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Human Capital, Intrinsic Motivation and Poverty of Elderly in Later Life

(Modal insan, motivasi intrinsik dan kemiskinan warga tua.)

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ABSTRACT

This research analyzes the linkage between elderly poverty and children's human capital level via income distribution, in which fertility decisions are motivated by old-age support with the assumption that the parents are non-altruistic. The paper provides a mathematical model looking at the relationship of old age poverty and the children human capital level with intrinsic motivation. Under the assumption that the parents depend solely on the children as the source of income in old age, the paper applies the overlapping generation model by incorporating intrinsic motivation. This study shows that, in the absence of public education, parents who choose not to invest in the private education of their children will be locked in a poverty trap. However, children with higher motivation show an improvement in their human capital evolution, and, thus, help the parents escape the poverty trap regardless of the parental economic status.

Keywords: Old age poverty; human capital; intrinsic motivation; education

ABSTRAK

Kajian ini menganalisis hubungan antara kemiskinan usia tua dan tahap modal insan anak-anak melalui pengagihan pendapatan,dimana keputusan kesuburan adalah didorong oleh pemberian bantuan kepada ibubapa yang telah tua dengan andaian bahawa ibubapa bukan altruistik. Makalah ini menyediakan model matematik mengkaji hubungan kemiskinan diusia tua dan tahap modal insan anak-anak dengan bermotivasi intrinsik. Di bawah anggapan bahawa ibubapa bergantung semata-mata kepada anak-anak sebagai sumber pendapatan diusia tua, kajian ini menggunakan model generasi bertindan denga nmengambil kira motivasi intrinsik. Kajian ini menunjukkan bahawa, dalam ketiadaan pendidikan awam, ibubapa yang memilih untuk tidak melabur dalam pendidikan anak-anak akan terkunci dalam perangkap kemiskinan. Walaubagaimanapun, anak-anak yang mempunyai motivasi intrinsik yang tinggi didapati menunjukkan peningkatan evolusi modal insan dan dengan itu ianya dapat membantu membebaskan ibubapa daripada perangkap kemiskinan tanpa mengira jenis ekonomi ibubapa itu.

Kata kunci: Kemiskinan usia tua; modal insan; motivasi intrinsik; pendidikan

INTRODUCTION

Based on the Report on Aging Population by the Committee on Aging Issues 2006, it was projected that, in 2030, Singapore would witness an unprecedented age shift. The number of residents aged 65 years or older will multiply threefold from 296,900 in June 2005 to 873,300 along with the Total Fertility Rate (TFR) below the replacement level. Masud et al. (2006) reported that the poverty in Malaysian elderly is expected to increase, particularly among the women. This worry is confirmed by a recent study (Mohd et al. 2016) that found that poverty amongst elderly women in Malaysia increased during the period of 2009-2012. This could lead to the conclusion that the majority of the Malaysian elderly women are poor.

Although elderly poverty is profound in developing countries (Barikdar et al. 2016; Barrientos et al. 2003), it is also a worldwide issue. The OECD (2016) has reported very high elderly poverty rates in Korea, Australia, and Switzerland. According to Zaidi (2006), it is estimated that about 13 million elderly people are at risk of poverty in 25 European Union (EU) member states, which is about one-in-six of 74 million elderly people living in the EU in the early 21st century. The results were obtained by calculation using the 60% median income poverty threshold for each respective country. In the United States alone, the National Council on Aging (NCOA factsheet 2015) reports that more than 25 million elderly Americans aged 60+ are considered poor, living at or below 250% of the federal poverty level.

Moreover, in the European Union, the Eurostat's population structure and Aging (2016) reports that the population of working age is expected to decline steadily, while older persons will likely account for an increasing share of the total population – those aged 65 years or above will account for 28.7% of the EU-28's population by 2080 (18.9% in 2015). This is the result of the increased longevity and drop in fertility over many years. The proportion of people of working age is shrinking while the relative number of those retired is expanding. This will, in turn, lead to an increased burden on those of working age to provide for the social expenditure required by the aging population for a range of related services. The consistently low fertility rate and higher life expectancy, therefore, will raise several issues concerning the old age support system, family institution, poverty among the elderly people, and government public policies.

For any given level of life expectancy at birth, and age of retirement, a reduction in the fertility rate gives rise to an increase in the dependency ratio. Since the state has taken over from the family institution the responsibility to support the elderly in almost all developed countries, any increase in the dependency ratio will definitely increase the public expenditures, especially pension expenditure. Developed countries, such as Sweden, South Korea, Japan, Taiwan, Germany and many more, are practicing a universal public pension system. This public pension system covers almost all the citizens. Under the universal public pension system, the benefits to the current elderly are paid out from the contributions of the current young workers. However, in an aging society, whenever the fertility rate is decreasing sharply, the number of young contributors to the system is projected to decline drastically, and, therefore, would cause the universal public pension system's failure to sustain for a longer period. In South Korea, for example, it has been projected that in 2049, their universal public pension system will probably face bankruptcy if the government is unable to find any good reform of the system immediately. If this system collapses, then there is a possibility that the elderly would end up in poverty in their later life.

These adverse effects are exacerbated if the decline in fertility is accompanied by the increase in life expectancy. Therefore, there appears to be a consensus that pension expenditure has risen, partly because fertility has fallen, and the conventional response to this consensus argument is to raise the contribution and reduce benefits (see Cigno 2009). However, raising the contribution and reducing the benefits will place the government in a dilemma since the implementation is politically difficult and almost impossible. The current workers will resist the idea of raising the contributions since it is equal to imposing a higher income tax, which, consequently, reduces their disposable income. Meanwhile, it is extremely difficult to reduce the benefits due to the political issues, and it seems that this is not a good recommendation (see Bonoli 2003). Another suggestion is to change from an unfunded system to one that is fully funded. However, this will reduce the welfare and would not be socially optimal because the cost of covering the previous cohort is too high (Cigno 2009). In addition, he also suggests that if the government provides subsidies, such as child benefits and education subsidies, it will probably increase the number of children, which, subsequently, might increase the number of contributors to the pension system.

