# Two Period Model of Bank Lending Channel: Basel II Regulatory Constraints 

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#### Abstract

This paper predicts the dynamic model of the bank lending channel under Basel II regulatory constraint with monopolistic competition. Two period model has been chosen in order to see the effect of new Basel capital constraints on the risks of banks assets in the both periods and the amount of equity in the second period. The prediction of period one and two are shown to have the same effect and the only difference is the constraint. The regulatory constraint in periods one and two are predicted depending on the regulatory parameters and constraints for both periods. Thus, the effect of optimal rates to the policy rate is felt greater or less in the first period than the second period which means tightening capital requirement increases or decreases the risks of assets and banks taking a higher or lower risk in the first period than second period.


Key words: Basel II, Bellman equation, two period model, bank lending channel

## INTRODUCTION

The main objective of this paper is to predict the dynamic model of the bank lending channel under Basel II regulatory constraint with monopolistic competition, which originally was analysed by Kishan and Opiela (2000) and Baglioni (2007) using a static model. Two period model has been chosen in order to see the effect of new capital constraint on the risks of banks assets in the both periods and the amount of equity in the second period.

This paper extends in two important directions: i) it explicitly considers the time period of banks by choosing interest rate charged on loans, interest rate paid to deposits and how much to borrow from the money market. ii) it predicts the different impact of optimal rates on the effect of new regulatory constraint, monetary policy and credit risks for each period.

Malaysian banks implemented the new Basel in the early part of 2008. Nevertheless, the first phase of the new Basel is the standard Internal Rating Based (IRB) approach. Under this standard IRB approach, exposure to banking institutions shall be accorded risk weights based on their external credit ratings which can be in the form of either long-term or short-term ratings. ${ }^{1}$ Banks have always borrowed from the interbank market since the constraint of capital has restricted the amount of loans and securities that can be offered to borrowers. The main role of the Basel regulatory constraint is to prevent banks from any difficulties. On the other side, if banks always borrow from interbank market it can only expose banks to a higher risk if banks unable to repay the borrowing.

Our main contribution is in the literature of the dynamic model of bank lending channel under the new Basel regulatory constraints. This paper predicts theoretically the impact of the banks' rates on the monetary policy and regulatory constraint in two period models with monopolistic competition.

The plan of the paper is as follows: Section 2 contains the literature review; and section 3 shows the theoretical model and the predictions. Finally, section 4 concludes the overall findings of the paper. The appendix contains all proofs and tables.

## LITERATURE REVIEW

The dynamic model in the lending channel has not been discussed in details. A transmission mechanism of monetary policy is important in determining the behaviour of the bank lending channel. Banks set their own interest rates as they behave like monopolistic competition. Two period model has been chosen in order to see the effect of new capital constraint on the risks of banks assets in the both periods and the amount of equity in the second period.

It is vital to highlight the role of the new Basel regulatory constraint in this model because it may leave a different impact on the banks' balance sheets. Why do banks need to be regulated? Banks are exposed to credit and liquidity risks. Banks are also faced by the possibility of the borrowers defaulting on loan repayments, besides not having enough cash to meet deposit withdrawals. The higher the risk of a bank's assets, for example the ratio of loans to total assets, the more vulnerable the banks are likely to be.

The papers most closely related to ours are those Kishan and Opiela (2000) and Baglioni (2007), and Honda (2004) using of static model of bank lending channel under the old Basel Accord. Whereas Jaques (2008), Ahmad (2006) and Kashyap and Stein (2004) analyse with the adverse macroeconomics effect of Basel, especially with its procyclicality and its neglect of endogeneity of financial risk. Similar to Jaques (2008) develops a theoretical model to examine how commercial loans of varying credit quality are likely to respond to an adverse capital shock under the revised Accord. The results of his study suggest that with the increased differentiation of credit risk introduced by the new Basel II (revised standards), low credit risk loans may actually increase. Ahmad (2006) who concludes that the new capital requirements can have both good and bad effects on the targeted financial institutions and markets. The recent study made by Boivin, Kiley and Mishkin (2010) review the empirical evidence on the changes in the effect of monetary policy actions on real activity and inflation and present new evidence, using both a relatively unrestricted factor-augmented vector autoregression (FAVAR) and a DSGE model. They have found a notable changes in policy behaviour (with policy more focused on price stability) and in the reduced form correlations of policy interest rates with activity in the United States. Both approaches yield similar results. Besides, under the competition on the assets side Repullo and Suarez (2004) argue that banks eligible for the IRB approach have a competitive advantage in the provision of low-risk loans (IRB approach has a lower capital requirement), while the less sophisticated banks have a competitive advantage in the provision of high risk loans (standardised approach has a lower capital requirement).

Our research makes a different point by starting from a setup that differs in several important respects from those used by Jaques (2008), Ahmad (2006), Kashyap and Stein (2004), Repullo and Suarez (2004), and Kishan and Opiela (2000). First, they analysed the bank lending channel by assuming that banks operate in an imperfect-competitive market. According to their assumption, the correct bank strategic variable is quantity instead of price. In other words, each bank decides its optimal volume of loans, taking as given the volumes supplied by the other banks. The equilibrium price is the one equating the aggregate supply and demand for loans. However, our research is different from their studies since we assume that each of the banks behave as if in monopolistic competition (an assumption inspired by Baglioni (2007) and Boivin, Kiley and Mishkin (2010)). This market structure is suitable for describing the market for bank loans, despite the presence of many players in the market, in which each of them retains the power of setting its own price at the desired level. The reason for choosing monopolistic competition over imperfect competition markets in this analysis is because loans are not perfect substitutes to borrowers (it can be differentiated). Each bank has some market power in the market for loans (faces downward-sloped demand for loans with finite elasticity) and time deposits. The difference with the analysis made by Boivin, Kiley and Mishkin (2010) is that we use disaggregated data of banks and analyse the behaviour of banks by changes in the policy rate. Whereas, they are more concentrate on the changes of monetary policy actions on real activity and inflation without looking into behaviour of banks individually.

Second, Jaques (2008) models bank competition on the asset side and ignore the competition on the liabilities side. However, in our analysis we will consider the competition on the both side assets and liabilities of banks' balance sheets. In other words, whether small or large banks (bank size) become more or less competitive in engaging a higher or lower risk of loans and securities, and whether high or low risk loans/securities are more competitive under the new Basel Accord.

Third, a two period model has been chosen in order to see the effect of the bank lending channel by setting bank prices as the optimal decisions in the different time periods. The chosen of two period model over infinite model in order to provide a clear proof whether banks are holding more risky assets or less risky assets in the first and second periods when new regulatory constraints is imposed at the start of period. This is essential especially to determine the amount of equity in the second period. If banks held more risky assets (less risky assets) in the first period therefore the amount of equity in second period will increase (decrease) if the investment is success. For example, if banks are assumed to impose new capital requirement at the start of the period, tightening the requirement decreases the risk of assets (Blum (1999)) or otherwise increases the risk of assets (Ahmad (2006)) depending whether the new requirements motivate banks from taking a lower risk or a higher risk in the first and second periods. Thus, the chosen two periods are sufficient enough to show banks' investment decisions in the first and all the cost are paid and returns are received in the second period.

