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Detecting Structural Breaks in Cryptocurrency Market

(Mengesan Perubahan Struktur dalam Pasaran Mata Wang Kripto)

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ABSTRACT

This paper aims to compare the empirical performance of two approaches in detecting structural breaks and outliers due to the significant frequent price changes seen in cryptocurrencies. The two approaches are indicator saturation (IS) and Bai and Perron (BP). The cryptocurrency data employed in this study are Bitcoin and Ethereum. In comparing the performance of the two approaches, this study performed multiple empirical comparisons using various significant levels, different data frequencies, as well as the original and log series (price). The findings showed that the prices contained structural breaks and outliers and that the IS approach performed significantly better than the BP test in terms of the identified structural breaks as well as outliers across different settings. The contribution of this study is providing empirical comparisons between IS and BP approaches using cryptocurrency data. These findings are important to the potential stakeholders, in particular, for quality control in industries, for setting price targets, and for confirming trading signals to reduce potential losses.

Keywords: Bai and Perron; indicator saturation; structural breaks; outliers; cryptocurrency

ABSTRAK

Kertas kerja ini bertujuan untuk membandingkan prestasi empirikal pendekatan tepu penunjuk (IS) dan Bai dan Perron (BP) untuk mengesan perubahan struktur dan kesan pesisih akibat daripada perubahan harga yang ketara dalam mata wang kripto. Untuk mencari perubahan struktur dan kesan pesisih dalam harga Bitcoin dan Ethereum, kajian ini menggunakan metodologi IS dan BP, dan untuk membandingkan kedua-dua ujian ini, kajian ini melakukan pelbagai perbandingan empirikal seperti menggunakan pelbagai tahap kesignifikanan, frekuensi data yang berbeza, dan data harga asal serta data log. Penemuan kajian menunjukkan bahawa harga mengandungi perubahan struktur dan kesan pesisih dan pendekatan IS jauh lebih baik daripada ujian BP dari segi mengenalpasti perubahan struktur dan kesan pesisih untuk keseluruhan perbandingan. Oleh kerana belum ada perbandingan empirikal kedua-dua ujian tersebut, kertas ini mengisi jurang tersebut dengan menggunakan data mata wang kripto. Penemuan kajian ini penting kepada pihak berkepentingan yang berpotensi, khususnya, dalam membuat kawalan kualiti industri, untuk menetapkan sasaran harga, dan untuk mengesahkan isyarat perdagangan bagi mengurangkan potensi kerugian.

Kata kunci: Bai dan Perron (BP); tepu penunjuk (IS); perubahan struktur; kesan pesisih; matawang kripto
JEL: C12, C18, C22, C58, C87, G10
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INTRODUCTION

A cryptocurrency is a form of digital money that can be exchanged via a computer system and is managed decentralized. Bitcoin was created in 2008 by Satoshi Nakamoto who used it for the first time (Nakamoto 2008). Bitcoin was developed primarily as a decentralized digital currency for peer-to-peer transactions, whereas Ethereum was developed in 2015 as a platform for the development of decentralized apps and the use of smart contracts. Ethereum and Bitcoin both rely on blockchain technology. Since cryptocurrencies are rapidly evolving, and highly volatile, investors, traders, and researchers are paying close attention to them. The cryptocurrency market is subject to unanticipated changes in investor sentiment, market acceptance, and regulatory developments. For instance, the price of a bitcoin increased from around 36 cents in 2010 to $9 in 2011, $1000 in 2013, $20,000 in

2017, and $69,000 in 2021. The average cost of Ethereum in 2017 was $10. In that year, the price has significantly climbed, reaching roughly $400.

The cryptocurrency market has experienced significant fluctuations, and several researchers have shown the existence of structural breaks (Canh et al. 2019; Mandaci & Cagli 2022; Sahoo 2021; Yen et al. 2022). Yen et al. (2022) used the BP test and discovered that there is a "year-end" influence on the cryptocurrency market since there are cyclical shifts in price at the beginning and end of each year. Canh et al. (2019) discovered that structural breaks are present in all of the well-known cryptocurrencies, and the shifts moved from smaller to larger cryptocurrencies (in terms of market capitalization). In addition, Dutta and Bouri (2022) stated that, except for Bitcoin, there is no empirical proof of the existence of outliers in the top cryptocurrencies. While Song and Kang (2021) found the existence of some outlying observations in the Bitcoin return series. Therefore it is still an open discussion regarding cryptocurrency price behavior, and the present work focuses on uncovering both breaks and outliers for Bitcoin and Ethereum prices.