In general, in order to reverse the fertility, government family policies can be divided into two: (1) cash benefit policies (family and child cash allowances, tax credits, and tax reduction), and (2) non-cash benefit policies (childcare provision, subsidized services for children and families, and maternity and parental leave). Australia for example, introduced a Baby Bonus in July 2004, in order to boost the fertility rate among Australian women (Langridge et al. 2012). As a welfare state, Sweden also put effort into encouraging a fertility boost by reducing the childcare cost (Mork et al. 2013). However, a few studies have reported that although these policies may have some effect on the fertility rates, the magnitude of that impact is still an issue (Whittington et al., 1990). Moreover, based on the empirical evidence, the impact of the policies on fertility provide mixed conclusions (Gauthier 2007), and the effectiveness of these policies remains an open question (Chen 2011).

On the other hand, in developing countries, not all the citizens are covered by the public pension system. Those people who worked with the government when they were young, would receive an amount of pension benefit based on their employment history. Meanwhile, those people working in the private sector must allocate a portion of their income as saving for the consumption during their period of retirement. However, many have reported that the low-income workers in the public sector in developing countries receive inadequate pension benefits, and low-income workers in the private sector do not have adequate savings. Therefore, both types of worker are vulnerable and are exposed to the old age poverty trap.

In the near future, either in developed countries or developing countries, the probability of the elderly falling into the poverty trap is high. In the developed countries, the system provides adequate benefits, but there is a tendency that the system would collapse due to the shortage of the labor force. In contrast, in the developing countries, the benefit itself is inadequate to provide a better life for the elderly. Since there is a problem with the public pension system in both the advanced economy and developing economy, the need to revisit the family system as an alternative to the old age social security system is now crucial.

At the theoretical level, a number of studies related to old age poverty exist (see Azarnert 2010; Azariadis & Drazen 1993; Becker 1974; Chakrabarti 1999; Ehrlich & Lui 1991; Morand 1999). In general, this literature can be divided into two major strands. The first strand is the overlapping generation model with endogenous fertility, and the fertility decision is motivated by the parent's altruism. The second strand is the overlapping generation model with endogenous fertility, and the fertility decision is motivated by the old-age support.

This paper mainly contributes to the second strand of the literature and can be viewed as an extension of the work of Morand (1999) and Azarnert (2010). Morand (1999) discusses the poverty trap and argues that an economy populated by identical agents evolves from the poverty trap if and only if the per capita human capital level is above the critical threshold. Moreover, in the economy populated by the heterogeneous agents, if the number of highly skilled agents is too low, then the economy will be locked in the poverty trap. Azarnert (2010) focuses more on free education and claims that parental human capital levels are crucial for determining the effect of free education.

The main objective of this paper is to study the effects of the education history of the extended family of an agent and to find the relationship of the human capital level of the children and the poverty of the elderly parent. In light of the objective, we will closely follow Morand (1999) and Azarnert (2010) by incorporating the concept of the intrinsic motivation of the children in pursuing knowledge so that it would be able to capture the internal effects consistent with what Lucas (1988) explained in his paper.

The rest of the article is outlined as follows. In section 2, we introduce the basic model. In section 3, we discuss the outcomes of the model, and, finally, in section 4, we make our conclusions.

THE MODEL

Consider a small, open, overlapping-generations economy, where individuals have identical preferences, and their lives can be divided into three parts: childhood, adulthood, and old age. During childhood, we assume that children do not make any economic decisions. When a child becomes an adult, they will start working and supplying inelastically one unit of labor to the labor market, and, in return, they will receive an amount of income. In the absence of capital markets, when the children are the only asset that is available and the only source of income for the elderly parent in the old age, then it is essential for the parent to allocate some positive amount of their income to raise the children and may also opt to invest in the children's education in addition to free public education. In return, the children will also allocate a fraction of their income to support their elderly parents.

HUMAN CAPITAL FORMATION AND THE LEARNING TECHNOLOGY WITH INTRINSIC MOTIVATION

In this economy, an adult in period t is attributed to a level of human capital, h_t, and his income is associated with his level of human capital and is denoted as, $h_t w$. He, therefore, may decide to use some fraction of his income to invest in the children's education in order to enhance their human capital level. The human capital level of each child, for a given public educational level, will depend on the parent's human capital level, h_t, the fraction of parent's investment in education, e_t, the level of their intrinsic motivation to acquire skills, m_t, and on the average level of human capital of the parent $\bar{h}_t$. (The derivations of the model are in the appendices.)

The learning technology of each child is therefore described as follows:

$$h_{t+1} = m_t(\bar{h}_t + \tilde{h}_t + e_t h_t)^{\gamma} \tag{1}$$

and will capture an internality and two main externalities: local externality and global externality. Local externality refers to the home environment provided by the parent to the children. A better home environment will provide a good atmosphere for motivating children to study, and all these externalities will be captured by the properties of the learning technology with respect to h_t. On the other hand, the global externality refers to the environment outside the house that could contribute to enhancing the level of skills of the children. This externality will be captured mainly by the properties of the human capital production function with respect to $\bar{h}_t$, the average level of human capital of parents; $\tilde{h}_t$ the level of public educational; and, e_t the level of private education.

Meanwhile, the internality effect will be captured by m_t which, in this model, we define as an intrinsic motivation of the children to pursue knowledge. We closely follow the definition of intrinsic motivation as given by Frey and Jegen (2001), i.e., someone doing something because it is interesting and enjoyable, and the incentives are coming from within the person. By incorporating intrinsic motivation in the learning technology, we are able to implant the sense of humanity in economic theory. Being a human, we not only do things because of external monetary or non-monetary rewards or punishments, but we also do things because we love to do them. As illustrated in Ryan and Deci (2000), someone with a high intrinsic motivation is more creative, and is able to create a novelty in knowledge and skills. Consequently, this behavior will affect performance, persistence, and well-being across life's epochs. Therefore, by introducing the intrinsic motivation in our learning technology, the children will not merely enhance their skills or knowledge by inheriting it from society or their parents but from within themselves, where there is a great power that is able to drive them to achieve a greater level of human capital.

THE OPTIMIZATION PROBLEM OF PARENTS

In this model, individuals are assumed to have identical preferences, and their life can be divided into three parts: childhood, adulthood, and old age. During childhood, it is assumed that each individual cannot make any economic decisions and they acquire human capital. When they reach adulthood, each individual starts to work and supply inelastically one unit of labor to the labor market, and, in return, he or she will receive an amount of income, for which allocation is made between consumption, child rearing, and old-age support. In old age, the parents will spend their life using the old-age support given by the living children.

