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Development of a Weighted Fractional Time-Varying Discrete Grey Model for CO2 Emissions Forecasting

Pembangunan Model Kelabu Diskret Wajaran Pecahan Berubah Masa bagi Ramalan Pelepasan CO2

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ABSTRACT

The data accumulation operators are the basic elements of the grey system theory and they help in transforming the raw data sequences into other forms whereby more clear patterns might be understood. Nonetheless, each of the accumulation methods is usually applicable to particular forms of time series activity. In order to exploit the advantages of different accumulation techniques even better, especially those which are in a position to detect nonlinear patterns, this paper presents a new model named the Time-Varying Discrete Grey Model (TDGM) with Weighted Fractional Accumulation. The best parameter values are determined effectively and prediction errors are reduced with the help of the Quantum Particle Swarm Optimization (QPSO) algorithm. To test the effectiveness of the model, four case studies of CO2 emission in Indonesia, Malaysia, Singapore, and Thailand were used. These findings are clear to hasten that the new model is capable of producing better prediction and produce lower error rates of previous models and able to identify complex trend nonlinearity in a real environmental data.

Keywords**:** CO2 emission, Grey model, ARIMA, SVM, forecasting

ABSTRAK

Operator pengakumulasian data merupakan unsur asas dalam teori sistem kelabu yang membolehkan penukaran urutan data mentah kepada bentuk lain untuk mendedahkan corak yang lebih jelas. Namun begitu, setiap kaedah pengakumulasian lazimnya bersesuaian dengan jenis siri masa tertentu. Bagi memanfaatkan kelebihan pelbagai teknik pengakumulasian, terutamanya yang mampu mengesan corak bukan linear, kajian ini memperkenalkan model baharu iaitu model kelabu diskret berubah masa (TDGM) dengan pengakumulasian pecahan wajaran. lai parameter optimum ditentukan dengan efisien, manakala ralat ramalan diminimumkan menggunakan algoritma Quantum Particle Swarm Optimization (QPSO). Prestasi model diuji melalui empat kajian kes mengenai pelepasan CO2 di Indonesia, Malaysia, Singapura, dan Thailand. Keputusan kajian menunjukkan bahawa model TDGM yang dicadangkan bukan sahaja memberikan ketepatan ramalan yang lebih tinggi dengan kadar ralat lebih rendah berbanding model terdahulu, malah mampu mengenal pasti corak bukan linear yang kompleks dalam data alam sekitar sebenar.

Kata kunci**:** Pelepasan CO2, model kelabu, ARIMA, SVM, ramalan

INTRODUCTION

The impact of global warming is real and is threatening human existence as it is already taking its toll in terms of societies, economic, and natural ecosystems due to climate-related calamities like rise in sea level, droughts, and freak floods (Lu 2024). [1]. This threat is mostly attributed to the emission of carbon dioxide (CO2) in the atmosphere as a result of human induced activities (Hosseini *et al*., 2019). Since the last decades, the intensity of CO2 emission has grown considerably due to intensifying industrialization, fast urbanisation and the high dependence of fossil fuel (Shang & Luo, 2021; Sun *et al*., 2024). In order to undo the scenario, the international community has embarked on several efforts to curb climate change, such as coming up with strategic plans, and deployment of regulatory and policy structures to steer the climate change cause. Realistic models of forecast also need to be created in order to accurately estimate the future trends of CO2 emissions development in order to create more efficient environmental policy and sustainable behaviour (Acheampong & Boateng, 2019).

There has been an array of methods to predict CO2 emissions, like the traditional estimation approaches like the autoregressive integrated moving average (ARIMA) (Liu et al. 2014; Pao & Tsai, 2011), logistic growth models (Piecyk & McKinnon, 2010), and regression-based predictions (Hosseini et al., 2019). Although these statistical models are very good, they usually need big data sets that can conform to tight statistical premises, something that may not be readily achieved in real life (Lee &Tong, 2011). As the information technology developed, artificial intelligence (AI) algorithms: artificial neural networks (Nie et al. 2023), support machine, extreme learning machine, have been more frequently applied to forecast CO2 emissions (Mason et al., 2018; Wen & Yuan, 2020). The AI methods can capture nonlinear and complex relationships in data and provide high flexibility in modelling. Nevertheless, they tend to be accurate only when many other influencing factors are considered. Such extensive data may be expensive, require time and in some instances, just not feasible or literally impossible because of absence of data (Pi *et al*. 2010).

Grey prediction models can be used explicitly to establish patterns within a system, even with limited information without necessarily using external variables to aid in the prediction. Grey models, due to their simplicity, ease of use, and good predictive capability have emerged as a favoured option in the estimation of CO2 emissions in a number of countries, industries and regions. GM(1,1) is a first-order univariate model that is simple and easy to implement as it is the basic model of grey theory. It is usually not sufficient in complex scenarios; however, it loses its effectiveness when it comes to being less flexible and inconsistently accurate in providing predictions. As a countermeasure to these drawbacks, Xie and Liu (2009) have suggested Discrete Grey Model (DGM(1,1)) to balance these errors generated by the transformation of discrete data into continuous counterparts. This was subsequently surpassed by Xie *et al.* (2014), who modeled the Non-Homogeneous Discrete Grey Model (NDGM(1,1)) as an improvement in modeling performance. Zhang and Liu (2016) later worked on this direction by developing the Time-Varying Discrete Grey Model (TDGM(1,1)) to represent systems with dynamic behaviours more appropriately. The latter was especially efficient in addressing real-life data that show fluctuations and nonlinearity making the inclusion of time varying into the grey model valid.