This operation will continue over time if we assume the model to be in the n-period or infinite horizon. However, we do not pursue this as a two period model, which can sufficiently prove the main objective. Miyake and Nakamura (2007) conclude that the timing of the introduction of tight regulations is important. If the regulations become tighter when a negative productivity shock occurs, the economy falls into a long and severe slump. This is consistent with the Japanese economy after experiencing the bubble economy. In addition, Naceur (2009) investigated the effects of capital regulations on the cost of intermediation and profitability. He found a higher capital adequacy increase in the interest rate of shareholders in managing banks' portfolios. The reduction in economic activity had opposite effects on banks' profitability.

## THEORETICAL MODEL

This paper studies a dynamic balance sheet model of bank lending and portfolio decision. Banks invest in loans and securities and obtain funds from deposits, own capital, and the money market. In their decisions they are bound by the regulatory capital requirements and risk weights imposed by the Basel accords. Banks act in a partial equilibrium monopolistic competition environment, where they set their interest rate on time deposits and on loans, given their conjecture of the equilibrium interest rate in both markets. In this setting we examine the transmission of monetary policy into bank lending and portfolio composition.

The model developed here is a two period extension of previous work by Kishan and Opiela (2000) and Baglioni (2007). Banks start their first period with an exogenous own capital endowment. The amount of equity in the second period is not fixed, but can actually be influenced through the investment decision in the first period. By decreasing (increasing) risk today the banks have a lower (higher) amount of equity available tomorrow in case of success. Therefore, the introduction of a new capital requirement for today induces a lower risk (higher risk) tomorrow depending whether banks are taking a risk to maximize profit tomorrow.

At the first period, each bank then observes the realization of the demand for deposits and the demand for loans directed at their own bank. Based on this information they decide how much to borrow from the money market and how much to reward deposits and charge for loans, effectively choosing their preferred position in these idiosyncratic demand curves.

At the second period loans are repaid (or defaulted upon) and risky securities yield their return, and banks pay back their depositors and also what they borrowed from the money market. If any money is left over, that constitutes the own capital of the bank for next period's exercise. ${ }^{2}$

## The Balance Sheet Constraint

The model is built on the definition of the balance sheet of each bank, which equates the following assets and liabilities:
$R_{j t}+S_{j t}+L_{j t}=D_{j t}+T_{j t}+B_{j t}+K_{j t}$
On the asset side R denotes required reserves, S denotes securities, and L denotes loans. On the liability side, D denotes demand deposits, T denotes time deposits, K denotes the bank's own capital, and B denotes the interbank borrowing. All items have subscripts ( $j t$ ) as we will be working with a panel structure, where the subscript $(j)$ identifies the bank and the subscript $(t)$ identifies the period.

This equality is an ex-ante definition: At the first period banks were given the choice of choosing time deposit and loan rates, and the choice of how much to borrow from the money market, the amount of loans issued, the amount of securities bought and reserves set aside must obey this relationship. At the end of the period (second period), some loans may be defaulted, and the amount of equity is not fixed, but can actually be influenced through the investment decision in the first period. Therefore, the equality of this equation will no longer hold. But it must hold ex-ante.

## Reserves

Banks do not hold excess reserves, only required reserves. As in Kishan and Opiela (2000) required reserves are assumed to be a constant fraction $\alpha$ of demand deposits at each period $t=1,2$ period:
$R_{j t}=\alpha D_{j t}$

The reserve requirement fraction $\alpha$ is set by the central bank at 4 percent of demand deposits. Required reserves receive no return.

## Securities

Banks also hold marketable financial assets such as government and private bonds and bills. Banks are assumed to hold securities if it is costly to liquidate loans in the short run as opposed to Kashyap and Stein (1995) or banks may hold a buffer stock of securities to insulate themselves, at least partially Stein (1998). The rate of return on securities, $r_{S t}$ is given by:
$r_{S t}=e_{0}+e_{1} i_{t}+v_{t}$
The current inter-bank or money market rate $i_{t}$ has a direct influence on the current rate of return on securities, where $\mathrm{e}_{0}$ and $\mathrm{e}_{1}$ are parameters and $v_{t}$ is a random error term which summarizes all other factors influencing the rate of return. These can be changes in the total factor productivity of firms. ${ }^{3}$ The money market rate $i_{t}$ is observed at the start of the period before decisions are taken but the error term is only realized after decisions are taken, at the end of the period.

## Loans

The loan market is in monopolistic competition, where each bank sets its own loan interest rate, $r_{L j t}$, taking as given the 'market' interest rate $r_{L j t}$. The demand for loans faced by bank j in period t is given by
$L_{j t}=b_{0}-b_{1}\left(r_{L j t}-\bar{r}_{L t}\right)-b_{2} \bar{r}_{L t}+v_{j t}$
where $v_{j t}$ is an error term. Individuals and firms demand loans based on the loan rate and have some cost of changing banks which generates a local monopoly power for each bank. The error term is not correlated with other variables and is observed at the start of each period, so that the exact location of the demand curve is known and can be explored by the bank. The monopolistic competition assumption follows Baglioni (2007).

Loans are subject to ex-post default. The default rate is a random variable with expected value (1-q). The bank therefore expects to recover qL (performing loans) of the loans made.

## Deposits

The banks' sources of funds are deposits, equity, and the money market borrowing. We separate demand and time deposits, and assume the demand deposits are out of any bank's control. All deposits are returned to customers at the end of the period.

The demand for demand deposits at each period faced by bank j is inversely related to the interbank rate and varies over time by an error term $\varepsilon_{t}$, the realization of which is observed at the start of each period. ${ }^{4}$
$D_{j t}=c_{0}-c_{1} i_{t}+\varepsilon_{t}$
The interest rate paid on demand deposits, $r_{D t}$ is determined as given by interbank rate or mean market interest rate and is exogenous to commercial banks. Every bank has the same interest rate on demand deposits.

The demand for time deposits directed at bank j is a function of the spread between banks j 's rate, $r_{T j t}$, and an average rate on the market. This demand varies over time by an error term $\omega_{j t}$, the realization of which is known at the start of each period before decisions are taken. If banks want to attract more time deposits, they have to raise interest rates to increase their market share. The demand for time deposits faced by the bank is given by

$$
\begin{equation*}
T_{j t}=d_{0}+d_{1}\left(r_{T j t}-\bar{r}_{T t}\right)+d_{2} \bar{r}_{T t}+\omega_{j t} \tag{6}
\end{equation*}
$$

## Borrowing

Banks' borrowing is understood as borrowing from the interbank market or money market. The cost of borrowing is assumed to be depends on the policy rate since the policy rate is operated in the interbank market and risk free market (repos).