In this model, the parents choose a current consumption, c_t, the number of children, n_t, and the children are taken to be identical. The representative individual entering the working period at time, t, faces the following simple log-linear utility form:

$$U_t(c_{1,t}, c_{2,t+1}) = \ln c_{1,t} + \frac{1}{1+\rho} \ln c_{2,t+1} \quad (2)$$

Choices of $c_{1,t}, c_{2,t+1}, n_t$, and e_t are made simultaneously and subject to the following budget constraints:

$$c_{1,t} + \alpha_1 wh_t + (e_t + a_t)n_t wh_t = wh_t \quad (3)$$

The right-hand side of equation (3) is an adult's labor income, which is allocated among current consumption, $c_{1,t}$, elderly old-age support in the form of a fraction α_i, $i = 1,2$ of income, and the costs of raising and educating children, , as shown on the left-hand side of the equation. Here, we define the second period of consumption as $c_{2,t+1} = \alpha_2 wh_{t+1} n_t$. In the second period, the adult retires from work and will consume some part of the children's income. The wage per efficiency unit of labor, w, is fixed over time and is assumed to be determined outside of this model, for instance through the small open-economy assumption.

THE INVESTMENT DECISION AND THE NUMBER OF CHILDREN

For future life consumption, parents make two decisions. The first decision is the number of children. The second decision is the amount to invest in education in order to enhance the quality of the children so that the children are able to give old-age support to the parents. Therefore, we set up a Lagrangian model and solve this optimization problem as follows:

$$L = \ln c_{1,t} + \frac{1}{1+\rho} \ln c_{2,t+1} + \lambda\big[wh_t - c_{1,t} - \alpha_1 wh_t - (e_t + a_t)\, n_t\, wh_t\big] \quad (4)$$

By solving the above optimization problem, the investment in quantity, or choice of an optimal number of children, n_t, we computed $\frac{\partial L}{\partial C_{1,t}} = \frac{1}{C_{1,t}} - \lambda = 0$ and $\frac{\partial L}{\partial n_{1,t}} = \frac{1}{1+\rho} \frac{1}{C_{2,t+1}} \frac{\partial C_{2,t+1}}{\partial n_t} - \lambda(e_t + a_t)wh_t = 0$. By these, we obtained the following first-order condition:

$$\frac{\partial U}{\partial c_{2,t+1}} \alpha_2 wh_t = \frac{\partial U}{\partial c_{2,t+1}} (e_t + a_t)wh_t \quad (5)$$

Or, in another form, after algebraic manipulation,

$$\frac{\partial U}{\partial c_{2,t+1}} \frac{c_{2,t+1}}{n_t} = \frac{\partial U}{\partial c_{1,t}}(e_t + a_t)wh_t \quad (6)$$

For the second decision, i.e., investment in quality, or the optimal choice of investment in children's education, e_t we computed $\frac{\partial L}{\partial C_{1,t}} = \frac{1}{C_{1,t}} - \lambda = 0$ and $\frac{\partial L}{\partial e_{1,t}} = \frac{1}{1+\rho} \frac{1}{C_{2,t+1}} \frac{\partial C_{2,t+1}}{\partial n_{t+1t}} \frac{\partial h_{t+1}}{\partial e_t} - \lambda n_t wh_t = 0$. By these, we obtained the following first-order condition:

$$\frac{\partial U}{\partial c_{2,t+1}} a_2 wn_t \frac{\partial h_{t+1}}{\partial e_t} = \frac{\partial U}{\partial c_{1,t}} n_t wh_t \quad (7)$$

Or, in another form, after some manipulation of equation (7), we obtained:

$$\frac{\partial U}{\partial c_{2,t+1}} \frac{c_{2,t+1}}{h_{t+1t}} \frac{\partial h_{t+1}}{\partial e_t} = \frac{\partial U}{\partial c_{1,t}} n_t wh_t \quad (8)$$

Denoting $R_t(h)$ as the rate of return on human capital for a given number of children, and $R_t(n)$ as the rate of return on children quantity for a given level of parental investment in education, and using the first-order conditions (6) and (8), $R_t(h)$ and $R_t(n)$ are given as follows:

$$R_t(n) = \frac{c_{2,t+1}}{(e_t + a_t)wh_t n_t}, \quad (9)$$

$$R_t(h) = \frac{c_{2,t+1}}{n_t wh_t h_{t+1}} \frac{\partial h_{t+1}}{\partial e_t}, \quad (10)$$

In order to find the optimal interior solutions for n_t, and e_t, the rates of return on quantity and the rates of return on quality, respectively, they must be equal. By equation (9) and (10), the optimal interior solution is therefore given by:

$$h_{t+1} = (e_t + a_t) \frac{\partial h_{t+1}}{\partial e_t}, \quad (11)$$

The next subsection discusses the solutions to the parents' optimization problem for a particular form of learning technology.

CHOICE OF FERTILITY AND INVESTMENT IN EDUCATION FOR A PARTICULAR LEARNING TECHNOLOGY

In order to fully understand the choice of fertility and private investment in education, we need to study a particular learning technology. By studying a particular learning technology, we are able to set up a threshold level that could be treated as a benchmark point to measure the human capital level. Throughout this paper, human capital is assumed to be acquired by private education investment in addition to the formal public schooling. To capture the possible externalities and the internality, we use the learning technology postulated in equation (1).

This particular learning technology implies that the average human capital is increasing over time, and for any given level of free public education, this particular learning technology will be able to capture the effect of the extra private educational investment, that is, parents' choice concerning the children's human capital evolution.

From equations (1) and (11), we divide the economy into two types based on the public education. The first type is the economy without public education, and the second type is the economy with public education. The general form of human capital level of the parents for both economies is given as follows:

$$h_t^P = \frac{\bar{h}_t + \tilde{h}_t}{\alpha_t\gamma + (\gamma - 1)e_t}, \tag{12}$$

$$h_t^{NP} = \frac{\bar{h}_t}{\alpha_t\gamma + (\gamma - 1)e_t}, \tag{13}$$

Equation (12) is the human capital level of the parents in the economy with public education, and equation (13) is the human capital level of parents in the economy without public education. It is obvious from the above equations that the level of human capital of the parents in the economy with public education is higher than that of the parents in the economy without public education. This led us to the following proposition:

Proposition 1 *Let h_t^P be the level of human capital of the parents in the economy with public education and h_t^{NP} be the level of human capital of the parents in the economy without public education. If $\frac{e_t}{\alpha_t + e_t}$, $< \gamma < 1$, and $h_t^{\%} < 0$ then $h_t^P > h_t^{NP}$.*