The GM(1,1) model operates by performing a first-order accumulated generating operation (1-AGO) upon the original data sequence $x^{(0)}$, resulting in a new sequence $\mathrm{x}^{(0)}$that is more monotonic than $x^{(0)}$. Though the classical 1-AGO approach showed good predictive power in its initial iterations, it also has a number of quite significant shortcomings. Among the main weaknesses is the equal emphasis given to each of the data points, which may diminish the accuracy of prediction, especially in those systems where more proximate observations are accorded greater importance (Zeng & Meng, 2015). To solve this problem, scholars have suggested numerous improved means of accumulation to facilitate the transformation process and increase the validity of the entire model.

Recent years major developments on methodologies of accumulation have been central in increasing the efficiency of grey models particularly when encountering dynamic and volatile time series data. One uniquely important step forward involves fractional-order accumulation (F-AGO), which has received significant attention due to its potential to enhance the predictive ability of its GM(1,1) model even further. The F-AGO method was first developed by Wu et al. (2013) in 2013, and was introduced to enhance grey models by providing a more detailed representation of data trends that can minimise prediction errors thus making the grey model more versatile and usable in a wider variety of situations. Among the main advantages of F-AGO are the fact that local or recent observations are weighted more heavily, which contributes to greater flexibility and shorter lags in modeling dynamic or changing situations. Based on this assumption, multiple fractional accumulation-based grey models have been suggested, and were effectively used in forecast practice. They include the fractional GM(q,1) model (Mao et al., 2016; Li et al., 2023), Fractional Accumulated Non-Homogeneous Grey Model (FANGBM(1,1)) (Wu et al., 2013), fractional non-homogeneous grey model (Wu et al., 2020), and discrete fractional grey model (Gao et al., 2015).

The effectiveness of grey models employing fractional accumulation is heavily influenced by the parameter r, which directly controls both the fitting accuracy and prediction performance. Therefore, enhancing the accumulation operator is essential for improving the overall performance of grey models. In response to this need, Shen et al. (2020) proposed a novel operator known as the Weighted Accumulated Generating Operator (W-AGO), which builds upon the principles of F-AGO and emphasizes the importance of incorporating new information. Unlike traditional approaches, W-AGO introduces two adjustable parameters that enable flexible weighting of recent data and allow dynamic adjustment of the summation order based on the characteristics of the input sequence. Shen et al. (2020) incorporated the W-AGO into the GM(1,1) model and validated its effectiveness through four real-world case studies. However, the integration of W-AGO into the Time-Varying Discrete Grey Model (TDGM) remains largely unexplored. To address this gap, the present study proposes a novel hybrid model—referred to as WTDGM—by combining W-AGO with TDGM to enhance forecasting performance. The optimal parameters of the model are determined using a nonlinear optimization approach based on Quantum Particle Swarm Optimization (QPSO). The proposed WTDGM model is applied to forecast annual CO_2 emissions and is validated through comparative experiments against DGM(1,1), TDGM(1,1), ARIMA, and Support Vector Machine (SVM) models, demonstrating its effectiveness and practical applicability.

RELATED WORK

Forecasting time series data is a crucial analytical method for predicting future outcomes based on historical records. In the context of environmental studies, predicting carbon dioxide (CO_2) emissions has become increasingly significant due to global climate change concerns and the need for sustainable development. Various forecasting techniques have been explored in the literature. Traditional statistical approaches, including linear regression (Wang et al., 2019), ridge regression (Qian et al., 2020), and time-series models like the ARIMA (Ning et al., 2021; Yang & O'Connell, 2020), are widely employed for CO_2 emission predictions. Nevertheless, classical statistical methods often depend on assumptions such as independence and normality, which, if violated, may reduce their reliability and forecasting accuracy (Gallo et al., 2014).

With advancements in computational methods, machine learning (ML) has emerged as a flexible alternative capable of capturing nonlinear and complex relationships. Stamenković et al. (2015) demonstrated that back-propagation and general regression neural networks outperform multiple linear regression for methane emission forecasting. Similarly, Saleh et al. (2016) applied support vector regression (SVR) to predict CO_2 emissions in Yogyakarta, achieving low errors. Acheampong and Boateng (2019) developed artificial neural networks (ANNs) for multiple countries to forecast carbon intensity. Other advanced ML methods, including deep forest regression (dos Santos Coelho et al., 2024), recurrent neural networks (RNNs) (Singh & Dubey, 2021), and long short-term memory networks (LSTM) (Wu et al., 2023), have also been applied. Despite their advantages, these models require large datasets and careful parameter tuning, and may overfit when sample sizes are small (Cang et al., 2024).