## Capital

The initial level of equity $K_{j t}$ is exogenously determined, either derived from retained earnings or capital injections and profits of previous period. This equity capital must obey a capital requirement regulation imposed by the Basel accords which limits the bank's exposure to non performing loans and securities. The reserves requirement is the amount that should be fulfilled by the banks before loans and securities. We will see below that this constraint always binds. The capital constraint is given by
$K_{j t} \geq \mu\left(R_{j t}+\delta_{S} S_{j t}+\delta_{L} L_{j t}\right)$
This equation states that banks are subject to risk-based capital requirements, where $\mu$ measures the minimum capital requirements for reserves, securities and loans. According to the Basel Capital Accord of 1988, or Basel I, all loans and securities in the private sector are given the average capital requirement $\mu=0.08$. However, under the Basel II Accord there is a different calculation of riskweighted assets across loans and securities, which depends on the borrowers' ratings or quality of portfolio held by the bank. Therefore, we assume that $\delta_{S}$ and $\delta_{L}$ are the risk weights on securities and loans. ${ }^{5}$ These risk factors are essential to banks exposed to the level of risk for loans and securities. The crucial property here is that:
$\delta_{S}, \delta_{L} \geq 1$

## Profit Maximization in Period 2

At the start of each period banks choose the interest rate they offer on time deposits, $r_{T j 2}$, and charge on loans, $r_{L j 2}$, as well as how much to borrow from the money market, $B_{j 2}$. We solve the model backwards and so we look first at the decision at the start of period 2 . Given positive capital and positive demand for deposits (no bank run), the bank chooses ( $r_{T j 2}, r_{L j 2}, B_{j 2}$ ) to maximize profits subject to the constraints and relationships given above. Expected profits are given by
$\mathrm{E} \pi_{j 2}=\mathrm{E}_{q} r_{L j 2} q L_{j 2}+E_{r S} r_{S 2} S_{j 2}-r_{D 2} D_{j 2}-r_{T j 2} T_{j 2}-i_{2} B_{j 2}-\mathrm{E}_{1-q}(1-q) L_{j 2}$
The balance sheet relationship is an identity and we replace it in the objective function to eliminate S . The Basel constraint has the associated Lagrange multiplier lambda, $\lambda_{j 2}$. Define $\mathcal{L}=$ $\left(\pi_{j 2}+\lambda_{j 2} C\right)$, The first order conditions for this problem, derived in the appendix are:
$\frac{\partial \mathcal{L}_{j 2}}{\partial r_{L j 2}}=q L_{j 2}-q b_{1} r_{L j 2}+r_{S 2}^{e} b_{1}+(1-q) b_{1}-\lambda_{j 2} b_{1}\left(\delta_{L}-\delta_{S}\right)=0$
$\frac{\partial \AA_{j 2}}{\partial r_{T j 2}}=d_{1} r_{S 2}^{e}-T_{j 2}-d_{1} r_{T j 2}+\lambda_{j 2} d_{1} \delta_{S}=0$
$\frac{\partial \mathcal{L}_{j 2}}{\partial B_{j 2}}=r_{S 2}^{e}-i_{2}+\lambda_{j 2} \delta_{S}=0$
and
$\frac{\partial \delta_{j 2}}{\partial \lambda_{j 2}}=\left(\alpha+(1-\alpha) \delta_{S}\right) D_{j 2}+\left(\delta_{L}-\delta_{S}\right) L_{j 2}+\delta_{S} T_{j 2}+\delta_{S} B_{j 2}-\left(\frac{1}{\mu}-\delta_{S}\right) K_{j 2}=0$
The first condition determines the value of the Lagrange multiplier. It is given by
$\lambda_{j 2}=\frac{i_{2}-r_{S 2}^{e}}{\delta_{S}}<0$
and is negative because we assume that the expected return on securities is higher than the policy rate $\mathrm{i}_{2}$. This shows that only borrowing depends on the capital, since banks always borrow from the money market.

The first condition for the constraint is given by:
$B_{j 2}=\left(\frac{1}{\mu}-\delta_{S}\right) \frac{K_{j 2}}{\delta_{S}}-\left(\alpha+(1-\alpha) \delta_{S}\right) \frac{D_{j 2}}{\delta_{S}}-\left(\delta_{L}-\delta_{S}\right) \frac{L_{j 2}}{\delta_{S}}-\delta_{S} \frac{T_{j 2}}{\delta_{S}}$
The Lagrange multiplier of constraint implies that interbank borrowing at period two depends linearly on the capital at the same period. This shows that banks will decide to borrow from the interbank market after observing the level of capital. The constraint of capital will determine the amount of borrowing in the money market. This has a strong implication that the regulatory constraint always binds. If the bank has enough own capital to invest, it will still borrow from the interbank market until it has bought enough securities and issued enough loans such that it becomes constrained by Basel. This is in fact not an unreasonable description of the risk taking behaviour observed recently throughout the international banking system: cheap money and toxic assets. ${ }^{6}$

The second implication of the solution is that the interbank rate affects the model only via the Lagrange multiplier. This is because the policy rate will always be less than the expected rate of return on securities or debt and the banks always borrow since they assume the rate of returns are profitable.

After substitution of terms we obtain the following formulas for the optimal interest rates:

$$
\begin{align*}
& r_{L j 2}^{*}=\frac{\left[b_{0}+\left(b_{1}-b_{2}\right) \bar{r}_{L 2}+v_{j 2}\right] q-\lambda_{j 2} b_{1}\left(\delta_{L}-\delta_{S}\right)+b_{1} r_{S 2}^{e}+(1-q) b_{1}}{2 q b_{1}}  \tag{16}\\
& r_{T j 2}^{*}=\frac{\left[r_{S 2}^{e}+\lambda_{j 2} \delta_{S}\right] d_{1}-d_{0}+\left(d_{1}-d_{2}\right) \bar{r}_{T 2}-\omega_{j 2}}{2 d_{1}} \tag{17}
\end{align*}
$$

and note that
$\frac{\partial r^{*}{ }_{L j 2}}{\partial i_{2}}=\frac{e_{1}}{2 q}-\frac{\left(\delta_{L}-\delta_{S}\right)}{2 q} \frac{\partial \lambda_{j 2}}{\partial i_{2}}$
$\frac{\partial r^{*} T j 2}{\partial i_{2}}=\frac{e_{1}}{2}+\frac{\delta_{S}}{2} \frac{\partial \lambda_{j 2}}{\partial i_{2}}$
The Lagrange multiplier shows a negative derivative $\left(\frac{\partial \lambda_{j 2}}{\partial i_{2}}=\frac{\left(1-e_{1}\right)}{\delta_{S}}\right)$ since the policy rate is always less than the expected return on securities. The Lagrange multiplier in period 2 shows the effect of regulatory constraint is always negative. Therefore the reaction of the optimal rate on loans from the change in policy rate is positive if $\delta_{L}>\delta_{S}$, or otherwise if $\delta_{S}>\delta_{L}$. This shows that when the credit risk of loans is more than the credit risk of securities the rate on loans positively reacts on the change of policy rate, as investment in loans are more risky than securities. Thus, an increasing rate of loans will reduce the amount of loans, as investment in securities will give more return. Otherwise, if credit risk on securities is more than loans, the rate on loans will increase as policy rate decreases. This is due to the fact that increasing credit risk of securities gives more exposures of risk on investment in securities, therefore, it is more rational for banks to decrease the rate on loans as policy rate increases, as the level of risk on loans is less than securities and they will prefer to reduce the cost of loans in order to increase the amount of loans. Besides, the reaction of optimal rate on time deposits is always negative if the Lagrange multiplier is always negative.

In addition, the binding of capital rule has always decreased the amount of risky loans and securities to be invested. Thus, this will decrease the profit of banks, due to the limited amount of assets that can be invested. Therefore, banks are less exposed to any risky and default assets.