Proof Proposition is proven directly. Assume that $\frac{e_t}{\alpha_t + e_t} < \gamma < 1$ and $\tilde{h}_t > 0$ From equation (12), $h_t^P = \frac{\bar{h}_t + \tilde{h}_t}{\alpha_t\gamma + (\gamma - 1)e_t}$. It follows that $h_t^P = \frac{\bar{h}_t}{\alpha_t\gamma + (\gamma - 1)e_t} + \frac{\tilde{h}_t}{\alpha_t\gamma + (\gamma - 1)e_t}$, and from equation (13), it implies that $h_t^P - h_t^{NP} = \frac{\tilde{h}_t}{\alpha_t\gamma + (\gamma - 1)e_t}$. Since $\frac{e_t}{\alpha_t + e_t} < \gamma < 1$, and $\tilde{h}_t > 0$, then $h_t^P > h_t^{NP} > 0$. Therefore, $h_t^P > h_t^{NP}$. **QED**

AN ECONOMY WITHOUT PUBLIC EDUCATION

In the economy without public education, with the assumption that all agents are identical, then all the parents in the economy are either investing in the children's education or choose not to invest in the quality of their children. Therefore, in order to study how the children of these parents behave, it is crucial for us to set up a threshold level as follows:

$$\hat{h}_1 = \frac{\tilde{h}_t}{\alpha_t\gamma}. \tag{14}$$

If the level of human capital of the parents is below the threshold level, then the level of human capital of their children will be as follows:

$$h_{t+1}^{NP-E} = m_t(\bar{h})^{\gamma} \tag{15}$$

Equation (15) shows that the children in this economy do not merely inherit some amount of society's skill level, but that it depends more on the level of their own intrinsic motivation of acquiring knowledge from the society. In this economy, the optimal number of children is:

$$h_t^{NP-E} = \frac{1 - \alpha_1}{(2 + \rho)a_t} \tag{16}$$

On the other hand, if the level of human capital of the parents in the economy is above the threshold level, then it is optimal for them to invest in their children's education. Therefore, the optimal level of investment in education is as follows:

$$e_t = \frac{1}{1 - \gamma}\left[\gamma a_t - \frac{\bar{h}_t}{h_t}\right] \tag{17}$$

so that, according to equation (1), the level of human capital of their children is:

$$h_{t+1}^{NP+E} = m_t\left\{\frac{\gamma}{1 - \gamma_t}\left[a_t h_t - \bar{h}_t\right]\right\} \tag{18}$$

and the optimal number of children is:

$$n_t^{NP+E} = \frac{(1 - \alpha_1)(1 - \gamma)h_t}{(2 + \rho)(a_t h_t - \bar{h}_t)} \tag{19}$$

Equation (17) shows that a parent's optimal choice of investment in the children's education is positively related to the parent's own human capital level. Equation (18) exhibits the children's human capital level and is consistent with the empirical findings with regards to the positive correlation between the human capital level of the parents and the children.

Equation (19), *ceteris paribus*, explains the negative relationship between the choice of fertility and parental human capital level. It also explains the negative relationship between the choice of fertility and the costs of rearing children, and also between the choice of fertility and the size of contribution to the children's grandparents. We summarize the results in the following propositions:

Proposition 2 *Let $\tilde{h}_t = 0$. Then it is optimal for a parent to invest in education if and only if the level of human capital of the parent is, $h_t > \frac{1}{a_t\gamma}\bar{h}_t$.*

Proof Let $h_{t+1} = m_t\left\{\frac{\gamma}{1 - \gamma_t}\left[a_t h_t - \bar{h}_t\right]\right\}^{\gamma}$. From the assumption, we know that, $h_{t+1} > 0$. Therefore $a_t h_t - \bar{h}_t > 0$. Thus, it implies that $h_t > \frac{1}{at\gamma}\bar{h}_t$. **QED**

Proposition 3 *Let h_{t+1}^{NP+E} be the level of human capital of the children with education investment in the economy without public education and h_{t+1}^{NP-E} be the level of human capital without education investment in the economy without public education. If $e_t h_t > 0$, then $h_{t+1}^{NP+E} > h_{t+1}^{NP-E}$.*

Proof The proof is straightforward from the definition. From equation (1), the level of human capital of the children with education investment in the economy without public education can be rewritten as $h_{t+1}^{NP+E} = (\bar{h}_t + e_t h_t)^{\gamma}$. This implies that $\left(h_{t+1}^{NP+E}\right)^{1/\gamma} = m_t^{1/\gamma}\bar{h}_t + m_t^{1/\gamma} e_t h_t$. Therefore, from equation (15), $\left(h_{t+1}^{NP+E}\right)^{1/\gamma} - \left(h_{t+1}^{NP-E}\right)^{1/\gamma} = m_t^{1/\gamma} e_t h_t$. Since $e_t h_t > 0$, then $\left(h_{t+1}^{NP+E}\right)^{1/\gamma} - \left(h_{t+1}^{NP-E}\right)^{1/\gamma} > 0$. Thus, $h_{t+1}^{NP+E} > h_{t+1}^{NP-E}$. **QED**

Proposition 4 *Let n_t^{NP+E} be the number of children for a parent with the human capital level above the threshold level $\hat{h}_1$ and n_t^{NP-E} be the number of children for a parent with the human capital level below the threshold level $\hat{h}_1$. If $h_t = \hat{h}_1$, then n_t^{NP-E} , n_t^{NP+E}, and if $h_t > \hat{h}_1$, then $n_t^{NP-E} < n_t^{NP+E}$.*

Proof The proof is straightforward. From equation (19), $n_t^{NP-E} = \dfrac{(1-\alpha_1)(1-\gamma)h_t}{(2+\rho)(a_t h_t - \bar{h}_t)}$, then it is equivalent to: $n_t^{NP+E} = \dfrac{(1-\alpha_1)(a_t h_t - a_t\gamma h_t)}{(2+\rho)a_t(a_t h_t - \bar{h}_t)}$.

From equation (16), it implies that $n_t^{NP+E} = n_t^{NP-E} \dfrac{a_t h_t - a_t\gamma h_t}{a_t h_t - \bar{h}_t}$. Given $\hat{h}_1 = \dfrac{\bar{h}_t}{a_t\gamma}$. If $h_t = \hat{h}_1$, it follows that $\dfrac{a_t h_t - a_t\gamma h_t}{a_t h_t - \bar{h}_t} = \dfrac{a_t\hat{h}_1 - a_t\gamma\hat{h}_1}{a_t\hat{h}_t - a_t\gamma\hat{h}_t} = 1$. Hence, $n_t^{NP+E} = n_t^{NP-E}$.