Grey prediction techniques have gained widespread use in forecasting CO2 emissions due to their simplicity, ease of implementation, and reliable performance, particularly in scenarios with limited or incomplete datasets (Pao et al., 2012; Lu et al., 2009; Lotfalipour et al., 2013; Lin et al., 2011). Unlike traditional statistical or machine learning models that often require large datasets and complex parameter tuning, grey models can capture underlying trends and dynamics with relatively small sample sizes. Beyond CO2 emission forecasting, grey models have demonstrated applicability across diverse domains, including electricity demand and power system analysis (Wang et al., 2019), renewable energy consumption prediction (Wu et al., 2013), natural gas and tight gas production (Qian et al., 2020; Nyoni & Bonga, 2019), air pollution monitoring (Yang & O'Connell, 2020), traffic flow forecasting (Ning et al., 2021), industrial growth and development (Lotfalipour et al., 2013), landslide susceptibility assessment (Lin et al., 2011), and epidemiological modeling such as COVID-19 spread (He et al., 2022; Guo et al., 2021). These applications highlight the versatility and practical relevance of grey models for dynamic systems, making them a valuable tool for both environmental and socioeconomic forecasting. However, the standard GM(1,1) model assigns equal weight to old and new information, limiting adaptability and accuracy in dynamic systems.

To improve performance, several enhanced grey models have been proposed. Wu et al. (2013) introduced fractional accumulation in FANGBM(1,1), Şahin (2020) added seasonal adjustments in OFANGBM(1,1), and Xie et al. developed CCFNGBM(1,1) for CO2 emissions from fuel combustion. Other studies, including Wang & Wang (GFBGM(1,1, tα)), Guo et al. (2021), and He et al. (2022), incorporated dynamic weighting and optimization techniques to better utilize new information.

Building on these developments, this study proposes a grey prediction model with a new weighted fractional accumulation mechanism, aiming to enhance the influence of recent observations while maintaining model stability, thereby improving CO2 emission forecasting accuracy.

METHODOLOGY

This part shows the brief introduction of classical Discrete Grey Model (DGM(1,1)) and Time-Varying Discrete Grey Model (TDGM(1,1)).

1. DGM(1,1) Model

Let $x^{(0)} = \left\{x_1^{(0)}, x_2^{(0)}, \dots, x_k^{(0)}, \dots, x_n^{(0)}\right\}$ be a non-negative time series of length *n*. The first-order accumulation generating operator (1-AGO) transforms the original sequence $x^{(0)}$ into a new sequence $x^{(1)} = \left\{x_1^{(1)}, x_2^{(1)}, \dots, x_k^{(1)}, \dots, x_n^{(1)}\right\}$, where each term is defined as:

$x_k^{(1)} = \sum_{i=1}^{k} x_i^{(0)}$, for $k = 1, 2, \dots, n$.

The DGM(1,1) is given by [16]

$$x_{k+1}^{(1)} = a x_k^{(1)} + b \quad , \quad k = 1,2,\dots,n-1 \tag{1}$$

The parameters of the DGM(1,1) model can be estimated using the least squares method, as follows:

$$\begin{bmatrix} a \\ b \end{bmatrix} = (X^T X)^{-1} X^T Y$$

where

$$Y = \begin{pmatrix} x_2^{(1)} \\ x_3^{(1)} \\ \vdots \\ x_n^{(1)} \end{pmatrix} \text{ and } X = \begin{pmatrix} x_1^{(1)} & 1 \\ x_2^{(1)} & 1 \\ \vdots & \vdots \\ x_{n-1}^{(1)} & 1 \end{pmatrix} \tag{2}$$

The predicted values of DGM(1,1) model is given by

$$\hat{x}_1^{(0)} = \hat{x}_1^{(1)} \quad \text{and} \quad \hat{x}_{k+1}^{(0)} = \hat{x}_{k+1}^{(1)} - \hat{x}_k^{(1)}$$

where $\hat{x}_{k+1}^{(1)} = a x_k^{(1)} + \frac{1-a^k}{1-a} b \quad , \quad k = 1,2,\dots,n-1.$

2. TDGM(1,1) Model

The TDGM(1,1) is given by

$$x_{k+1}^{(1)} = (ak + b) x_k^{(1)} + ck + d \quad , \quad k = 1,2,\dots,n-1 \tag{3}$$

The parameters of the Time-Varying Discrete Grey Model (TDGM) can be estimated using the following least-squares estimation method:

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = (X^T X)^{-1} X^T Y$$

where

$$Y = \begin{pmatrix} x_2^{(1)} \\ x_3^{(1)} \\ \vdots \\ x_n^{(1)} \end{pmatrix} \text{ and } X = \begin{pmatrix} x_1^{(1)} & x_1^{(1)} & 1 & 1 \\ 2x_2^{(1)} & x_2^{(1)} & 2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ (n-2)x_{n-2}^{(1)} & x_{n-2}^{(1)} & n-2 & 1 \\ (n-1)x_{n-1}^{(1)} & x_{n-1}^{(1)} & n-1 & 1 \end{pmatrix}$$