## The Value Function

The method of dynamic programming as suggested by Bellman (1957) can be used to solve this problem. In order to solve the first period problem we need to write the value function for period 2, which is the maximized profit as a function of all state variables of the problem. Note that at the optimum the regulatory constraint binds and is not explicitly visible in the value function.

$$
\begin{equation*}
V_{j 2}\left(K_{j 2}, Z_{j 2}\right)=r_{L j 2} q L_{j 2}+r_{S 2}^{e} S_{j 2}-r_{D 2} D_{j 2}-r_{T j 2} T_{j 2}-i_{2} B_{j 2} \tag{20}
\end{equation*}
$$

where $Z_{j 2}=\left\{\omega_{j 2}, v_{j 2}, i_{2}, \varepsilon_{j 2}\right\}$ contains the realizations of shocks to time deposits, loan demand, interbank rate and demand deposits. We must replace securities with the balance sheet identity and obtain

$$
\begin{align*}
V_{j 2}\left(K_{j 2}, Z_{j 2}\right)= & \left(q r_{L j 2}-r_{S 2}^{e}\right) L_{j 2}+r_{S 2}^{e} K_{j 2}+\left(r_{S 2}^{e}-i_{2}\right) B_{j 2}+r_{S 2}^{e}\left[(1-\alpha) D_{j 2}+T_{j 2}\right]-r_{D 2} D_{j 2}-r_{T j 2} T_{j 2} \\
& \quad(1-q) L_{j 2} \tag{21}
\end{align*}
$$

where of course $B_{j 2}$ is a linear function of $K_{j 2}$.
More important, we can decompose the value function of period 2 in two components, separating the components that depend on $K_{j 2}$ from those that do not. Since only $B_{j 2}$ depends on $K_{j 2}$ we can write:
$V_{j 2}\left(K_{j 2}, Z_{j 2}\right)=K_{j 2}\left[r_{S 2}^{e}+\left(r_{S 2}^{e}-i_{2}\right)\left(\frac{1}{\mu}-\delta_{S}\right) \frac{1}{\delta_{S}}\right]\left(r_{S 2}^{e}-i_{2}\right)\left[\left(\alpha+(1-\alpha) \delta_{S}\right) \frac{D_{j 2}}{\delta_{S}}+\left(\delta_{L}-\delta_{S}\right) \frac{L_{j 2}}{\delta_{S}}+\right.$
$\left.\delta_{S} \frac{T_{j 2}}{\delta_{S}}\right]+\left(q r_{L j 2}-r_{S 2}^{e}\right) L_{j 2}+r_{S 2}^{e}\left[(1-\alpha) D_{j 2}+T_{j 2}\right]-r_{D 2} D_{j 2}-r_{T j 2} T_{j 2}-(1-q) L_{j 2}$
$V_{j 2}\left(K_{j 2}, Z_{j 2}\right)=\Gamma_{1} K_{j 2}+\Gamma_{2}$
where,
$\Gamma_{1}=\left[r_{S 2}^{e}+\left(r_{S 2}^{e}-i_{2}\right)\left(\frac{1}{\mu}-\delta_{S}\right) \frac{1}{\delta_{S}}\right]$
This is useful for the next section.

## Profit Maximization in Period 1

Profit function of period one is given by:
$\pi_{j 1}=\left(q r_{L j 1}-r_{S 1}-(1-q)\right) L_{j 1}+r_{S 1} K_{j 1}+\left(r_{S 1}-i_{1}\right) B_{j 1}+r_{S 1}\left[(1-\alpha) D_{j 1}+T_{j 1}\right]-r_{D 1} D_{j 1}-r_{T j 1} T_{j 1}$
and subject to constraint:
$K_{j 1} \geq \mu\left(R_{j 1}+\delta_{S} S_{j 1}+\delta_{L} L_{j 1}\right)$
We defined $V_{j 1}\left(K_{j 1}, Z_{j 1}\right)$ as the maximized value of the objective function at time 1 given an initial capital stock of assets $K_{j 1}$, then $V_{j 2}\left(K_{j 2}, Z_{j 2}\right)$ is the maximized value of the objective function at time 2 given an initial capital stock of assets $K_{j 2}$. In other words the objective function for the 2-period problem is defined at the start of period one and greatly simplified in a recursive form using the Bellman equation as shown below:
$V_{j 1}\left(K_{j 1}, Z_{j 1}\right)=\max _{r_{L j 1}, r_{T j 1}, B j_{1}} \pi_{j 1}+\beta V_{j 2}\left(K_{j 2}, Z_{j 2}\right)$
where of course the realized $\pi_{j 1}$ is equal to $K_{j 2}$.
This allows us to rewrite the Value Function in period 1 as
$V_{j 1}\left(K_{j 1}, Z_{j 1}\right)=\max _{r_{L j 1}, r_{T j 1}, B_{j 1}} \pi_{j 1}+\beta\left(\Gamma_{1} \pi_{j 1}+\Gamma_{2}\right)$
The next step is to maximize this objective function with respect to $r_{T j 1}, r_{L j 1}, B_{j 1}$. Then, take a derivative of the entire right hand side (RHS) of Bellman operator, where the solution maximization of the value function of period 1 is given as below:

$$
\begin{align*}
& \frac{\partial \pi_{j 1}}{\partial r_{L j 1}}=\left[\begin{array}{c}
b_{0} q+\left(b_{1}-b_{2}\right) \bar{r}_{L 1} q-2 b_{1} r_{L j 1} q+v_{j 1} q+b_{1} r_{S 1} \\
+(1-q) b_{1}-\left(\mu-\frac{1}{\delta_{S}}\right) b_{1} r_{S 1}\left(\delta_{L}-\delta_{S}\right)
\end{array}\right]\left[1+\beta \Gamma_{1}\right]=0  \tag{29}\\
& \frac{\partial \pi_{j 1}}{\partial r_{T j 1}}=\left[\begin{array}{c}
d_{1} r_{S 1}+\left(d_{1}-d_{2}\right) \bar{r}_{T 1}-d_{0}-2 d_{1} r_{T j 1}-\omega_{j 1} \\
+r_{S 1}\left(\mu-\frac{1}{\delta_{S}}\right) d_{1} \delta_{S}
\end{array}\right]\left[1+\beta \Gamma_{1}\right]=0  \tag{30}\\
& \frac{\partial \pi_{j 1}}{\partial B_{j 1}}=\left[\left(r_{S 1}-i_{1}\right)+r_{S 1}\left(\mu-\frac{1}{\delta_{S}}\right) \delta_{S}\right]\left[1+\beta \Gamma_{1}\right]=0 \tag{31}
\end{align*}
$$

The derivative of $r_{T j 1}, r_{L j 1}, B_{j 1}$ in the first period is realized with the addition of the expected value of the second period which depends on the first period. In this case $\beta$ is a discount factor that can be formulated as $=\frac{1}{i+\rho}$, in which $\rho$ is a premium rate and $i$ is an interbank rate.