Given $\hat{h}_1 = \dfrac{\bar{h}_t}{a_t\gamma}$, it follows from equation (16), that $n_t^{NP+E} = n_t^{NP-E}\dfrac{a_t h_t - a_t\gamma h_t}{a_t h_t - \bar{h}_t}$ is equivalent to $n_t^{NP+E} = n_t^{NP-E}(1-\gamma)(1-\gamma\hat{h}_1 h_t^{-1})^{-1}$. If $h_t > \hat{h}_1$ it implies that $(1-\gamma)(1-\gamma\hat{h}_1 h_t^{-1})^{-1} < 1$ and thus $\dfrac{n_t^{NP+E}}{n_t^{NP-E}} < 1$. Therefore, $n_t^{NP+E} < n_t^{NP-E}$. **QED**

Proposition 4, implies that the optimal choice of fertility depends on the parental human capital. It is optimal for a parent with a high human capital level to choose low fertility when compared to a parent with a low human capital level.

However, if the parental human capital level of the parent is equal to the threshold level, then it is optimal not to invest in children's education, and the fertility rate is equal for any parent.

Proposition 5 *The number of children for both types of parent is negatively related to the size of contribution to the elderly.*

Proof The proof is direct from equation (16) and (19). **QED**

This proposition implies that, if the parents have to distribute their income to support their elderly parents, then they have to make a tradeoff between the costs of rearing their own child and the costs of caring for their own elderly parents. If they spend more on their parents, then they must reduce the number of children, and, unconsciously, they reduce their future income for old age. Therefore, they must choose wisely in terms of the best amount or size of contribution that they should give to their own parents without losing too much of their future income.

AN ECONOMY WITH PUBLIC EDUCATION

On the other hand, in an economy with public education, we could set up the following threshold level to study the behavior of the parents and children in the economy:

$$\hat{h}_2 = \frac{\bar{h}_t + \tilde{h}_t}{\gamma\alpha_t}. \tag{20}$$

This threshold level is a necessary condition for the existence of the interior solution of the human capital level of which the rate of returns on quality and quantity are equal. Under the assumption that all agents are identical, the threshold level divides the parents in this economy into two types. This division is based on the level of their human capital, either lower or above the threshold level $\hat{h}_2$. Interestingly, comparing the threshold levels in equations (20) and (14) gives us a fascinating result that leads us to the following proposition:

Proposition 6 *If $\hat{h}_2$ is the threshold level of the economy without public education, and $\hat{h}_2$ is the threshold of the economy with public education, then $\hat{h}_2 > \hat{h}_1$.*

Proof Let the threshold level of human capital in the economy without public education be $\hat{h}_2 = \dfrac{\bar{h}_t}{\gamma\alpha_t}$. If the threshold level of the human capital in the economy with public education is $\hat{h}_2 = \dfrac{\bar{h}_t + \tilde{h}_t}{\gamma\alpha_t}$, and $\hat{h}_2 = \dfrac{\bar{h}_t + \tilde{h}_t}{\gamma\alpha_t} = \dfrac{\bar{h}_t}{\gamma\alpha_t} + \dfrac{\tilde{h}_t}{\gamma\alpha_t} = \hat{h}_2 = \dfrac{\bar{h}_t}{\gamma\alpha_t}$, then $\hat{h}_2 = \hat{h}_1 = \dfrac{\tilde{h}_t}{\gamma\alpha_t}$. Since $\dfrac{\tilde{h}_t}{\gamma\alpha_t} > 0$, hence, it implies that $\hat{h}_2 > \hat{h}_1$. **QED**

If the parent's human capital level is below the threshold level $\hat{h}_2$, then the level of human capital of their children is as follows:

$$h_{t+1}^{P-E} = m_t(\bar{h}_2 > \tilde{h}_1)^{\gamma} \tag{21}$$

The children will inherit the society human capital in addition to the given level of public education. Moreover, the size of inheritance strongly depends on the attitude and intrinsic motivation of the children themselves toward the knowledge. Even though a parent does not invest in the children's education since it is not optimal to invest, the level of human capital of the children in this economy is higher than that of the children whose parents do not invest in the children's education in the economy without public education. The following proposition illustrates this statement:

Proposition 7 *Let h_{t+1}^{P-E} be the human capital level of the children in the economy with public education without parental investment on education, and h_{t+1}^{NP-E} be the human capital level of the children in the economy without public education and without private education investment. For a given $\tilde{h}_1 > 0$ and $m_t > 0$, then $h_{t+1}^{P-E} >$ h_{t+1}^{P-E}.*

Proof This proposition is proven directly. From equation (21), $h_{t+1}^{P-E} = m_t(\bar{h}_2 > \tilde{h}_1)^{\gamma}$. It is equivalent to write it as $\left(h_{t+1}^{P-E}\right)^{\frac{1}{\gamma}} = m_t^{\frac{1}{\gamma}}\bar{h}_2 + m_t^{\frac{1}{\gamma}}h_t^{\%}$. By using equation (15), it follows that $\left(h_{t+1}^{P-E}\right)^{\frac{1}{\gamma}} = \left(h_{t+1}^{NP-E}\right)^{\frac{1}{\gamma}} + m_t^{\frac{1}{\gamma}}\tilde{h}_t$. Since $\tilde{h}_t > 0$ and $m_t > 0$, then $\left(h_{t+1}^{P-E}\right)^{\frac{1}{\gamma}} = \left(h_{t+1}^{NP-E}\right)^{\frac{1}{\gamma}} > 0$. Hence, $h_{t+1}^{P-E} > h_{t+1}^{NP-E}$. **QED**

If the parental human capital level is above the threshold level $\hat{h}_2$, then it is optimal for the parents to invest in the education of the children. The optimal choice for investment is as follows:

$$e_t = \frac{1}{1-\gamma_t}\left(a_t\gamma - \frac{\bar{h}_t + \tilde{h}_t}{h_t}\right) \tag{22}$$

so that according to equation (1), the level of human capital for the children is, therefore,

$$h_{t+1}^{P+E} = m_t\left\{\frac{\gamma}{1-\gamma_t}\left[a_t h_t - (\bar{h}_t + \tilde{h}_t)\right]\right\}^{\gamma} \tag{23}$$

and

$$n_t^{P+E} = \frac{(1-\alpha_1)(1-\gamma)h_t}{(2+\rho)\left[a_t h_t - (\bar{h}_t + \tilde{h}_t)\right]} \tag{24}$$