(4)

The predicted values of the Time-Varying Discrete Grey Model (TDGM) can be expressed as follows:

$$\hat{x}_{k+1}^{(0)} = \hat{x}_{k+1}^{(1)} - \hat{x}_k^{(1)}$$

(5)

where $\hat{x}_{k+1}^{(1)} = (ak+b)\hat{x}_k^{(1)} + ck + d \ , \quad k = 1,2,\dots,n-1$

3. Weighted Accumulation Generating Operator of Time-Varying Discrete Grey Model

The Weighted Accumulation Generating Operator (W-AGO) is defined as follows:

$$x_k^{(w_1 w_2)} = \sum_{i=1}^{k} \binom{w_1}{k-i} w_2^{k-1} x_i^{(0)} , k = 1,2,\dots,n \tag{6}$$

where $\binom{r}{n} = \frac{r((r+1)\dots(r+n-1)}{n!}$ and $\binom{r}{0} = 0$.

The WTDGM(1,1) is given by

$$x_{k+1}^{(w_1 w_2)} = (ak+b)x_k^{(w_1 w_2)} + ck + d \ , \quad k = 1,2,\dots,n-1$$

(7)

The parameters of the WTDGM(1,1) model can be estimated using the following least-squares formulation:

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = (X^T X)^{-1} X^T Y \tag{8}$$

where

$$Y = \begin{pmatrix} x_2^{(w_1 w_2)} \\ x_3^{(w_1 w_2)} \\ \vdots \\ x_n^{(w_1 w_2)} \end{pmatrix} \text{ and } X = \begin{pmatrix} x_1^{(w_1 w_2)} & x_1^{(w_1 w_2)} & 1 & 1 \\ 2x_2^{(w_1 w_2)} & x_2^{(w_1 w_2)} & 2 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ (n-2)x_{n-2}^{(w_1 w_2)} & x_{n-2}^{(w_1 w_2)} & n-2 & 1 \\ (n-1)x_{n-1}^{(w_1 w_2)} & x_{n-1}^{(w_1 w_2)} & n-1 & 1 \end{pmatrix}$$

The prediction function of the model is developed as

$$\hat{x}_k^{(0)} = \sum_{i=1}^{k} \begin{bmatrix} -w_1 \\ k-i \end{bmatrix} w_2^{k-1} \hat{x}_i^{(w_1 w_2)} \tag{9}$$

where $\hat{x}_{k+1}^{(w_1 w_2)} = (ak+b)\hat{x}_k^{(w_1 w_2)} + ck + d\ , \quad k = 1,2,\ldots,n-1$

(10)

By setting the weight parameters to a fixed value $w_1 = w_2 = 1$, the WTDGM model simplifies to the classical TDGM(1,1) model. On the other hand, when $w_2 = 1$ and $0 < w_1 < 1$, the model becomes the fractional-order time-varying discrete grey model (FTDGM). Therefore, the WTDGM model is a generalized and unified model that encompasses both TDGM(1,1) and FTDGM as special instances.

4. Parameters Estimation of WTDGM model

In particular, the weights $w_1$and w_2 have to be specified in advance to form the weighted accumulated generating sequence (W-AGO). When the W-AGO sequence is determined, it becomes possible to estimate model parameters a, b, c and d based upon which the tendency of the system development can be determined with the help of the least-squares method in accordance with Eq. (8). These parameters have a great effect on the model to make forecasts. In determining the best combination of parameters that will create the highest prediction accuracy, Mean Absolute Percentage Error (MAPE) will be used a measurement. Thus, the optimum value of the $w_1$and w_2 can be found through the solution of the following minimization problem:

$$\min MAPE = \frac{1}{n}\sum_{k=1}^{n} \frac{\left|\hat{x}_k^{(0)} - x_k^{(0)}\right|}{x_k^{(0)}} \times 100\%$$

$$s.t. \begin{cases} 0 < w_1 \leq 1 \\ 0 < w_2 \leq 1 \\ \begin{bmatrix} a \\ b \end{bmatrix} = (X^T X)^{-1} X^T Y \\ \hat{x}_{k+1}^{(w_1 w_2)} = (ak+b)\hat{x}_k^{(w_1 w_2)} + ck + d \\ \hat{x}_k^{(0)} = \sum_{i=1}^{k} \begin{bmatrix} -w_1 \\ k-i \end{bmatrix} w_2^{k-1} \hat{x}_i^{(w_1 w_2)} \\ k = 1,2,\ldots,n \end{cases} \tag{11}$$

Generally speaking, the optimization problem presented in Equation (11) is nonlinear, non-differentiable, and it is difficult to solve analytically. The difficulty of optimization landscape can make the traditional techniques inefficient in finding solutions of high quality. In order to successfully estimate the optimal $w_1$and w_2 this paper is going to use the Quantum Particle Swarm Optimization (QPSO) algorithm. QPSO is an innovative next generation of the conventional PSO, and has improved with the concepts of quantum mechanics, and has been shown to have better global searching abilities than the conventional PSO on a wide spectrum of benchmark optimization tasks (Xi et al., 2008; Liu et al., 2010).