Optimal interest rates in period 1
$r_{L j 1}^{*}=\frac{\left.\left[b_{0} q+\left(b_{1}-b_{2}\right) \bar{r}_{L 1} q+v_{j 1} q+b_{1} r_{S_{1}+(1-q) b_{1}-\left(\mu-\frac{1}{\delta_{S}}\right)}\right) b_{1} r_{S_{1}}\left(\delta_{L}-\delta_{S}\right)\right]}{2 b_{1} q}$
$r_{T j 1}^{*}=\frac{\left[\left(d_{1}-d_{2}\right) \bar{r}_{T 1}-d_{0}-\omega_{j_{1}}+d_{1} r_{S_{1}}+r_{\left.S_{1}\left(\mu-\frac{1}{\delta_{S}}\right) d_{1} \delta_{S}\right]}^{2 d_{1}}\right.}{2}$
and note that

$$
\begin{align*}
& \frac{\partial r_{L j 1}^{*}}{\partial i_{1}}=\frac{e_{1}}{2 q}-e_{1}\left(\mu-\frac{1}{\delta_{S}}\right)\left(\frac{\left(\delta_{L}-\delta_{S}\right)}{2 q}\right)=0  \tag{36}\\
& \frac{\partial r_{T j 1}^{*}}{\partial i_{1}}=\frac{e_{1}}{2}+e_{1}\left(\mu-\frac{1}{\delta_{S}}\right)\left(\frac{\delta_{S}}{2}\right)=0 \tag{37}
\end{align*}
$$

The effect of regulatory constraint in period one is shown by $e_{1}\left(\mu-\frac{1}{\delta_{S}}\right)$. Since $\mu=0.08$ is the capital adequacy ratio and is a small percentage if compared with $\frac{1}{\delta_{s}}$, thus the effect of regulatory constraint is always negative. In this first period, the constraint of capital plays an important role in influencing the response of the interest rates on loans and time deposits. If we assume regulatory constraint is always negative, the response of interest rate on loans in period 1 to a policy rate has a positive effect if $\delta_{L}>\delta_{S}$, otherwise if $\delta_{L}<\delta_{S}$. The first period predictions are similar to the second period predictions without taking into account the constraint, however only the constraint has made the predictions difference. In addition, if we assume the effect of regulatory constraint is negative, the response of interest rate on time deposits in period 1 to policy rate is negatively predicted. Only the risk factor of securities influences the optimal rate of time deposits.

The regulatory constraint in periods 1 and 2 are predicted depending on the regulatory parameters and constraints for both periods. The impact of interest rates on loans and time deposits to a policy rate are more or less in the first period, when banks face a shock of capital rule from the start of period 1.

## CONCLUSIONS

The overall result of predictions shows how essential it is to analyse in detail a dynamic model of a bank lending channel under monopolistic competition market. In period one the regulatory constraint is also shown negative value. This implication is true for both time of period that we predicted. In other words, the prediction of period one and two are shown to have the same effect and the only difference is the constraint. The impact of interest rates on loans and time deposits to a policy rate are more or less in the first period, when banks face a shock of capital rule from the start of period 1 . Thus, the effect of optimal rates to the policy rate is felt greater or less in the first period than the second period which means tightening capital requirement increases or decreases the risks of assets and banks taking a higher or lower risk in the first period than second period. This is consistent with the case analysed by Ahmad (2006).

The limitations of this paper are;first, we do not calibrating or estimating data by the predictions found in this paper. However, we will expand this research by calibrating and estimating a banks data from the first period of implementation of Basel II until to the recent years in order to see the effect of new regulatory constraints on the optimal decisions predicted in this paper. The effect of new regulatory constraints will be depends on the regulatory parameters predicted in the first and second period. Second, this is only a partial equilibrium model. Therefore, it is more interesting if we could include the role of other agents such as government, households, and firms in order to get a full general equilibrium.

## APPENDIX A

## Profit Maximization

Profit Maximization of Second Period, Given $\boldsymbol{K}_{\boldsymbol{j} 2}>0$
Rewrite the objective function as

$$
\mathcal{L}_{j 2}=\left[\begin{array}{c}
\mathrm{E}_{q} r_{L j 2} q L_{j 2}+E_{r S} r_{S 2} S_{j 2}-r_{D 2} D_{j 2}  \tag{39}\\
-r_{T j 2} T_{j 2}-i_{2} B_{j 2}-\mathrm{E}_{1-q}(1-q) L_{j 2} \\
+\lambda_{j 2}\left\{\mu_{j 2}\left(R_{j 2}+\delta_{S} S_{j 2}+\delta_{L} L_{j 2}\right)-K_{j 2}\right\}
\end{array}\right]
$$

where $\lambda_{j 2} \leq 0$. Now use the balance sheet equality to eliminate securities:

$$
\begin{equation*}
S_{j 2}=(1-\alpha) D_{j 2}+T_{j 2}+K_{j 2}+B_{j 2}-L_{j 2} \tag{40}
\end{equation*}
$$

Capital constraint becomes;

$$
\begin{array}{r}
\frac{K_{j 2}}{\mu} \geq \alpha D_{j 2}+(1-\alpha) \delta_{S} D_{j 2}+\delta_{S} T_{j 2}+\delta_{S} K_{j 2}+\delta_{S} B_{j 2}-\delta_{S} L_{j 2}+\delta_{L} L_{j 2} \\
\left(\alpha+(1-\alpha) \delta_{S}\right) D_{j 2}+\left(\delta_{L}-\delta_{S}\right) L_{j 2}+\delta_{S} T_{j 2}+\delta_{S} B_{j 2} \leq\left(\frac{1}{\mu}-\delta_{S}\right) K_{j 2} \tag{42}
\end{array}
$$

Therefore:
$\mathcal{L}_{j 2}=\left[\begin{array}{c}\mathrm{E}_{q} r_{L j 2} q L_{j 2}+E_{r S} r_{S 2} S_{j 2}-r_{D 2} D_{j 2}-r_{T j 2} T_{j 2}-i_{2} B_{j 2} \\ -\mathrm{E}_{1-q}(1-q) L_{j 2}+\lambda_{j 2}\left\{\begin{array}{c}\left(\alpha+(1-\alpha) \delta_{S}\right) D_{j 2}+\left(\delta_{L}-\delta_{S}\right) L_{j 2} \\ +\delta_{S} T_{j 2}+\delta_{S} B_{j 2}-\left(\frac{1}{\mu}-\delta_{S}\right) K_{j 2}\end{array}\right\}\end{array}\right]$
And now take first order conditions:
$\frac{\partial \mathcal{L}_{j 2}}{\partial r_{L j 2}}=q L_{j 2}-q b_{1} r_{L j 2}+r_{S 2}^{e} b_{1}+(1-q) b_{1}-\lambda_{j 2} b_{1}\left(\delta_{L}-\delta_{S}\right)=0$
$\frac{\partial \mathcal{L}_{j 2}}{\partial r_{T j 2}}=d_{1} r_{S 2}^{e}-T_{j 2}-d_{1} r_{T j 2}+\lambda_{j 2} d_{1} \delta_{S}=0$
$\frac{\partial \mathcal{L}_{j 2}}{\partial B_{j 2}}=r_{S 2}^{e}-i_{2}+\lambda_{j 2} \delta_{S}=0$
and
$\frac{\partial \mathcal{L}_{j 2}}{\partial \lambda_{j 2}}=\left(\alpha+(1-\alpha) \delta_{S}\right) D_{j 2}+\left(\delta_{L}-\delta_{S}\right) L_{j 2}+\delta_{S} T_{j 2}+\delta_{S} B_{j 2}-\left(\frac{1}{\mu}-\delta_{S}\right) K_{j 2}=0$
Capital constraint is always binding, $\lambda_{j 2} \neq 0$ :

$$
\begin{equation*}
r_{L j 2}^{*}=\frac{\left[b_{0}+\left(b_{1}-b_{2}\right) \bar{r}_{L 2}+v_{j 2}\right] q-\lambda_{j 2} b_{1}\left(\delta_{L}-\delta_{S}\right)+b_{1} r_{S 2}^{e}+(1-q) b_{1}}{2 q b_{1}} \tag{48}
\end{equation*}
$$

