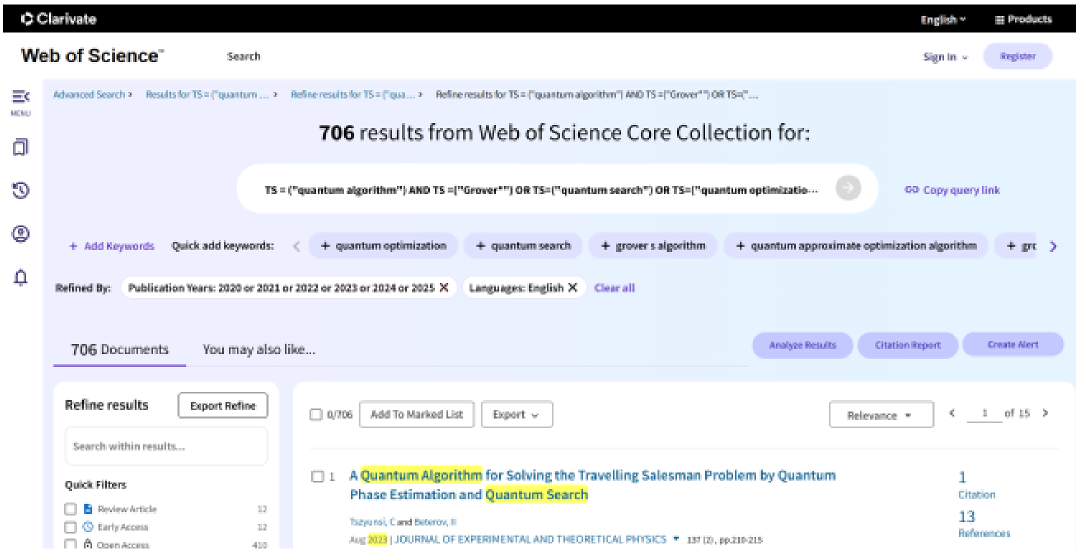


FIGURE 1. Search results in Scopus and WoS

II. Screening

Screening, as the name suggests, is the process of vetting indexed article items sorted from the previous stage. This systematic literature review examined article items about Grover’s algorithm extensions that had been published between 2019 and 2025. The maturity of the subject was determined to be acceptable for the six-year period, granted that the systematic searching strategy was performed relatively early in 2025 (Kraus et al., 2020).

The early screening process using a title and abstract review resulted in 1,134 of the 1,317 article items being excluded. In the later screening stage, the 183 remaining article items were validated to ensure that the inclusion and exclusion criteria were met. These criteria were determined as the subject matter, literature type, language, publication year, and specifically defined terms. Table 2 exhibits the inclusion and exclusion criteria.

TABLE 2. Inclusion and Exclusion Criteria

Inclusion Criteria	Exclusion Criteria
Article items specifically on the subject matter of “quantum algorithm”, “Grover’s algorithm”, and “quantum search”	Article items with specifically defined terms: “non-oracular”, “quantum walk”, and “quantum optimization”
Indexed article items in Scopus and WOS	Article items published before 2019
Article items written in English	Article items with undefined formulation of Grover’s algorithm extensions

An additional important precautionary step was that only article items written in English were considered, in order to avoid misunderstandings and mistranslation. Article items unrelated to the specified criteria were excluded from this systematic literature review. During this stage, 161 remaining article items met the inclusion and exclusion criteria.

III. Eligibility

In the eligibility stage, the remaining 161 article items retained from the previous stage were thoroughly reviewed again for the systematic literature review. In this stage, the items were re-reviewed for their suitability. It was determined that several article items were offtopic in relation to the theme of Grover's algorithm. After this stage, 142 article items remained and were prepared for quality appraisal.

D. Quality appraisal

The remaining 142 article items were presented to a field expert for quality appraisal to ensure the article items were of sufficient quality to be used in the systematic literature review. According to (Petticrew & Roberts, 2006), only article items ranked as moderate- and high-quality can be included in such a review. Low-quality article items should be excluded to preserve the standard of the systematic literature review. Following the appraisal, the field expert accepted 40 article items for the review. The entire article item selection process is shown in Figure 2.