The QPSO algorithm iteratively shifts the position of the particles according with quantum rules of behaviour until it reaches an optimal or near-optimal solution of the weights parameters. The purpose of doing this is to reduce the MAPE and improve the accuracy of forecasting the WTDGM(1,1) model. The following outlines the step-by-step procedure used in the optimization process:

Step 1: Initialize the optimization process by starting with the original data series $x^{(0)}$, and assign initial values to the fractional-order parameters $w_1$and w_2 .
Step 2: Construct the Weighted Accumulated Sequence
Apply the W-AGO to the original data series $x^{(0)}$to obtain the accumulated sequence. This transformation incorporates the fractional-order parameters $w_1$and w_2 , emphasizing recent data and capturing nonlinear trends. The WTDGM model equation is then constructed as:

$$\hat{x}_{k+1}^{(w_1 w_2)} = (ak + b)\hat{x}_k^{(w_1 w_2)} + ck + d.$$

where $\hat{x}_{k+1}^{(w_1 w_2)}$ is the next predicted accumulated value based on current inputs.
Step 3: Estimate Model Parameters
Utilize the least-squares method (as defined in Equation (13)) to estimate the parameter vector $\beta = [a, b, c, d]$in the WTDGM model equation.
Step 4: Generate Predicted Values
Compute the fitted accumulated values $\hat{x}_{k+1}^{(w_1 w_2)}$, and subsequently derive the predicted values of the original series $\hat{x}^{(0)}$ using the inverse W-AGO process (as shown in Eq. (14)).
Step 5: QPSO-Based Optimization of $w_1$and w_2
Employ the QPSO algorithm to minimize the MAPE and determine the optimal values of the fractional orders.
Step 6: Finalize Optimal Parameters and Predictions
Identify the optimal fractional parameters $w_1$and w_2 compute the corresponding fitted accumulated series $\hat{x}_k^{(w_1 w_2)}$ and the predicted values $\hat{x}^{(0)}$.

5. Evaluation of the Model Performance

To test the proposed model accuracy in prognostication, well-known model evaluation measures were used as mean absolute percentage error (MAPE). These indicators of model evaluation are determined as follows:

$$MAPE = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{x_i^{(0)} - \hat{x}_i^{(0)}}{x_i^{(0)}}\right|$$

where $x_i^{(0)}$ is the original series, and $\hat{x}_i^{(0)}$ is the fitted or predicted series. The metric that will be used to measure the accuracy of the predictions is the MAPE. The level of interpretation of the MAPE values is taken as the benchmark criteria suggested by Lewis [21], as illustrated in Table 1.

TABLE 1. Benchmark of modelling accuracy evaluation

MAPE	<10%	10%-20%	20%-50%	>50%
Evaluation Criteria	Very High Accuracy	Good Accuracy	Moderate/Acceptable	Poor Accuracy

RESULTS AND DISCUSSIONS

The amounts of CO2 emissions in Indonesia, Malaysia, Singapore and Thailand were taken into account in the present paper on annual basis. The data of CO2 emissions were acquired at Our World in Data.We work with the total capacity of CO2 production over the period 2000-2023, where data between 2000 and 2019 serve as a model training data, and the remaining four entries are intended to verify the accuracy of various grey models by validating them (testing). The subsets of raw data of four countries are illustrated in the sequel as the Fig. 1. At the aim of receiving the best emerging coefficients $w_1$andw_2 of the WTDGM(1,1) model, the algorithm QPSO realizes in the course of this experiments. Afterwards, the W-AGO coefficients are quantified using the minimum MAPE associated with the algorithm.

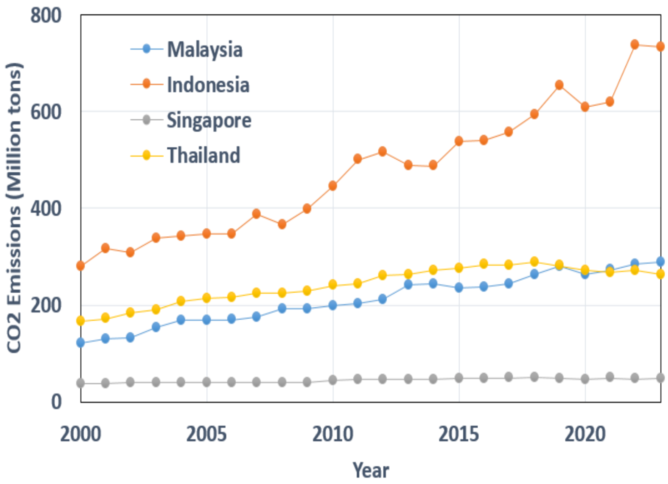

FIGURE 1. CO2 Emissions Trends in Malaysia, Indonesia, Singapore and Thailand

In this paper, the results of carbon emissions of four representative countries were used to assess and compare the predictive ability of five different models. The models will be asked to forecast the annual carbon emissions of a four-year dataset spanning 2020 to 2023.The three grey forecasting techniques contained in these models consist of Discrete Grey Model (DGM(1,1)), Time-Varying Discrete Grey Model (TDGM(1,1)) and the proposed Weighted Time-Varying Discrete Grey Model (WTDGM(1,1)), whereas Autoregressive Integrated Moving Average (ARIMA) and Support Vector Machine (SVM) are two extensively studied methods in the traditional statistical.