From equations (18) and (23), the human capital level of the children in the economy with public education who the parents choose to invest in education is higher than that of the children in the economy without public education in which the parents choose to invest in education. This led us to the following proposition:

Proposition 8 *Let h_{t+1}^{P+E} be the level of human capital of the children with education investment in the economy with public education, and h_{t+1}^{P-E} be the level of human capital without education investment in the economy with public education. If $e_t h_t > 0$, then $h_{t+1}^{P+E} > h_{t+1}^{P-E}$.*

Proof The proof is straightforward from the definition. From equation (1), the level of human capital of the children with education investment in the economy without public education can be rewritten as $h_{t+1}^{P+E} = m_t(\bar{h}_t + \tilde{h}_1 + e_t h_t)^{\gamma}$. This implies that $\left(h_{t+1}^{P+E}\right)^{\frac{1}{\gamma}} = m_t^{\frac{1}{\gamma}}\bar{h}_t + m_t^{\frac{1}{\gamma}}\tilde{h}_t + m_t^{\frac{1}{\gamma}}e_t h_t$. Therefore, from equation (21), $\left(h_{t+1}^{P+E}\right)^{\frac{1}{\gamma}} - \left(h_{t+1}^{P-E}\right)^{\frac{1}{\gamma}} = m_t^{\frac{1}{\gamma}}e_t h_t$. Since $e_t h_t > 0$, then $\left(h_{t+1}^{P+E}\right)^{\frac{1}{\gamma}} - \left(h_{t+1}^{P-E}\right)^{\frac{1}{\gamma}} > 0$. Thus, $h_{t+1}^{P+E} > h_{t+1}^{P-E}$. **QED**

With regards to the optimal number of children, the following proposition is obtained:

Proposition 9 *Let h_{t+1}^{NP+E} be the level of human capital of the children with education investment in the economy without public education and h_{t+1}^{P-E} be the level of human capital without education investment in the economy with public education. Then:*

1. *If $\tilde{h}_1 > e_t h_t$, then $h_{t+1}^{P-E} > h_{t+1}^{NP+E}$.*
2. *If $\tilde{h}_1 < e_t h_t$, then $h_{t+1}^{NP+E} > h_{t+1}^{P-E}$.*

Proof The proof is straightforward. From the definition, $h_{t+1}^{P-E} = (\bar{h}_t + \tilde{h}_t)^{\gamma}$, it follows that $\left(h_{t+1}^{P-E}\right)^{\frac{1}{\gamma}} = m_t^{\frac{1}{\gamma}}\bar{h}_t + m_t^{\frac{1}{\gamma}}\tilde{h}_t$. Since $m_t^{\frac{1}{\gamma}}\bar{h}_t = \left(h_{t+1}^{NP+E}\right)^{\frac{1}{\gamma}} - m_t^{\frac{1}{\gamma}}e_t h_t$; hence, $\left(h_{t+1}^{P-E}\right)^{\frac{1}{\gamma}} - \left(h_{t+1}^{NP+E}\right)^{\frac{1}{\gamma}} = m_t^{\frac{1}{\gamma}}\left(\tilde{h}_t - e_t h_t\right)$. Therefore, if $\tilde{h}_t > e_t h_t$, then $h_{t+1}^{P-E} > h_{t+1}^{NP+E}$, and if $\tilde{h}_t < e_t h_t$, then $h_{t+1}^{NP+E} > h_{t+1}^{P-E}$. **QED**

Lemma 1 It is optimal for the parents who choose to invest in the education of their children in addition to the public education to have the highest human capital level among four types of parent, and also their children's human capital level. However, in terms of fertility, their fertility is the lowest among the four types of parent.

LATER LIFE POVERTY TRAPS

The main objective of this paper is to determine which type of parent will suffer in poverty in later life and which type of parent will probably not. In the earlier discussion, in the absence of the capital market, it is a great task for the children to finance the consumption of their parents in later life.

Since the income level of the children is determined by their level of human capital, it is important to study the law of motion on how the children's human capital level evolves. The analysis is conducted in two ways. Firstly, is to assume that the economy is populated by identical agents, and, secondly, is to assume that the economy is populated by heterogeneous individuals.

ECONOMY WITH IDENTICAL AGENTS

As previously discussed, in this simple model with the assumption that economy is populated by identical agents, the economy will be divided into two types. In each economy, there will be two types of parent, and it can be summarized as follows:

1. An economy without public education and the parent chooses not to invest in education.
2. An economy without public education and the parent chooses to invest in education.
3. An economy with public education and the parent chooses not to invest in education.
4. An economy with public education and the parent chooses to invest in education.

Among these four types of parent, there will be those who are going to suffer in later life due to the inadequate financial resources, and there will those who will survive in later life. Therefore, the following determines the results:

Lemma 2 The parents who choose not to invest in the children's education in an economy without public education will be locked in the poverty trap. Meanwhile, for the parents who choose not to invest in children's education in an economy with public education, will be locked in poverty trap if and only if the ratio of human capital of parent to the grandparent is equal or less than one. However, there is a chance to escape from future poverty if and only if all the generations possess high intrinsic motivation.

Proof This lemma is proved directly. In general, the human capital of children is given by $h^{*}_{t+1} = m_t \left\{ \frac{\gamma}{1-\gamma_t} \left[a_t h_t - (\bar{h}_t + \tilde{h}_t)\right] \right\}^{\gamma}$. By using this general equation for the optimal human capital level of the children, we can divide it into four as follows:

Type 1 Children with no parental investment in education in the economy without public education, $h_t^{NP-E} = m_{t+1}(\bar{h}_t)^{\gamma}$.

Type 2 Children with parental investment in education in the economy without public education, $h_{t+1}^{NP+E} = m_{t+1} \left\{ \frac{\gamma}{1-\gamma_t} \left[a_t h_t - \bar{h}_t\right] \right\}^{\gamma}$.

Type 3 Children with no parental investment in education in the economy with public education, $h_{t+1}^{NP+E} = m_{t+1}(\bar{h}_t + \tilde{h}_t)^{\gamma}$.