$r^{*}{ }_{T j 2}=\frac{\left[r_{S 2}^{e}+\lambda_{j 2} \delta_{S}\right] d_{1}-d_{0}+\left(d_{1}-d_{2}\right) \bar{r}_{T 2}-\omega_{j 2}}{2 d_{1}}$
$\lambda_{j 2}=\frac{i_{2}-r_{S 2}^{e}}{\delta_{S}}<0$
And note that
$\frac{\partial r^{*}{ }^{*} j 2}{\partial i_{2}}=\frac{e_{1}}{2 q}-\frac{\left(\delta_{L}-\delta_{S}\right)}{2 q} \frac{\partial \lambda_{j 2}}{\partial i_{2}}$
$\frac{\partial r^{*} T j 2}{\partial i_{2}}=\frac{e_{1}}{2}+\frac{\delta_{S}}{2} \frac{\partial \lambda_{j 2}}{\partial i_{2}}$

## Consistency

This is important to check for consistency. Are these rates positive and is $r_{L j 2}>r_{T j 2}$ ? This is reasonable assumption that banks always offer the rate of loans more than rate of time deposits to ensure that banks have sufficient income or return to run their operation.

$$
\begin{gathered}
r_{L j 2}=\frac{\left[b_{0}+\left(b_{1}-b_{2}\right) \bar{r}_{L 2}+v_{j 2}\right] q-\lambda_{j 2} b_{1}\left(\delta_{L}-\delta_{S}\right)+b_{1} r_{S 2}^{e}+(1-q) b_{1}}{2 q b_{1}} \\
r_{T j 2}=\frac{\left[r_{S 2}^{e}+\lambda_{j 2} \delta_{S}\right] d_{1}-d_{0}+\left(d_{1}-d_{2}\right) \bar{r}_{T 2}-\omega_{j 2}}{2 d_{1}}
\end{gathered}
$$

First we need the market rates.
If we assume that all banks are equal except for their draws of the shock $\omega_{j 2}$, and $\mathrm{E}\left(\omega_{j 2}\right)=$ $\bar{\omega}_{j 2}$, we have for example:

$$
\begin{gathered}
\bar{r}_{T 2}=\frac{\left[r_{S 2}^{e}+\lambda_{j 2} \delta_{S}\right] d_{1}-d_{0}+\left(d_{1}-d_{2}\right) \bar{r}_{T 2}-\omega_{j 2}}{2 d_{1}} \\
\bar{r}_{T 2}=\frac{\left[r_{S 2}^{e}+\lambda_{j 2} \delta_{S}\right] d_{1}-d_{0}-\bar{\omega}_{j 2}}{d_{1}+d_{2}} \\
\bar{r}_{T 2}=\frac{\left[r_{S 2}^{e}+\lambda_{j 2} \delta_{S}\right] d_{1}}{d_{1}+d_{2}}-\frac{d_{0}+\bar{\omega}_{j 2}}{d_{1}+d_{2}} \\
\bar{r}_{T 2}=\frac{d_{1}}{d_{1}+d_{2}}\left[i_{2}\right]-\frac{d_{0}+\bar{\omega}_{j 2}}{d_{1}+d_{2}}
\end{gathered}
$$

where of course $r_{S 2}^{e}$ is the expected rate on securities.
We get the realized rate:

$$
\begin{gathered}
r_{T j 2}=\frac{\left[r_{S 2}^{e}+\lambda_{j 2} \delta_{S}\right] d_{1}-d_{0}-\omega_{j 2}}{2 d_{1}}+\frac{1}{2} \frac{\left(d_{1}-d_{2}\right)}{d_{1}+d_{2}}\left[r_{S 2}^{e}+\lambda_{j 2} \delta_{S}-\frac{d_{0}+\bar{\omega}_{j 2}}{d_{1}}\right] \\
r_{T j 2}=\left[\frac{d_{1}}{\left(d_{1}+d_{2}\right)}\right]\left[r_{S 2}^{e}+\lambda_{j 2} \delta_{S}-\frac{d_{0}}{d_{1}}\right]-\frac{1}{2 d_{1}}\left[\omega_{j 2}+\frac{\left(d_{1}-d_{2}\right)}{d_{1}} \bar{\omega}_{j 2}\right]
\end{gathered}
$$

We can do the same for the loan rate:

$$
\bar{r}_{L 2}=\frac{b_{0}+\left(b_{1}-b_{2}\right) \bar{r}_{L 2}+\bar{v}_{j 2}}{2 b_{1}}+\frac{r_{S 2}^{e}}{2 q}-\frac{\lambda_{j 2}\left(\delta_{L}-\delta_{S}\right)}{2 q}+\frac{(1-q)}{2 q}
$$

$$
\begin{gathered}
\bar{r}_{L 2}=\frac{\left[b_{0}+\bar{v}_{j 2}\right] q-\lambda_{j 2} b_{1}\left(\delta_{L}-\delta_{S}\right)+b_{1} r_{S 2}^{e}+(1-q) b_{1}}{q b_{1}+q b_{2}} \\
\bar{r}_{L 2}=\frac{\left[b_{0}+\bar{v}_{j 2}\right]}{\left(b_{1}+b_{2}\right)}-\frac{\lambda_{j 2}}{q} \frac{b_{1}\left(\delta_{L}-\delta_{S}\right)}{\left(b_{1}+b_{2}\right)}+\frac{b_{1} r_{S 2}^{e}}{\left(b_{1}+b_{2}\right)}+\frac{(1-q) b_{1}}{\left(b_{1}+b_{2}\right)}
\end{gathered}
$$

Finally we are able to go back, plug these expressions in, and see whether the realized interest rates chosen by the bank are both positive and whether (or under what conditions) the loan rate is bigger than the time deposit rate. We need to make sure that this is true.

We get the realized rate:

$$
r_{L j 2}=\frac{b_{1}}{\left(b_{1}+b_{2}\right)}\left[\frac{b_{0}}{b_{1}}-\frac{\lambda_{j 2}}{q}\left(\delta_{L}-\delta_{S}\right)+r_{S 2}^{e}+(1-q)\right]+\left[\frac{1}{2 b_{1}}\left(v_{j 2}+\frac{\left(b_{1}-b_{2}\right)}{\left(b_{1}+b_{2}\right)} \bar{v}_{2}\right)\right]
$$

First note that if $\left(\delta_{L}>\delta_{S}\right)$ and $i_{2}<r_{S 2}^{e}$, then $\bar{r}_{L 2}>0$. But on the other hand $\bar{r}_{T 2}$ will be positive if $\mathrm{d}_{0}$ is small enough. It looks like both rates will be positive without much problem. Now, is $r_{L 2}>r_{T 2}$ ?

$$
\begin{gathered}
\bar{r}_{L 2}-\bar{r}_{T 2}=\left[\frac{\left(b_{0}+\bar{v}_{2}\right)}{\left(b_{1}+b_{2}\right)}+\frac{d_{0}+\bar{\omega}}{d_{1}+d_{2}}\right]-\lambda_{j 2}\left[\frac{\left(\delta_{L}-\delta_{S}\right)}{q} \frac{b_{1}}{\left(b_{1}+b_{2}\right)}+\delta_{S} \frac{d_{1}}{d_{1}+d_{2}}\right] \\
+\left[\frac{b_{1}}{\left(b_{1}+b_{2}\right)}-\frac{d_{1}}{d_{1}+d_{2}}\right] r_{S 2}^{e}+\left[\frac{b_{1}}{\left(b_{1}+b_{2}\right)}(1-q)\right]
\end{gathered}
$$

and we see that this is the case as long as $\left[\frac{b_{1}}{\left(b_{1}+b_{2}\right)}-\frac{d_{1}}{d_{1}+d_{2}}\right]$ is not too negative we should have the desired result.