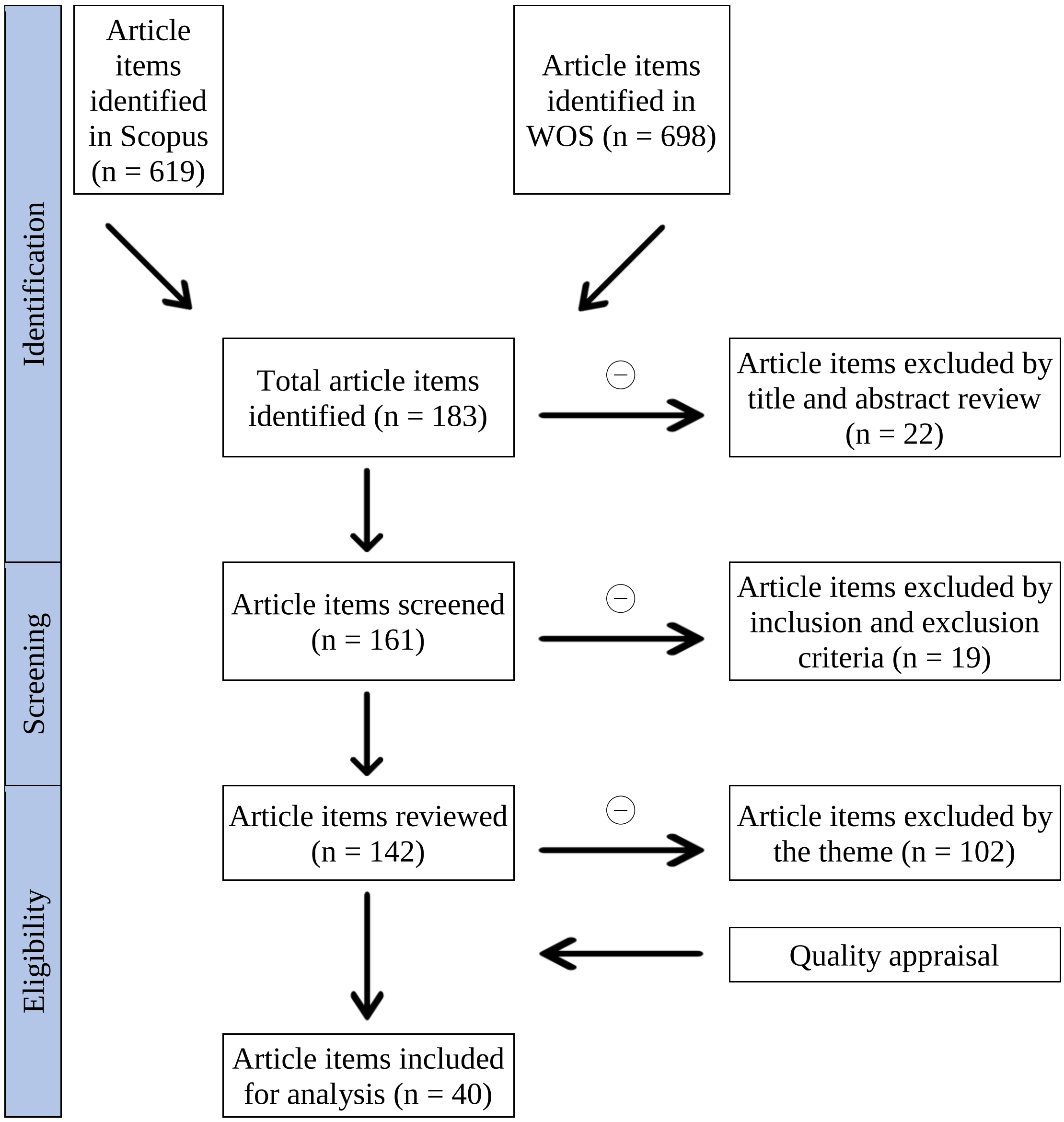


FIGURE 2. Article item selection process

E. Data extraction and analysis

To extract relevant data from the selected article items, an in-depth analysis process was done. This started with analysis of each abstract, then of the discussion and conclusion sections, and finally of the body of the article item. The extracted data were then tabulated and recorded in a Microsoft Word file. The article items could be primarily divided into seven classes based on the publication year (see Figure 3). The main theme extracted from the article items was Grover’s algorithm extensions.

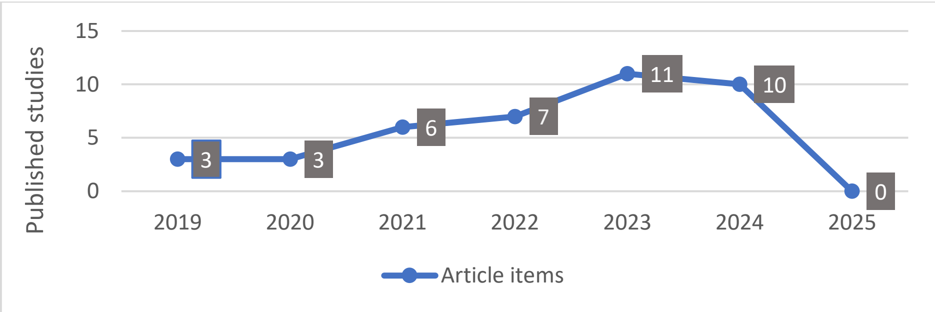


FIGURE 3. Article items classified by year of publication

Based on Figure 3, three selected article items were published in 2019, three items in 2020, six items in 2021, seven items in 2022, 11 items in 2023, 10 items in 2024, and none in 2025. It is important to note that the systematic searching was done early in 2025. While several article items were accepted in the quality appraisal stage, those items from 2025 were excluded from the systematic literature review as they were ranked as low-quality by the expert. Additionally, the figure does not indicate an uptrend in work on Grover’s algorithm extensions, but rather a variation throughout the period. Another important point is the inclusion and exclusion criteria set in the previous screening stage. These definitely influenced the distribution of the classes shown.