Type 4 Children with parental investment in education in the economy with public education,

$$h_{t+1}^{P+E} = m_{t+1} \left\{ \frac{\gamma}{1-\gamma_t} \left[a_t h_t - (\bar{h}_t + \tilde{h}_t)\right] \right\}^{\gamma}.$$

Here, we have four types of children based on their parent choices in parental investment in education. We verify the evolution of children's human capital level in Type 1 and Type 3.

The level of human capital of the parent for the children of Type 1, and Type 3 are below the threshold level $\hat{h}_1$ and $\hat{h}_2$, respectively. In the case of children of Type 1, since $h_t^{NP-E} < \hat{h}_1$, then the parents who do not invest in private education, implies that the children will inherit a portion of the parental human capital. Because $\bar{h}_t = h_t$, then, in this case, it implies that, $h_t^{NP-E} = m_{t+1}(h_t)^{\gamma}$. Since $h_t^{NP-E} = m_t(h_{t+1})^{\gamma}$, then $h_t^{NP-E} = [m_{t+1}(h_{t-1})^{\gamma}]^{\gamma}$, or equivalently $h_{t+1}^{NP-E} = m_{t+1}\, m_t^{\gamma-1} h_t^{NP-E}$. Since the agents are identical, then $m_t = m_{t+1}$. This implies that both parents and children have the same human capital level, and, therefore, are locked in poverty. However, in the case of children of Type 3, it is a bit different. If $m_t = m_{t+1}$, and $\sigma h_t^{P-E} = h_{t+1}^{NP-E}$, and $\sigma = \frac{(\bar{h}_t + \tilde{h}_t)^{\gamma}}{(\bar{h}_{t-1} + \tilde{h}_{t-1})^{\gamma}}$ the evolution of the children's human capital depends on the ratio σ. If $\sigma < 1$, then both parent and children will be trapped in poverty, but if $\sigma > 1$, then there is a possibility that the children's human capital level will evolve and surpass the threshold level.

Since the proofs for the children with the human capital level of Types 2 and 4 are similar to the children of Type 3, then we will omit the proof. They will be locked in poverty if and only if the value of $\sigma < 1$ and the human capital level will decrease and eventually cross the threshold level backward.

However, if all these identical agents possess high intrinsic motivation, all types of children will eventually evolve cross the threshold level and maintain the momentum forwards. **QED**

ECONOMY WITH HETEROGENEOUS AGENTS

In the previous section, we already explained that four types of parent exist, i.e., Type 1, Type 2, Type 3, and Type 4. If the agents are identical, the analysis is quite simple because, for any type of economy, there is only one type of parent. However, when we want to discuss the economy with heterogeneous agents, then the analysis would be a little bit complex. There are two ways to discuss: Analyzing the agents based on the average level of society human capital level or analyzing the agents based on the level of intrinsic motivation.

However, based on our learning technology augmented with the intrinsic motivation, analyzing the evolution of human capital level based on the average society human capital level will not give us a very significant result since the effect is very much similar to that discussed in the aforementioned section. In contrast, if we analyze the human capital evolution based on the level of intrinsic motivation of the agents, then the result would be significant.

Therefore, based on the intrinsic motivation level of the agents, an economy will evolve according to one of the following three cases:

1. Regardless of the parental human capital level, all agents have high intrinsic motivation.
2. Regardless of the parental human capital level, all agents have low intrinsic motivation.
3. Regardless of the parental human capital level, agents are divided into two groups. One group of agents has low intrinsic motivation level, and another group has high intrinsic motivation level.

The analysis of case 1 and case 2 are very easy because it is very similar to the analysis of the economy for the identical agents and the analysis yields a similar result to the Lemma 2.

Unlike case 1 and 2, case 3 is a bit more complex and interesting. In this case, four types of parent and four types of children exist in an economy. The government provides public education, but does not cover all the population. There is a group of people who can access the public education, and some cannot due to some difficulties, such as geographical problems. These people are also free to choose to either invest or not invest in the private education of the children, and yet the most important thing is that the level of intrinsic motivation of each family is different. This led us to the following Lemma.

Lemma 3 *If the agents are heterogeneous, then regardless of the parental and societal human capital level, the individual that possess high intrinsic motivation will always be free from the poverty trap even though the parent is locked in poverty.*

This lemma implies that, even though a parent is poor, if the child has high intrinsic motivation, then the child will be able to get out of the poverty trap. This Lemma reflects the reality that some children exist who come from a poor family but later on are able to achieve a human capital level much higher than their parent's human capital, or even higher than the human capital level of the society.

CONCLUSION

The main contribution of this paper is the development of a theoretical framework that relates the human capital level of the children and poverty in elderly parents by introducing the concept of intrinsic motivation of pursuing the knowledge. As a result, two types of economy are discussed in this paper. One economy has free public education, and the other does not have free public education. In each economy, parents either choose to invest in extra private education or choose not to invest. This setup, therefore, induces two different levels of human capital threshold. We, therefore, summarize all the important results in this paper by establishing several propositions and lemmas.

With regards to the relationship between the choice of fertility and parental human capital level, the parents whose human capital level is above the threshold, choose to invest in private education in both economies, and they prefer fewer children. In other words, they choose quality over quantity; this result is consistent with Morand (1999). Moreover, as one of the important results, as stated in Lemma 1, the children of the parent with human capital above the threshold level in the economy with public education possess the highest human capital level. However, that parent has a tendency to choose the least number of children.

The main objective of this paper is to understand how a lack of education can trap the elderly into poverty and how they could avoid being locked in the trap. In an economy populated by identical agents, and where all agents possess low intrinsic motivation, the elderly in the economy without public education and in the previous period who did not invest in the private education of their children will be locked in the poverty trap. This scenario will happen because of three reasons: (1) the children inherited only a portion of the parental and social human capital level; (2) since the parental and the society human capital level is below the threshold level, hence recursively all future generations inherit this low-level human capital level, thus all generation are locked in poverty; and (3) since the agents are identical, then the intrinsic motivation is also similar, and thus implies no paradigm shift in the level of intrinsic motivation. Therefore, if the children are also locked in poverty, then how could they help their parents? Unless all the agents are identical, and they possess high intrinsic motivation, then there is a possibility that the whole generation will eventually evolve and surpass the threshold level, and, therefore, save all the future generations from the poverty trap. On the other hand, under the assumption that the agents are heterogeneous, the results are more interesting. The result implies that, even though some children come from a poor family, if they have high intrinsic motivation, eventually, they will be able to achieve a human capital level much higher than their parent's human capital, or even higher than the human capital level of the society.