## Optimal Borrowing

We can now use the binding constraint and the balance sheet to find the expression for optimal borrowing:

$$
\begin{gathered}
\frac{K_{j 2}}{\mu}=R_{j 2}+\delta_{S} S_{j 2}+\delta_{L} L_{j 2} \\
S_{j 2}=(1-\alpha) D_{j 2}+T_{j 2}+K_{j 2}+B_{j 2}-L_{j 2}
\end{gathered}
$$

and we get

$$
\begin{gathered}
B_{j 2}=\left(\frac{1}{\mu}-\delta_{S}\right) \frac{K_{j 2}}{\delta_{S}}-\left(\alpha+(1-\alpha) \delta_{S}\right) \frac{D_{j 2}}{\delta_{S}} \\
-\left(\delta_{L}-\delta_{S}\right) \frac{L_{j 2}}{\delta_{S}}-\delta_{S} \frac{T_{j 2}}{\delta_{S}}
\end{gathered}
$$

where T and L are functions of the optimal rates which in turn are functions of the Lagrange Multiplier. It is useful to see if suitable values for the different variables ( $\mathrm{R}, \mathrm{S}, \mathrm{L}, \mathrm{D}, \mathrm{T}, \mathrm{K}, \mathrm{B}$ ) yield acceptable values for the missing parameters ( $\delta_{S}, \delta_{L}$ ). In fact, we have some freedom since we have only one equation which must be satisfied, which is the Basel constraint, but we have as much as two parameters $\left(\delta_{S}, \delta_{L}\right)$ to do it. The following table illustrates two combinations of deltas that are consistent with the Basel constraint for a given configuration of the balance sheet: ${ }^{7}$

## Implied Securities

Given B we can construct S :

$$
S_{j 2}=(1-\alpha) D_{j 2}+T_{j 2}+K_{j 2}-L_{j 2}+\left(\frac{1}{\mu}-\delta_{S}\right) \frac{K_{j 2}}{\delta_{S}}-\left(\alpha+(1-\alpha) \delta_{S}\right) \frac{D_{j 2}}{\delta_{S}}-\left(\delta_{L}-\delta_{S}\right) \frac{L_{j 2}}{\delta_{S}}-\delta_{S} \frac{T_{j 2}}{\delta_{S}}
$$

$$
S_{j 2}=\left[\frac{1}{\mu} \frac{1}{\delta_{S}}\right] K_{j 2}-\left[\frac{\alpha}{\delta_{S}}\right] D_{j 2}-\left[\frac{\delta_{L}}{\delta_{S}}\right] L_{j 2}
$$

and what determines the size of $S$ is how big loans are since the first 3 terms are positive.
This quantity will be positive because there are combinations of the (b,c,d) parameters such that we can generate the balance sheet configurations similar to those in Table 1. In such a case, S will take that value by consistency.

## The Value Function of Second Period

The value function for period 2, which B and the optimal rates are linear functions of the policy rate.
$V_{j 2}\left(K_{j 2}, Z_{j 2}\right)=r_{L j 2} q L_{j 2}+r_{S 2}^{e} S_{j 2}-r_{D 2} D_{j 2}-r_{T j 2} T_{j 2}-i_{2} B_{j 2}-(1-q) L_{j 2}$
where $Z_{j 2}=\left\{\omega_{j 2}, v_{j 2}, i_{2}, \varepsilon_{j 2}\right\}$ contains the realizations of shocks to time deposits, loan demand, interbank rate and demand deposits. We must replace securities with the balance sheet identity and obtain

$$
\begin{align*}
V_{j 2}\left(K_{j 2}, Z_{j 2}\right)= & \left(q r_{L j 2}-r_{S 2}^{e}\right) L_{j 2}+r_{S 2}^{e} K_{j 2}+\left(r_{S 2}^{e}-i_{2}\right) B_{j 2}+r_{S 2}^{e}\left[(1-\alpha) D_{j 2}+T_{j 2}\right]-r_{D 2} D_{j 2}-r_{T j 2} T_{j 2}  \tag{54}\\
& -(1-q) L_{j 2}
\end{align*}
$$

where of course $B_{j 2}$ is a linear function of $K_{j 2}$.
More important, we can decompose the value function of period 2 in two components, separating the components that depend on $K_{j 2}$ from those that do not. Since only $B_{j 2}$ depends on $K_{j 2}$ we can write:
$V_{j 2}\left(K_{j 2}, Z_{j 2}\right)=K_{j 2}\left[r_{S 2}^{e}+\left(r_{S 2}^{e}-i_{2}\right)\left(\frac{1}{\mu}-\delta_{S}\right) \frac{1}{\delta_{S}}\right]\left(r_{S 2}^{e}-i_{2}\right)\left[\left(\alpha+(1-\alpha) \delta_{S}\right) \frac{D_{j 2}}{\delta_{S}}+\left(\delta_{L}-\delta_{S}\right) \frac{L_{j 2}}{\delta_{S}}+\right.$
$\left.\delta_{S} \frac{T_{j 2}}{\delta_{S}}\right]+\left(q r_{L j 2}-r_{S 2}^{e}\right) L_{j 2}+r_{S 2}^{e}\left[(1-\alpha) D_{j 2}+T_{j 2}\right]-r_{D 2} D_{j 2}-r_{T j 2} T_{j 2}-(1-q) L_{j 2}$

$$
\begin{equation*}
V_{j 2}\left(K_{j 2}, Z_{j 2}\right)=\Gamma_{1} K_{j 2}+\Gamma_{2}(56) \tag{55}
\end{equation*}
$$

where,

$$
\Gamma_{1}=\left[r_{S 2}^{e}+\left(r_{S 2}^{e}-i_{2}\right)\left(\frac{1}{\mu}-\delta_{S}\right) \frac{1}{\delta_{S}}\right]
$$