RESULTS AND DISCUSSION

This section discusses the identified theme of Grover’s algorithm and its extensions in previous studies (see Table 3). A proposed conceptual framework of Grover’s algorithm extensions was developed, and it is presented here as a reference for future works (see Figure 4).

A. Grover’s algorithm and its extensions

Grover's algorithm is known as a quantum search algorithm that provides quadratic speedup compared to classical search methods. The algorithm was designed to find a target marked item within an unstructured database of N-elements in $O(\sqrt{N})$ time, surpassing the classical search in $O(N)$. This capability is enabled through a quantum amplitude amplification, which increases the probability of measuring the desired solution state systematically. Grover’s algorithm is valuable in problem-solving, particularly in relation to optimization and data retrieval in an unstructured space, by dramatically improving its efficiency.

Grover’s algorithm operates within the framework of quantum mechanics, exploiting the principles of superposition and entanglement. Instead of checking each and every item one by one like a classical computer, Grover’s algorithm uses key principles to consider all possible options at the same time and determine the correct one. Given an unstructured database with N possible states, these states are encoded into a quantum system represented by a Hilbert space of dimension, N. The primary goal is to identify the target state $|\omega\rangle$ using a function called oracle, O, which applies a phase shift to the marked state, flipping its sign and thus highlighting the marked state. The quantum algorithm relies on the Grover's operator, $G = DO$, where D is the diffusion operator. The repeated application of Grover's operator, G amplifies the amplitude of the marked state while reducing the remaining state, ensuring that it is measured with high probability after approximately $O(\sqrt{N})$ iterations.

Grover's algorithm consists of several key components that enable the search process, including 1) initialization, 2) oracle, 3) diffusion operator, 4) iterations, and 5) measurement. In simple terms, these stages narrow down the targeted state from a large set of possibilities. Firstly, in the initialization stage, the algorithm begins with an equal superposition of all possible states by applying the Hadamard transform to an n-qubit register initialized in the $|0\rangle$ state. Each state is guaranteed to have an equal probability amplitude, preparing the quantum system for amplitude amplification. Next, in the oracle stage, the oracle function, often implemented as a black box, flips the identified target state by applying a phase flip or phase shift. Mathematically, the target state, $|\omega\rangle$ is marked as $O|\omega\rangle = -|\omega\rangle$, while the other states in the system remained unchanged. This selective inversion process is crucial for the iterative amplification process.

Next, the diffusion operator stage, also known as the amplitude amplification stage, increases the marked state's amplitude by reflecting all states about their average amplitude. This step gradually concentrates the probability mass toward the desired solution, increasing the chance of successful measurement. The iterations stage, as the name suggests, is Grover's operator repetitions process. The application of oracle followed by the diffusion operator represents a single Grover iteration. The number of iterations required depends on the total number of states, N and the number of marked items M, and is approximately $r \approx \frac{\pi}{4}\sqrt{\frac{N}{M}}$. In the special case of a single marked item ($M = 1$), this reduces to the familiar $\frac{\pi}{4}\sqrt{N}$ form. It is crucial to determine the optimal number of iterations as insufficient iterations could lead to a low probability of measuring the target state, while an excessive number of iterations could cause the amplitude to overshoot the desired state. Following this, in the measurement stage, a quantum measurement is performed after the number of optimal iterations has been determined. The measurement probabilistically yields one of the possible states, with the marked state having the highest likelihood of being observed due to amplitude amplification. In the event of an incorrect result being obtained, the process can be repeated to increase the probability of success.

Despite its advantages over its classical counterparts, Grover's algorithm has several recognized limitations, which has stimulated further research into its extensions. A major challenge is the requirement for an efficiently implementable oracle as many practical problems lack a well-defined, universal quantum oracle function. Additionally, the algorithm does not guarantee an exact solution in a single run as it provides a probabilistic advantage that still requires multiple repetitions. Furthermore, the performance of the algorithm is limited by the available quantum hardware, including noise and decoherence, impacting the precision of iterative amplitude amplification.

In the period defined in the previous section, various extensions of Grover's algorithm have been proposed in previous studies. These extensions aim to improve the efficiency of the original Grover's algorithm, expanding its capability and applicability to broader domains. Table 3 shows the Grover's algorithm extensions outlined in previous studies.