Traditional methods of identifying breaks and outliers based on well-known real events, like Jiun (2019), who divided the sample based on a known break date, are not precise in the sense that estimates may be inaccurate. Because it involves stochastic processes, they may only be defined using econometric methods rather than by visual inspection. Moreover, the most crucial problems that need to be resolved are computing break/outlier dates, testing structural breaks/outliers, and estimating the number of breaks/outliers in financial data (Bai & Perron 2003). When dealing with structural changes in the dataset, researchers now use various competing methods that deal with outliers and breaks. The breakpoints must typically be calculated from the data because they are rarely given exogenously (Zeileis et al. 2003). A war, a significant shift in governmental policy, or another equally abrupt event may cause structural breaks and outliers. Weideman et al. (2017) conducted a BP test and showed that most of the captured breaks are caused by actual events. So, researchers need to be able to understand how to relate breaks to macroeconomic or social events (Fattori & Carniel 2008). Because the locations of breaks frequently refer to political or economic factors, detecting structural changes offers a better understanding of the dynamics that lead to regime shifts in time series (Telli & Chen 2020). Extreme events (outliers), such as financial crises and wars, have an impact on financial time series and can change the estimation parameter (breaks) (Ismail & Nasir 2018).

A structural break refers to a change in the behavior of a variable over time, such as a spike in the money stock (Castle & Hendry 2019). Outliers are referred to as data points that do not fit in with the pattern of the other observations and are a long way from the fitted model (Brooks 2019). This paper evaluates the empirical performance of the Bai and Perron tests and the indicator saturation approach and identifies their strengths and drawbacks. The question is which of the two methods is more effective at identifying the most frequent breaks and outliers in cryptocurrency. Empirical research helps academics comprehend actual phenomena so they can draw conclusions with application to the real world. Bai and Perron (1998) considered multiple structural changes when evaluating a linear model using least squares. This test is referred to as BP. On the other hand, the indicator saturation (IS) approach by Hendry (1999) improves the basics of regression analysis by identifying outliers and structural breaks in the regression specification. It is useful when there are several unknown outliers or shifts at any time in the sample and does not require prior knowledge of the numbers, signs, timings, magnitudes, or durations of the breaks. Numerous scholars have investigated structural breaks in financial data using Bai and Perron tests see (Bouri et al. 2019; Mensi et al. 2019; Tan et al. 2019; Telli & Chen 2020; Weideman et al. 2017; Wu 2021; Zainudin & Shah Shaharudin 2011). BP test cannot detect more than 5 breaks (Lee et al. 2022; Muthuramu & Maheswari 2019).

Additionally, numerous works that applied or assessed the efficacy of indicator saturation techniques using simulation studies have focused impulse indicator saturation approach (IIS) and discovered that IIS as an illustration of IS approach performed well and helpful tool for locating outliers. These studies include Castle et al. (2015), Castle and Hendry (2014), Che Rose et al. (2021), Ericsson (2017), Marczak and Proietti (2016), Muhammadullah et al. (2022), Nasir and Ismail (2020). Results from simulations might not be applicable to actual circumstances. Some other researchers compared the BP test to other tests to show its performance (Enders & Holt 2012; Zarei et al. 2015; Yasir & Önder 2021). Thus, these two tests were used independently in several research. Since there have not been any empirical performance comparisons of the two tests, we fill this gap using cryptocurrency data, specifically Bitcoin (BTC) and Ethereum (ETH) monthly and weekly observations for each, the original and log series for each, and 4 or 5 significant levels for each. This study aims firstly to examine the empirical performance of the BP and IS tests for spotting breaks and outliers in cryptocurrency data. Secondly, to specifically compare the performance of the BP Test and SIS. Thirdly, to highlight the impact of significant levels and frequencies on the performance of each test.

The remainder of the paper is structured as follows. Section 2 presents the development of structural change tests in the literature. Section 3 discusses the theoretical background of the models used and the methodology followed, whereas Section 4 covers the results and discussion of the study. The researchers' conclusion and suggestions for further research are presented in section 5.

LITERATURE REVIEW

Structural breaks in a series can be found by assuming that the break dates are either known, allowing for exogenous detection, or unknown, allowing for endogenous detection. Chow (1960) pioneered structural break testing for regression models, developing the F-test for a single break, assuming that the break date is previously known under the null of no break. Quandt (1960) altered the Chow framework to consider the F-statistic with the highest value among all potential break dates to loosen the requirement that the candidate break date be known. Perron (1989) assumed that a break could happen based on significant economic events that are determined exogenously. These writers assumed that there was no breakpoint at the beginning or end of the period because they believed that the likelihood of a breakpoint arising at the terminal point was extremely low. Most studies that use these approaches assume that 15% of the time, there is no break at the beginning or end. Later its allowed for multiple breaks, particularly the Bai and Perron tests see Bai and Perron (1998, 2003). The consideration of linear regression with multiple (m) breakpoints and $m+1$ regimes was Bai and Perron's (1998) key contribution to the methodology. To study the case with breaks with uncertain break dates, Ohara (1999) also employed a strategy based on sequential t-tests of Zivot and Andrews (1992). Also, Papell and Prodan (2003) proposed a test based on restricted structural change that expressly permits two offsetting structural changes.