This is useful for the next section.
Profit Maximization in Period 1, Given $\boldsymbol{K}_{\boldsymbol{j} 1}>0$
Profit function of period one is given by:
$\pi_{j 1}=\left(q r_{L j 1}-r_{S 1}-(1-q)\right) L_{j 1}+r_{S 1} K_{j 1}+\left(r_{S 1}-i_{1}\right) B_{j 1}+r_{S 1}\left[(1-\alpha) D_{j 1}+T_{j 1}\right]-r_{D 1} D_{j 1}-r_{T j 1} T_{j 1}$
and subject to constraint:
$K_{j 1} \geq \mu\left(R_{j 1}+\delta_{S} S_{j 1}+\delta_{L} L_{j 1}\right)$
Now use the balance sheet equality to eliminate securities:

$$
S_{j 1}=(1-\alpha) D_{j 1}+T_{j 1}+K_{j 1}+B_{j 1}-L_{j 1}
$$

The constraint in (3.58) becomes:
$K_{j 1}=\left(\mu-\frac{1}{\delta_{S}}\right)\left[\left(\alpha+(1-\alpha) \delta_{S}\right) D_{j 1}+\left(\delta_{L}-\delta_{S}\right) L_{j 1}+\delta_{S} T_{j 1}+\delta_{S} B_{j 1}\right]$
The objective function for the 2-period problem defined at the start of period one is then given by the Bellman equation:
$V_{j 1}\left(K_{j 1}, Z_{j 1}\right)=\max _{r_{L j 1}, r_{T j 1}, B_{j 1}} \pi_{j 1}+\beta V_{j 2}\left(K_{j 2}, Z_{j 2}\right)$
where of course the realized $\pi_{j 1}$ is equal to $K_{j 2}$.
This allows us to rewrite the Value Function in period 1 as
$V_{j 1}\left(K_{j 1}, Z_{j 1}\right)=\max _{r_{L j 1}, r_{T j 1}, B_{j 1}} \pi_{j 1}+\beta\left(\Gamma_{1} \pi_{j 1}+\Gamma_{2}\right)$
The next step is to maximize this objective function with respect to $r_{T j 1}, r_{L j 1}, B_{j 1}$. Then, take a derivative of the entire right hand side (RHS) of Bellman operator, where the solution maximization of the value function of period 1 is given as below:

$$
\frac{\partial R H S}{\partial r_{L j 1}}, \frac{\partial R H S}{\partial r_{T j 1}}, \frac{\partial R H S}{\partial B_{j 1}}
$$

The whole thing reduces to

$$
\begin{aligned}
& \frac{\partial \pi_{j 1}}{\partial r_{L j 1}}\left(1+\beta \Gamma_{1}\right)=0 \\
& \frac{\partial \pi_{j 1}}{\partial r_{T j 1}}\left(1+\beta \Gamma_{1}\right)=0 \\
& \frac{\partial \pi_{j 1}}{\partial B_{j 1}}\left(1+\beta \Gamma_{1}\right)=0
\end{aligned}
$$

So, the derivatives are shown:
$\frac{\partial \pi_{j 1}}{\partial r_{L j 1}}=\left[\begin{array}{c}b_{0} q+\left(b_{1}-b_{2}\right) \bar{r}_{L 1} q-2 b_{1} r_{L j 1} q+v_{j 1} q+b_{1} r_{S 1} \\ +(1-q) b_{1}-\left(\mu-\frac{1}{\delta_{S}}\right) b_{1} r_{S 1}\left(\delta_{L}-\delta_{S}\right)\end{array}\right]\left[1+\beta \Gamma_{1}\right]=0$
$\frac{\partial \pi_{j 1}}{\partial r_{T j 1}}=\left[\begin{array}{c}d_{1} r_{S 1}+\left(d_{1}-d_{2}\right) \bar{r}_{T 1}-d_{0}-2 d_{1} r_{T j 1}-\omega_{j 1} \\ +r_{S 1}\left(\mu-\frac{1}{\delta_{S}}\right) d_{1} \delta_{S}\end{array}\right]\left[1+\beta \Gamma_{1}\right]=0$
$\frac{\partial \pi_{j 1}}{\partial B_{j 1}}=\left[\left(r_{S 1}-i_{1}\right)+r_{S 1}\left(\mu-\frac{1}{\delta_{S}}\right) \delta_{S}\right]\left[1+\beta \Gamma_{1}\right]=0$
The derivative of $r_{T j 1}, r_{L j 1}, B_{j 1}$ in the first period is realized with the addition of the expected value of the second period which depends on the first period. In this case $\beta$ is a discount factor that can be formulated as $=\frac{1}{i+\rho}$, in which $\rho$ is a premium rate and $i$ is an interbank rate.

## Optimal interest rates in period 1

$r_{L j 1}^{*}=\frac{\left[b_{0} q+\left(b_{1}-b_{2}\right) \bar{r}_{L 1} q+v_{j 1} q+b_{1} r_{S 1}+(1-q) b_{1}-\left(\mu-\frac{1}{\delta_{S}}\right) b_{1} r_{S 1}\left(\delta_{L}-\delta_{S}\right)\right]}{2 b_{1} q}$
$r_{T j 1}^{*}=\frac{\left[\left(d_{1}-d_{2}\right) \bar{r}_{T 1}-d_{0}-\omega_{j_{1}}+d_{1} r_{S_{1}}+r_{S_{1}}\left(\mu-\frac{1}{\delta_{S}}\right) d_{1} \delta_{S}\right]}{2 d_{1}}$
and note that

$$
\begin{align*}
& \frac{\partial r_{L j 1}^{*}}{\partial i_{1}}=\frac{e_{1}}{2 q}-e_{1}\left(\mu-\frac{1}{\delta_{S}}\right)\left(\frac{\left(\delta_{L}-\delta_{S}\right)}{2 q}\right)=0  \tag{67}\\
& \frac{\partial r_{T j 1}^{*}}{\partial i_{1}}=\frac{e_{1}}{2}+e_{1}\left(\mu-\frac{1}{\delta_{S}}\right)\left(\frac{\delta_{S}}{2}\right)=0 \tag{68}
\end{align*}
$$

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## Endnotes

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[^0]:    ${ }^{1}$ Sources: Prudential Financial Policy Department, Central Bank of Malaysia.
    ${ }^{2}$ If no money is left over, the bank is extinguished and the problem ends. There is limited liability so that bank owners are not forced to cover unfulfilled claims. However, we will sidestep this problem by assuming that the distributions of the relevant random variables are such that some positive profit will always occur in period one. This has been discussed further in the first chapter which shows the difference of banks' capital ratio and capital adequacy ratio (known as excess of capital) for all banks (to show whether banks are well-capitalized or less-capitalized) and also in appendix of descriptive analysis shows that all banks have a positive equity. If excess of capital is positive (negative) means banks are classified as well capitalized (less capitalized). However, it doesn't mean for those banks have negative excess of capital have a negative equity but, their capital ratio is less than capital adequacy ratio (8\%).
    ${ }^{3}$ We assume $v_{t} \sim \mathrm{~N}(0, \Omega)$; where $\Omega$ is a scalar. The interest rate on private securities reflects firstly the production possibilities frontier, but it is also assumed to respond to monetary policy via the interbank rate.
    ${ }^{4}$ The shock $\varepsilon_{t}$ does not include bank run, which the error only give a positive value. So the distribution only positive and $\varepsilon_{t} \sim N(0, \Omega)$ where $\Omega$ is a scalar.
    ${ }^{5}$ In Basel II delta can be varied because assets depend on credit risk, therefore $\delta_{L}, \delta_{S}>1$, which can be divided into high and low credit risks on loans and securities.
    ${ }^{6}$ Toxic asset is a popular term for certain financial assets whose value has fallen significantly and for which there is no longer a functioning market, so that such assets cannot be sold at a price satisfactory to the holder.
    ${ }^{7}$ This configuration is close to the average configuration of the balance sheet in our data. It is not identical because the balance sheet in the model is a simplification of the actual data.