TABLE 3. Grover's Algorithm Extensions Proposed in Previous Studies

Source / Indicator	Title	Method & Description	Cluster		
			T	O	C
(Gejea et al., 2023)	A Novel Approach to Grover's Quantum Algorithm Simulation: Cloud-Based Parallel Computing Enhancements	Develops a cloud-based parallel simulation of Grover's algorithm, improving efficiency through multi-core processing. Compares five cloud architectures and verifies performance, achieving up to 31-bit simulation.	*	*	
(Alves et al., 2019)	A Quantum Algorithm for Ray Casting using an Orthographic Camera	Uses Grover's algorithm for ray casting in rendering, achieving quadratic speedup. Simulated on quantum hardware but limited to simple geometry.	*		
(Bhattacharyya & Raina, 2022)	A quantum algorithm for syndrome decoding of classical error-correcting linear block codes	Proposes a Grover-inspired quantum search for error detection in linear block codes, optimizing syndrome decoding with probabilistic improvements.	*	*	
(Almuqbil & Felemban, 2024)	Discovery of Quantum Algorithms Using Genetic Algorithms: Exponential Speedup via Random Sampling	Uses Genetic Algorithms (GAs) to optimize oracle-based quantum algorithms, specifically Grover's search, by reducing the evaluation complexity through random sampling, making larger circuits feasible.	*	*	*
(Roy et al., 2022)	Deterministic Grover search with a restricted oracle	The D2p protocol modifies Grover's search to achieve deterministic success without user-controlled oracles, using generalized phase rotations while preserving quadratic speedup.	*	*	*
(Das & Sadhu, 2022)	Experimental study on the quantum search algorithm over structured datasets using IBMQ experience	Proposes a quantum search algorithm; applies state elimination and amplitude amplification to structured datasets. Executed on IBMQ real chips and simulators, it shows reduced quantum cost and better suitability for NISQ-era quantum devices compared to Grover's algorithm.	*	*	*
(Ominato et al., 2024)	Grover Adaptive Search With Fewer Queries	Enhances GAS by refining the query selection strategy in each iteration, removing the upper limit reset. Simulation results confirmed that the approach reduces the number of queries	*	*	*

Source / Indicator	Title	Method & Description	Cluster		
			T	O	C
		while maintaining quadratic speedup, improving efficiency in binary optimization problems.			
(Prokop et al., 2024)	Grover's oracle for the Shortest Vector Problem and its application in hybrid classical-quantum solvers	Constructs a quantum oracle for the Shortest Vector Problem (SVP) and integrates it into Grover's algorithm, combining quantum enumeration with classical solvers to improve efficiency.	*	*	*
(Ablayev et al., 2024)	Hybrid Classical–Quantum Text Search Based on Hashing	Proposes hybrid classical–quantum algorithm; integrates Grover's search with hashing techniques to find a substring in a text. It assumes a single occurrence of the substring and optimizes qubit usage through efficient hash-based memory management.	*	*	*
(Ardelean & Udrescu, 2024)	Hybrid quantum search with genetic algorithm optimization	Introduces Hybrid Quantum Algorithm with Genetic Optimization (HQAGO), a hybrid quantum genetic algorithm that improves the efficiency of the Reduced Quantum Genetic Algorithm (RQGA) by using classical bits to reduce the quantum search space. This results in a more scalable approach to solving NP-hard problems while preserving solution quality.	*	*	*
(Wang et al., 2021)	Implementing a quantum search algorithm with nonorthogonal states	Proposes a quantum search method using nonorthogonal states and demonstrates it experimentally in a linear optical system. The approach encodes a large search space into a single qubit, achieving high fidelity (>0.99). The method could be extended to multi-qubit systems and optimized versions of Grover's algorithm.		*	*
(Ambainis et al., 2023)	Improved Algorithm and Lower Bound for Variable Time Quantum Search	Introduces a simplified quantum algorithm for variable time search, using Grover diffusion steps with varying query times. It improves both upper and lower bounds for the problem and suggests further refinements for cases where query times are unknown.	*	*	*
(Barán & Villagra, 2019)	Multiobjective Optimization Grover Adaptive Search	Proposes the Multiobjective Optimization Grover Adaptive Search (MOGAS), a quantum multiobjective optimization algorithm using two types of quantum oracles. Benchmarked	*	*	*