Various grey forecasting models, including ARIMA and SVM, are used in this part to model CO2 emissions of Malaysia. The search of the optimal value of WTDGM(1,1) on the MAPE is based on the further data of the period 2000 to 2019 via the QPSO algorithm. When the best fractional order values have been found, the WTDGM(1,1) model may be built directly based on these parameters. Table 2 gives the best fractional value of WTDGM(1,1) model. Table 3 and Fig. 2 shows the results of the MAPE values of each of their models.

The training results of CO2 in Malaysia indicate the mean absolute per cent error of the performance of WTDGM, DGM, TDGM, ARIMA and SVM are 3.765, 3.333, 2.945, 6.206 and 4.463 respectively. The comparison shows that TDGM gives a better result in fitting the initial time series, whereas, DGM, WTDGM and SVM models, as well as ARIMA shows the highest MAPE. WTDGM gives the best prediction of testing data followed by SVM, ARIMA, TDGM and DGM. Moreover, histograms represented in Figure 2 reveal that although the

TDGM model has the lowest MAPE on training dataset, the WTDGM model has the lowest MAPE on the testing one. This result implies that WTDGM is far much better in terms of generalization capacity and it provides the best estimates on forthcoming situations.

The same argument is applied to analyze the CO2 emissions of the countries being Indonesia, Singapore and Thailand. We will initially use QPSO algorithm to find the optimum of weighted order of WFTDGM(1,1) model. Table 2 illustrates the best weighted series order of WFTDGM(1,1) model of the various data. $w_1$andw_2 values are practically equal to 1, as they are the optimal weighted values across the needles of the world. Table 3 reflects the training and testing of the various models.

TABLE 2. The optimal fractional order of WTDGM(1,1) model

Weighted	Malaysia	Indonesia	Singapore	Thailand
w_1	0.9988	0.9995	0.9992	0.9993
w_2	0.9960	0.9974	0.9944	0.9968

TABLE 3. MAPE values of training and testing between the five models

Data	Countries	DGM	TDGM	WTDGM	ARIMA	SVM
Training	Malaysia	3.333	**2.945**	3.765	6.206	4.463
	Indonesi	3.703	**3.689**	3.755	6.504	4.733
	Singapore	2.276	**2.156**	4.850	2.212	2.164
	Thailand	2.571	**1.546**	2.908	3.464	1.730
Testing	Malaysia	9.505	8.578	**1.768**	4.387	3.684
	Indonesi	5.516	7.012	**4.816**	4.980	7.301
	Singapore	9.815	12.424	**2.632**	4.283	2.842
	Thailand	19.740	9.129	**3.230**	7.306	3.756

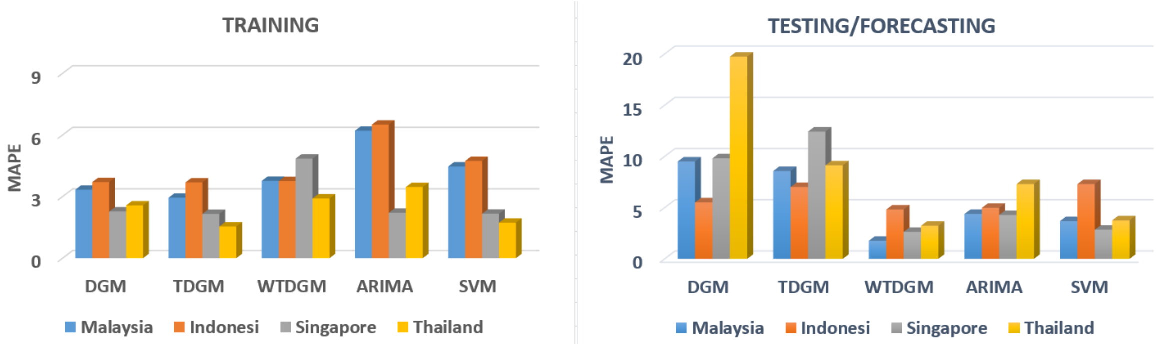

FIGURE 2. MAPE comparison of five models

Table 3 indicates, TDGM model in training data is more effective in all countries compared to other competitive models. ARIMA model has a low result in the case of Indonesia and Thailand countries and WTDGM Singapore country. All models had a MAPE that varied between [1.5-6.5]% less than 10%. Yet, during the testing phase, the optimized TWDGM model out of all models is the most accurate to all countries and its MAPE is also the lowest at around 3%. ARIMA model of Indonesia is characterized with the second lowest MAPE, and SVM model of Singapore and Thailand countries. These findings show that the TDGM model does not apply well to Indonesia and Singapore, whereas the DGM model is less accurate in Thailand. Irrespective of these shortcomings, the models tend to reflect the general CO2 emission patterns in all four countries. In the training dataset, both models have a MAPE of less than 7%, and during testing, the error was also less than 20%, meaning that both models have adequate forecasting results. It is important to note that the proposed WTDGM model has the lowest error rates in all countries, and the lowest MAPE standing at less than 5% in all countries, indicating a higher prediction potential. These findings substantiate the idea that weighted fractional accumulation improves the accuracy of forecasting substantially when it is introduced into the TDGM(1,1) framework.