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APPENDIX A

Let $U = \ln C_{1,t} + \dfrac{1}{1+\rho} \ln C_{2,t+1}$ and the budget constraint are as follows:

$c_{1,t} + \alpha_1 wh_t + (e_t = a_t)n_t wh_t = wh_t$ and $c_{2,t+1} = \alpha_1 wh_{t+1} n_t$. By setting up a Lagrangian, we obtain the following:

$$L = \ln c_{1,t} + \frac{1}{1+\rho} \ln c_{2,t+1} + \lambda[wh_t - c_{1,t} + \alpha_1 wh_t + (e_t = a_t)n_t wh_t].$$

By solving for the number of the children and the first period consumption, we obtain the following results:

1. $\dfrac{\partial L}{\partial C_{1,t}} \dfrac{1}{c_{1,t}} - \lambda = 0$

2. $\dfrac{\partial L}{\partial n_t} \dfrac{1}{1+\rho} \dfrac{1}{C_{2,t+1}} \dfrac{\partial C_{2,t+1}}{\partial n_t} - \lambda(e_t + a_t)wh_t = 0$

By solving (1) and (2), we will obtain the following result:

$$\frac{\partial U}{\partial C_{2,t+1}} \alpha_2 wh_{t+1} = \frac{\partial U}{\partial C_{1,t}} (e_t + a_t)wh_t$$

APPENDIX B

Let $U = \ln C_{1,t} + \dfrac{1}{1+\rho} \ln C_{2,t+1}$ and the budget constraint are as follows:

$c_{1,t} + \alpha_1 wh_t + (e_t = a_t)n_t wh_t = wh_t$ and $c_{2,t+1} = \alpha_1 wh_{t+1} n_t$. By setting up a Lagrangian, we obtain the following:

$$L = \ln c_{1,t} + \frac{1}{1+\rho} \ln c_{2,t+1} + \lambda[wh_t - c_{1,t} + \alpha_1 wh_t + (e_t = a_t)n_t wh_t].$$

By solving for the investment in education and the first period consumption, we obtain the following results:

1. $\dfrac{\partial L}{\partial C_{1,t}} \dfrac{1}{c_{1,t}} - \lambda = 0$

2. $\dfrac{\partial L}{\partial n_t} \dfrac{1}{1+\rho} \dfrac{1}{C_{2,t+1}} \dfrac{\partial C_{2,t+1}}{\partial h_{t+1}} - \lambda n_t wh_t = 0$

By solving (1) and (2), we will obtain the following result:

$$\frac{\partial U}{\partial C_{2,t+1}} \frac{\partial h_{t+1}}{\partial e_t} \alpha_2 wn_t = \frac{\partial U}{\partial C_{1,t}} n_t wh_t.$$

APPENDIX C

To determine the threshold level, we solve the equation (11) and (1) as follows:
Given the learning technology by $h_{t+1} = m_t(\bar{h}_t + h_t^{\%} + e_t h_t)^{\gamma}$. Then differentiate h_{t+1} respect to the investment of the education to obtain $\dfrac{\partial h_{t+1}}{\partial e_t} = \dfrac{\gamma h_t h_{t+1}}{\bar{h}_t + h_t^{\%} + e_t h_t}$.

Then, substitute the result into the equation (11) to get $\bar{h}_t + h_t^{\%} + e_t h_t = (e_t + a_t)\gamma h_t$. Hence, by an algebraic manipulation, we solve for h_t as follows:

$$h_t = \frac{\bar{h}_t + h_t^{\%}}{\gamma a_t + (\gamma - 1)\, e_t}.$$

Therefore, by setting $e_t = 0$, we are able to set up a threshold level for both types of economies as follows:

1. Threshold level of human capital in economy without public education is $\hat{h}_t = \dfrac{\bar{h}_t}{\gamma a_t}$.
2. Threshold level of human capital in economy with public education is $h_t = \dfrac{\bar{h}_t + h_t^{\%}}{\gamma a_t}$.

APPENDIX D

To obtain the optimal level of investment in education, we solve the equation (11) and (1) as follows:
Given the learning technology by $h_{t+1} = m_t(\bar{h}_t + h_t^{\%} + e_t h_t)^{\gamma}$. Then differentiate h_{t+1} respect to the investment of the education to obtain $\dfrac{\partial h_{t+1}}{\partial e_t} = \dfrac{\gamma h_t h_{t+1}}{\bar{h}_t + h_t^{\%} + e_t h_t}$.

Then, substitute the result into the equation (11) to get $\bar{h}_t + h_t^{\%} + e_t h_t = (e_t + a_t)\gamma h_t$. Hence, by an algebraic manipulation, we solve for e_t as follows:

$$e_t = \frac{1}{1-\gamma}\left(\gamma a_t - \frac{\bar{h}_t + h_t^{\%}}{h_t}\right).$$

APPENDIX E

To obtain the optimal number of children, we solve the equation (3), (8) and the budget constraint in equation (4).

From the equation (3) and (8), we obtain $\dfrac{1}{1+\rho}\dfrac{1}{n_t} = \dfrac{(e_t + a_t)wh_t}{C_{1,t}}$ and it is equivalent to $(1+\rho)(e_t + a_t)wh_t n_t = C_{1,t}$.

From equation (4), we get the following equation:

$$(1+\rho)(e_t + a_t)wh_t n_t = wh_t - \alpha_1 wh_t - (e_t + a_t)n_t wh_t$$

Then, by solving the above equation for n_t, we obtain the general form of the optimal fertility as follows:

$$n_t^* = \frac{1-\alpha_1}{(2+\rho)(e_t + a_t)}.$$

Since the number of the children is depends on the level of investment in education, therefore we could divide it into three cases:

Case 1: The parent chooses not to invest in education.

$$n_t^{NP-E} = n_t^{P-E} = \frac{1-\alpha_1}{(2+\rho)a_t}$$

Case 2: The parent chooses to invest in education in an economy without public education.

$$n_t^{NP+E} = \frac{(1-\alpha_1)(1-\gamma)h_t}{(2+\rho)(a_t h_t - \bar{h}_t)}$$

Case 3: The parent chooses to invest in education in an economy with public education.

$$n_t^{P+E} = \frac{(1-\alpha_1)(1-\gamma)h_t}{(2+\rho)[a_t h_t - (\bar{h}_t + h_t^{\%})]}$$