Source / Indicator	Title	Method & Description	Cluster		
			T	O	C
		against NSGA-II, it shows comparable or better results while optimizing the number of oracle queries.			
(Ablayev et al., 2021)	Quantum Algorithms for String Processing	Proposes quantum algorithms for string matching and string comparing by leveraging quantum parallelism and Grover's algorithm, achieving lower memory requirements and improved speed.	*	*	*
(Alasow et al., 2022)	Quantum Algorithm for Variant Maximum Satisfiability	Proposes a quantum oracle for MAX-SAT using Grover's algorithm with a quantum counter, reducing qubit overhead; tested on IBM Qiskit.	*	*	*
(Preston, 2023)	Applying Grover's algorithm to hash functions: a software perspective	Implements MD5, SHA-1, SHA-2, and SHA-3 as quantum oracles, validating them with Q# and estimating quantum resource needs.	*	*	*
(Wang et al., 2020)	A Generic Variable Inputs Quantum Algorithm for 3-SAT Problem	Proposes a variable-inputs approach to Grover's algorithm, combining it with Schöning's algorithm to solve 3-SAT more efficiently.	*	*	*
(Zhou et al., 2024)	Coherence fraction in Grover's search algorithm	Introduces a generalized Grover search algorithm to measure success probability. The coherence fraction corresponds to the highest success probability in the Grover search algorithm.		*	
(Denisenko & Nikitenkova, 2019)	Application of Grover's Quantum Algorithm for SDES Key Searching	Implements Grover's algorithm for an SDES key search with a minimal 19-qubit circuit, simulated using Quipper.		*	*
(Nakanishi et al., 2021)	Modeling Grover's Algorithm with Colored Petri Net	Uses Colored Petri Nets (CPN) to model Grover's algorithm. Simulations in CPN Tools demonstrated the model's usefulness for visualizing algorithm behavior and verifying correctness.			
(Shimizu & Mori, 2022)	Exponential-Time Quantum Algorithms for Graph Coloring Problems	Proposes an exponential-space QRAM-based algorithm ($O(1.9140^n)$) and a polynomial-space alternative ($O(1.9575^n)$) using quantum dynamic programming and Grover's search to speed up classical branching algorithms for the k-colorability problem.	*		
(Seo & Heo, 2022)	Analysis of Quantum Search Algorithm for Weighted Solutions: Simple Case	Uses amplitude expressions to derive a lower bound for the number of Grover operator applications (t) and optimize the probability ratio between high-		*	

Source / Indicator	Title	Method & Description	Cluster		
			T	O	C
		importance and low-importance target states, maximizing the search success for important data.			
(Um, 2021)	Discrete Time Multi-particle Grover Search	Generalizes Grover's algorithm by incorporating interactions between multiple searching particles in a discrete-time setting, using eigenvalue perturbation theory to derive an improved asymptotic time-complexity based on the number of interactions.	*	*	
(Lee & Nam, 2024)	Finding All Solutions with Grover's Algorithm by Integrating Estimation and Discovery	Proposes a two-step approach integrating solution estimation and discovery into Grover's algorithm, reducing the total iterations by capturing most solutions in the estimation phase and dynamically adjusting measurements to optimize the search efficiency.	*	*	
(Chowdhury et al., 2023)	Phase matching in quantum search algorithm	Generalizes Grover's algorithm using density matrix formalism and optimized phase angles (α, β) to improve the search efficiency for databases with an unknown number of marked states, ensuring a high success probability with fewer iterations.	*	*	
(He et al., 2020)	Quantum Search with Prior Knowledge	Modifies Grover's algorithm to incorporate contextual side information, optimizing the search success probability when prior knowledge about solution distribution is available.	*	*	
(Norimoto et al., 2023)	Quantum Algorithm for Higher-Order Unconstrained Binary Optimization and MIMO Maximum Likelihood Detection	Implements a GAS-based quantum search for real-valued HUBO, applies it to MIMO MLD, and analyzes the circuit complexity algebraically.	*	*	*
(Yukiyoshi & Ishikawa, 2024)	Quantum search algorithm for binary constant weight codes	Formulates a binary constant weight code search as a polynomial binary optimization problem, implements the Grover Adaptive Search (GAS), and optimizes performance using problem-specific upper and lower bounds to reduce overhead and qubit requirements.	*	*	*
(Guţoiu et al., 2024)	Simple exact quantum search	Optimizes Grover's algorithm by introducing a perturbed initial state and fixed reflector, eliminating search failures while maintaining computational	*	*	*