CONCLUSIONS

The contribution of this work was a weighted orders that had a time-variant discrete grey model which was abbreviated as WTDGM. Optimal values of two distinct weighted orders were then obtained using the QPSO algorithm. The effectiveness of QPSO-optimized WTDGM model has been presented through numerical experiments based on four real world data sets as compared to classical models, DGM(1,1), TDGM (1,1), ARIMA and SVM. The findings were nonspecific in that the entire time, the use of the proposed WTDGM has proven better in predicting accuracy in comparison to the other models. Also, the results point at the prospect of the use of the WTDGM framework as an active approach to CO2 emissions prediction, leading to the idea that it could be a possible feature of further progress in fractional gay modeling. It is a method which could be made a base to form higher-level grey models which are better in terms of predictive capacity.

REFERENCES

Acheampong, A. O., and E. B. Boateng. 2019. “Modelling Carbon Emission Intensity: Application of Artificial Neural Network.” *Journal of Cleaner Production* 225: 833–856.

Acheampong, A. O., and E. B. Boateng. 2019. “Modelling Carbon Emission Intensity: Application of Artificial Neural Network.” *Journal of Cleaner Production* 225: 833–856.

Cang, Y., J. Li, and X. Zhao. 2024. “Deep Learning Approaches for Carbon Emission Prediction: A Review.” *Environmental Modelling & Software* 173: 105956.

Chen, Z., Z. Zan, and S. Jia. 2022. “Effect of Urban Traffic-Restriction Policy on Improving Air Quality Based on System Dynamics and a Non-Homogeneous Discrete Grey Model.” *Clean Technologies and Environmental Policy* 24 (8): 2365–2384.

dos Santos Coelho, L., V. C. Mariani, and P. N. A. Batista. 2024. “Deep Forest Regression for Environmental Forecasting Problems.” *Applied Soft Computing* 147: 110715.

Gallo, C., A. Contini, and F. Neri. 2014. “Statistical Methods for Time Series Forecasting: Limitations and Applications.” *Environmental Modelling & Software* 51: 155–165.

Gao, M., S. Mao, X. Yan, and J. Wen. 2015. “Estimation of Chinese CO_2 Emission Based on a Discrete Fractional Accumulation Grey Model.” *Journal of Grey System* 27: 114–130.

Guo, J., Y. Li, and H. Zhang. 2021. “Grey Prediction Models for COVID-19 Epidemic Forecasting.” *Chaos, Solitons & Fractals* 142: 110479.

He, Y., J. Wang, and X. Li. 2022. “Forecasting COVID-19 Spread Using an Improved Grey Prediction Model.” *Applied Mathematical Modelling* 104: 273–285.

Hosseini, M., K. Y. Liu, and C. Xu. 2019. “Global and Regional Impacts of Climate Change at Different Levels of Global Temperature Increase.” *Climatic Change* 154 (1–2): 261–276.

Hosseini, S. M., A. Saifoddin, R. Shirmohammadi, and A. Aslani. 2019. “Forecasting of CO_2 Emissions in Iran Based on Time Series and Regression Analysis.” *Energy Reports* 5: 619–631.

Lee, Y.-S., and L.-I. Tong. 2011. “Forecasting Energy Consumption Using a Grey Model Improved by Incorporating Genetic Programming.” *Energy Conversion and Management* 52 (1): 147–152.

Li, H., Z. Wu, S. Qian, and H. Duan. 2023. “A Novel Fractional-Order Grey Prediction Model: A Case Study of Chinese Carbon Emissions.” *Environmental Science and Pollution Research* 30: 110377–110394.

Lin, C.-S., S. F. Liu, and H. Y. Fang. 2011. “Grey Forecasting Model for Landslide Prediction.” *Engineering Geology* 118: 29–35.

Liu, J., J. Sun, and W. Xu. 2010. “Quantum-Behaved Particle Swarm Optimization with Adaptive Mutation Operator.” *Applied Soft Computing* 10 (2): 529–537.

Liu, L., H. Zong, E. Zhao, C. Chen, and J. Wang. 2014. “Can China Realize Its Carbon Emission Reduction Goal in 2020: From the Perspective of Thermal Power Development.” *Applied Energy* 124: 199–212.

Lotfalipour, M. R., S. Falahi, and M. Ashena. 2013. “Economic Growth, CO_2 Emissions, and Fossil Fuels Consumption in Iran.” *Energy* 63: 23–30.