Source / Indicator	Title	Method & Description	Cluster		
			T	O	C
		efficiency, requiring at most one extra oracle call.			
(Seo et al., 2022)	Quantum Search Algorithm for Weighted Solutions	Extends Grover's search to incorporate solution weighting, enabling a reward-based probability distribution that improves the search efficiency in problems like Dynamic Spectrum Management (DSM).	*	*	*
(Wu et al., 2023)	Circuit optimization of Grover quantum search algorithm	The 2P-Grover algorithm is a two-stage, divide-and-conquer quantum search method that leverages block-level oracle optimization to reduce circuit depth and iterations, improving the search efficiency on NISQ devices.	*	*	*
(Roy & Kim, 2023)	Applying Quantum Search Algorithm to Select Energy-Efficient Cluster Heads in Wireless Sensor Networks	The Quantum Weighted Search Algorithm (QWSA)-based Cluster Head selection method integrates Grover's search into Wireless Sensor Networks (WSN) topology control, significantly reducing search times and energy consumption, making it a viable approach for scalable and efficient sensor networks.	*	*	*
(Soni & Rasool, 2020)	Pattern Matching: A Quantum Oriented Approach	The Combined Exact Quantum Algorithm (COEQUAL) quantum pattern matching algorithm integrates Grover's search logic with improvements for handling multiple occurrences, making it a competitive alternative to classical and quantum benchmarks in large-scale pattern matching.	*	*	*
(Yang et al., 2021)	Multiplicative inverse with quantum search algorithm under $\pi/18$ phase rotation	Proposes a quantum search algorithm with a $\pi/18$ phase shift for finding multiplicative inverses, achieving a 99.81% success probability while maintaining standard time-complexity.	*		
(Soni & Malviya, 2021)	Design and Analysis of Pattern Matching Algorithms Based on QuRAM Processing	Proposes QuESM, QuEMS, QuEMM, and QuAMM algorithms for pattern matching using QuRAM, leveraging Grover's search for efficiency. Simulations with QuEST confirmed their advantage over classical methods.	*	*	*
(Zhang et al., 2021)	Implementation of efficient quantum search algorithms on NISQ computers	Optimizes Grover's algorithm for NISQ devices using local diffusion operators, implementing three-qubit to		*	*

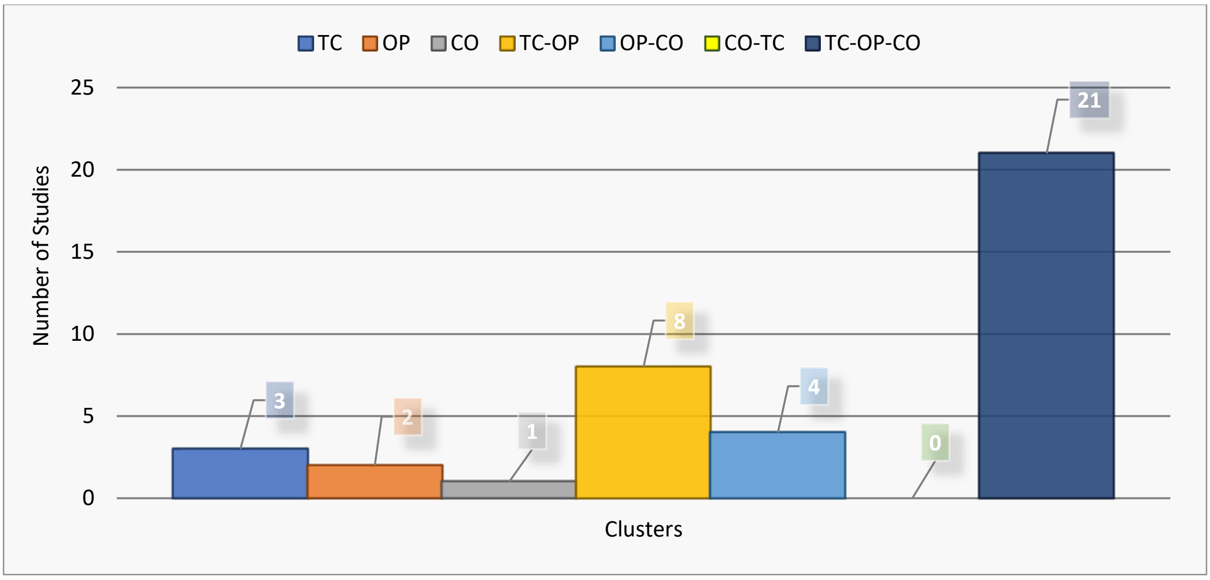
Source / Indicator	Title	Method & Description	Cluster		
			T	O	C
		five-qubit searches on IBM processors with higher success rates.			
(Ahmadkhaniha et al., 2023)	Performance Analysis of the Hardware-Efficient Quantum Search Algorithm	Analyzes a hardware-efficient quantum search algorithm for NISQ devices, optimizing circuit depth and error resilience, with noise effects evaluated via Qiskit and MATLAB.		*	*
(Bhuvaneswari et al., 2023)	Computational Analysis: Unveiling the Quantum Algorithms for Protein Analysis and Predictions	Applies Grover's algorithm to Protein-Protein Interactions (PPI) prediction, enhancing its accuracy over large datasets compared to classical methods. Discusses quantum hardware limits and future improvements for bioinformatics.		*	
(Huang & Pang, 2023)	Optimization of probabilistic quantum search algorithm with a priori information	Optimizes Grover's search by adjusting the initial state and diffusion operator based on a priori probability distributions, reducing the query complexity at the cost of a small failure probability.	*	*	
(Li & Li, 2023)	Deterministic quantum search with adjustable parameters: implementations and applications	Proposes a search framework with adjustable parameters (α, β) for Grover's algorithm, introducing the Fixed-Axis-Rotation (FXR) method to achieve deterministic searching without losing quadratic speedup.	*	*	

T = Time-complexity; O = Optimization; C = Cost (quantum cost)

Table 3 lists and briefly describes the Grover's algorithm extensions published in previous studies. The extension methods vary between modifications and adaptations of the algorithm, with their application and simulation being contingent on the specific scope of each study, with a wide range of practical applications in various disciplines. The literature analysis revealed that the methods utilized in these studies are independent of each other, with every study presenting unique approaches. In examining all the methods and approaches presented in these studies, the comparison assessment between these studies in the current paper is non-existential. It is unfair to express which method and approach is the best or which is not as they are all unique according to the particular scope and application. As mentioned above, this study merely serves as a guide for future works. However, in this systematic literature review, three critical aspects were considered and then grouped into clusters: time-complexity, optimization and cost (quantum cost), and its bi- and tri-combinations (see Figure 4). These core aspects were selected in this study, as it is well-known and well-acknowledged that these aspects directly determine algorithmic efficiency, feasibility, and scalability.

Time-complexity is defined as the execution time or the efficiency of an algorithm to execute queries faster. Classical algorithms exhibit exponential speed, such as $O(2^n)$, while quantum algorithms like Grover's algorithm achieve quadratic speedup with $O(\sqrt{N})$. The reduction in time-complexity significantly impacts the search-based related problems and others. Optimization focuses on enhancing the algorithm's performance using various approaches, including reducing operations, improving error tolerance, or leveraging hybrid techniques, thus bringing in the best of both worlds: classical-quantum methods. The primary objective is to discard or reduce unnecessary computing operations and optimize the algorithm while maintaining or improving its accuracy.

Quantum cost is another crucial aspect when evaluating the practical ability of quantum algorithms. It has multiple components. These include qubit numbers: qubit numbers are required for computation; gate requirements: the types and quantity of gates used; gate depth: the sequence of gate operations; the gate rotation: the use of gate rotation; T-quantum cost: the cost associated with T-gate; doubling strategy: doubling the number of qubits; iterations: the count of iterations required; quantum memory: the amount of memory required in an operation; circuit scale: the overall circuit size in an operation; and circuit depth: the depth of operation needed. These components are among the most important factors in determining the feasibility of an algorithm, influencing the practical adoption of near-term and future quantum hardware.



TC = Time-complexity; OP = Optimization; CO = Cost (quantum cost)

FIGURE 4. Aspect clusters

Based on Figure 4, three primary aspect clusters were determined: time-complexity, optimization and cost (quantum cost), and its bi- and tri-combinations. These clusters emerged as recurring themes in this literature as a foundation for scholars and researchers to use in enhancing or extending the original Grover's algorithm. The source/indicator label from Table 3 shows that the majority of the studies focused greatly on all three aspects, demonstrating a comprehensive approach in the extensions. However, a subset of some other studies directed the intention to only one or two of these aspects, depending on the direction of the study. Notably, only one outlier disregarded all the aspects, but this particular study was still included in the current review as the approach method utilized in its algorithm extension was deemed worth examining. It is important to note that neither time-complexity nor optimization and cost (quantum cost) are the definitive benchmarks for a successful Grover's algorithm extension. Instead, they serve as commonly adopted standard indicators for comparative assessments of extension methods.

B. Proposed conceptual framework of Grover's algorithm extensions

The Grover's algorithm extensions were thoroughly examined to capture the essence of the methods outlined in the previous subsection. The second purpose of this study was to propose a conceptual framework of Grover's algorithm extensions (see Figure 5), and this was developed in accordance with the specified methods established in previous studies. This proposed conceptual framework serves as a guide for future researchers aiming for comparative studies or to further extend Grover's algorithm, particularly in the aspects of time-complexity, optimization, cost (quantum cost), and its bi- and tri-combinations, or more. Future studies can directly map new extensions into the proposed conceptual framework, evaluating these aspects.