Lu, S. 2024. *The Reality of Climate Change: Evidence, Impacts and Engineering Solutions.*

Lu, Z., S. Liu, and Y. Yang. 2009. “Grey Forecasting Model for Energy Consumption.” *Energy Policy* 37: 3953–3959.

Mao, S., M. Gao, X. Xiao, and M. Zhu. 2016. “A Novel Fractional Grey System Model and Its Application.” *Applied Mathematical Modelling* 40 (7–8): 5063–5076.

Mason, K., J. Duggan, and E. Howley. 2018. “Forecasting Energy Demand, Wind Generation and Carbon Dioxide Emissions in Ireland Using Evolutionary Neural Networks.” *Energy* 155: 705–720.

Nie, W., O. Ao, and H. Duan. 2023. “A Novel Grey Prediction Model with a Feedforward Neural Network Based on a Carbon Emission Dynamic Evolution System.” *Environmental Science and Pollution Research* 30: 20704–20720.

Ning, Y., Q. Wang, and J. Li. 2021. “Traffic Flow Forecasting Based on Improved Grey Models.” *Expert Systems with Applications* 170: 114512.

Pao, H.-T., and C.-M. Tsai. 2011. “Modeling and Forecasting the CO_2 Emissions, Energy Consumption, and Economic Growth in Brazil.” *Energy* 36 (4): 2450–2458.

Pao, H.-T., H.-C. Fu, and C.-L. Tseng. 2012. “Forecasting CO_2 Emissions Using Grey Models.” *Energy Policy* 40: 276–285.

Piecyk, M., and A. C. McKinnon. 2010. “Forecasting the Carbon Footprint of Road Freight Transport in 2020.” *International Journal of Production Economics* 128 (1): 31–42.

Qian, X., H. Wang, and Y. Zhang. 2020. “Ridge Regression Analysis of Carbon Emissions in China.” *Energy Reports* 6: 250–257.

Saleh, C., S. H. Widyastuti, and R. Suryani. 2016. “Prediction of CO_2 Emissions Using Support Vector Regression.” *Procedia Environmental Sciences* 33: 369–375.

Şahin, U. 2020. “Seasonal Fractional Grey Model for Energy Demand Forecasting.” *Energy Sources, Part B* 15 (4): 212–223.

Shang, X., and Y. Luo. 2021. “The Role of Economic Governance in Food Production and Food-Related CO_2 Emissions in China.” *Journal of Environmental Management* 276: 111327.

Shen, Q. Q., Q. Shi, T. P. Tang, and L. Q. Yao. 2020. “A Novel Weighted Fractional GM(1,1) Model and Its Applications.” *Mathematical Problems in Engineering* 2020: Article 6570683.

Singh, A., and R. Dubey. 2021. “Carbon Emission Forecasting Using Recurrent Neural Networks.” *Energy Reports* 7: 350–360.

Stamenković, L. J., D. Z. Antanasijević, M. Đ. Ristić, A. A. Perić-Grujić, and V. V. Pocajt. 2015. “Modeling Methane Emissions Using Artificial Neural Networks.” *Environmental Science and Pollution Research* 22: 18849–18858.

Sun, Y., W. Chen, and J. Zhang. 2024. “Global CO_2 Emissions to Reach New High in 2024 Despite Slower Growth.” *Nature Communications* 15 (1): 1–9.

Wang, S., J. Zeng, and X. Liu. 2019. “Examining the Multiple Impacts of Technological Progress on CO_2 Emissions in China.” *Renewable and Sustainable Energy Reviews* 103: 140–150.

Wu, L. F., S. F. Liu, L. G. Yao, S. L. Yan, and D. L. Liu. 2013. “Grey System Model with the Fractional Accumulation.” *Communications in Nonlinear Science and Numerical Simulation* 18 (7): 1775–1785.

Wu, W., X. Ma, Y. Zhang, W. Li, and Y. Wang. 2020. “A Novel Conformable Fractional Non-Homogeneous Grey Model for Forecasting Carbon Dioxide Emissions of BRICS Countries.” *Science of the Total Environment* 707: 135447.

Wu, Y., H. Chen, and Z. Li. 2023. “Carbon Emission Prediction Using Long Short-Term Memory Networks.” *Energy* 263: 125650.

Xie, N.-M., and S.-F. Liu. 2009. “Discrete Grey Forecasting Model and Its Optimization.” *Applied Mathematical Modelling* 33 (2): 1173–1186

Xie, X., S. Liu, et al. 2014. “Non-Homogeneous Discrete Grey Model, NDGM(1,1).” *Neural Computing and Applications* 25 (5): 1215–1221.

Yang, Y., and R. O’Connell. 2020. “Air Pollution Prediction Using Grey System Theory.” *Atmospheric Environment* 223: 117277.

Zeng, B., and W. Meng. 2015. *Research on Fractional Accumulating Generation Operators and Grey Prediction Models.* Beijing: Scientific Press.

Zhang, Z., and S. Liu. 2016. “Time-Varying Discrete Grey Prediction Model TDGM(1,1).” *Journal of Grey System* 30 (3): 345–352.