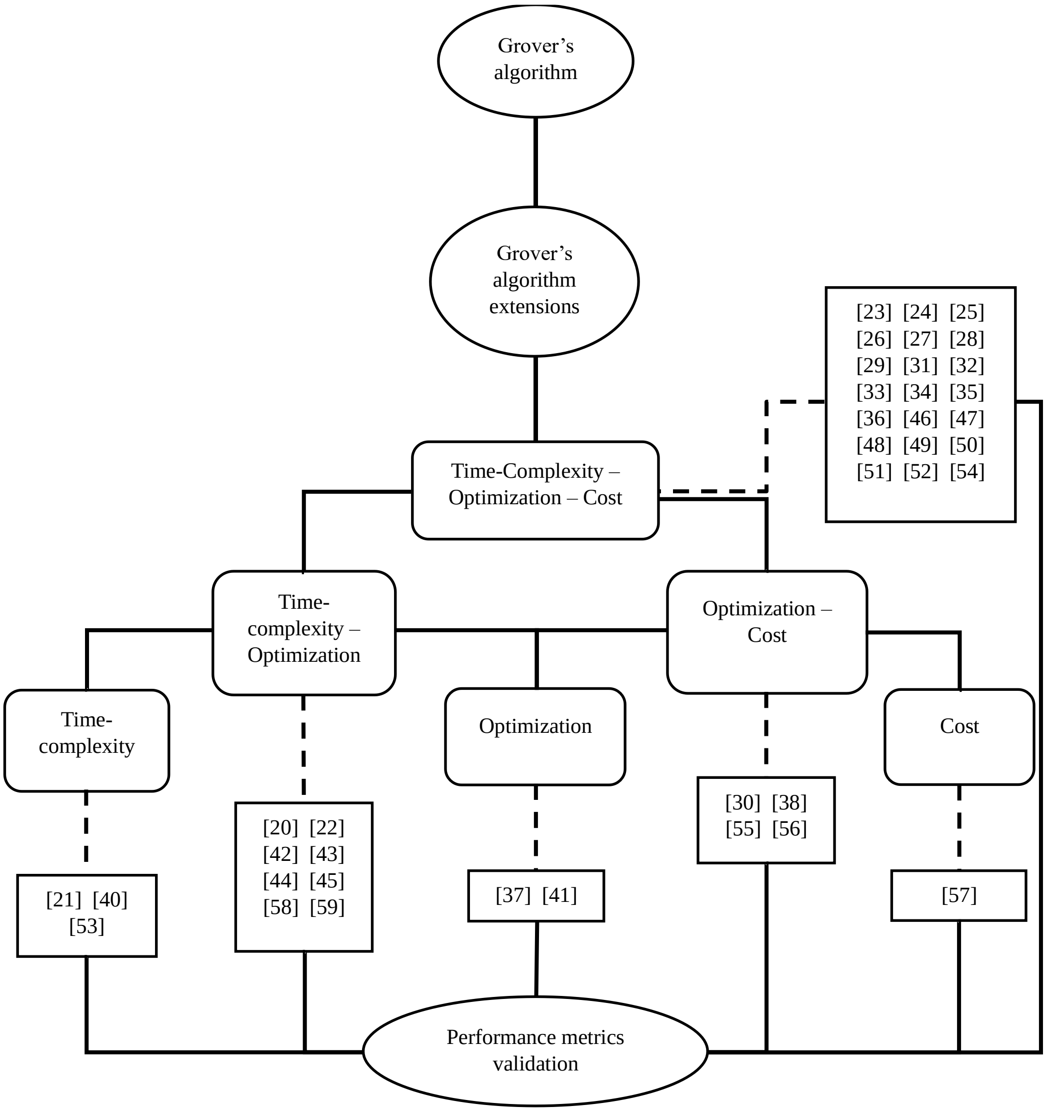


FIGURE 5. Proposed conceptual framework of Grover's algorithm extensions

CONCLUSION

This study presents a systematic literature review of Grover's algorithm and its extensions based on previous studies, specifically those published between 2019 and 2025. The analysis revealed three critical recurring aspects: time-complexity, optimization, and cost (quantum cost). These aspects revealed that the published extensions had different directions and purposes, with most studies directed their intention to all three. While some studies focused on reducing critical attributes like the time taken, number of iterations, and quantum resources, others focused on optimizing the efficiency for specific problems. The focus on time-complexity, optimization and quantum cost allows for a clearer synthesis of the current state of research and its future scalability. It is vital to affirm that this study has its limitations. It was confined to works within the inclusion and exclusion restrictions stated. These set of restrictions are mainly to ensure quality and consistency of the systematic literature review. However, they may exclude relevant contributions within the scope.

Based on the findings, a proposed conceptual framework of Grover's algorithm extensions is presented to summarize and organize the ideas related to these extensions. The proposed framework aims to illustrate clearly how the extensions were differently approached and, more importantly, to serve as guidance for future works, thus contributing to the ongoing development of quantum algorithm and quantum computing. Although this systematic literature review focused on selected aspects, future researchers could expand this study to include other different aspects, such as other existing performance metrics and practical application domains. The intentions behind this study were to assist future researchers in directing their Grover's algorithm research and to inspire potential advancements in this field.

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