Therefore, BP procedure is helpful for formal identification of structural breakpoints by some researchers see Enders and Holt (2012), Muthuramu and Uma Maheswari (2019), Tule et al. (2019), Yasir and Önder (2021). Many other studies have recently concentrated on Indicator Saturation (IS) approach. Hendry (1999) created the indicator saturation (IS) technique to evaluate parameter constancy. This strategy was later extended to the multiplicative indicator saturation (MIS) strategy, the trend indicator saturation (TIS) strategy, the step indicator saturation (SIS) strategy, and the impulse indicator saturation (IIS) strategy. Castle et al. (2012) identified the capability of the IIS by recognizing a similar number of breaks and timing in Bai and Perron (1998) approach. Additionally, Castle et al. (2012) demonstrated that IIS could identify several outliers and structural breaks, mainly when the breaks are at the start and end of the sample and can also adjust for non-normality. Later, Doornik et al. (2013) introduced step-indicator saturation (SIS), which predicts level shifts based on step interventions, to provide an expanded version of the IIS. Following that, Castle et al. (2015) discovered that SIS displays a higher power when location change happens than IIS. Ericsson et al. (2012) combined IIS and SIS to create the super-saturation indicator technique (SSI) to address the effects of outliers and breaks. Pretis et al. (2018) created indicator saturation in general-to-specific (gets) package which gives a methodological technique for building econometric models where the researcher starts with a comprehensive model and then narrows it down through hypothesis testing (Brooks 2019). Researchers such as Ghouse et al. (2022) and Tuan Anh et al. (2020) are currently using this technique.

Finally, the literature does not only consider these two tests to manage breaks and outliers; other scholars have also used other break tests. For instance, to study the fluctuations in stock market values, Wu and Ow (2021) created a novel sentiment classifier approach in machine learning algorithms. Tran (2022) used Hodrick-Prescott (HP) and Christiano-Fitzgerald (CF) filters to analyze turning points based on the bull and bear markets for stocks. Sahoo (2021) used a test known as Narayan and Popp (NP), which only considers structural breaks. Jiang et al. (2023) used BP and iterative cumulative sum of squares (ICSS) to spot breaks in cryptocurrency. Gałecki (2020) used the Perron test, Andrews and Zivot test, and BP tests to validate the presence of structural breaks. Finally, Pretis et al. (2018), the creators of the GETS package, provided a brief comparison of the software packages that implement the IS method and examined existing break detection algorithms like the BP test. They discovered that the model selection technique for change detection in the IS approach is the main difference. But until now, neither of these two tests has been empirically compared or taken into account cryptocurrency data, which this paper does. However, several researchers used these two tests separately in different financial data as multiple mean level changes, setting them apart from other tests. Furthermore, the IS test is effective at identifying breaks and outliers in the BTC and ETH simultaneously.

METHODOLOGY

This section presents the background of the BP and IS tests, the dataset used and the methodology. The Bai and Perron methodology is used to endogenously estimate breaks in a series without knowing the break date beforehand (Bai & Perron 1998, 2003). We consider a time series with $t = 1, 2, \ldots, T$ and m structural breaks to accommodate $m+1$ regimes (partitions). We set equations for each potential regime if we allow $T_1, T_2, \ldots, T_m$ signify the breakpoints and T represent the total number of observations in the series:

$$\begin{aligned} y_t &= x_t\beta + z_t\delta_1 + u_t \; for \; t = 1,2,..T_1 \\ y_t &= x_t\beta + z_t\delta_2 + u_t \; for \; t = T_1+1, T_1+2,..T_2 \\ y_t &= x_t\beta + z_t\delta_3 + u_t \; for \; t = T_2+1, T_2+2,..T_3 \\ &\vdots \\ y_t &= x_t\beta + z_t\delta_{m+1} + u_t \; for \; t = T_m+1, T_m+2,..T \end{aligned} \tag{1}$$

The matrix of the estimated coefficients for each partition is represented by δ_1 through δ_{m+1}, where β

denotes the matrix of coefficients that remain constant across all partitions. Both x_t and z_t are vectors of p and q dimensions, respectively. The above set of equations can, however, be represented as follows.

$$Y = X\beta + \bar{Z}\delta + U \quad (2)$$

Here, Y and X each include (y_1, y_2, y_3, y_t), while $\bar{Z}$ contains (z_1, z_2, z_3, z_t). where β denotes a matrix whose coefficients, from δ_1 to δ_{m+1}, are constant across all partitions and U are $iid(0, \sigma^2)$. This method uses Ordinary Least Square (OLS) to calculate the parameters, and the following function can be used to reduce the sum of squared errors:

$$(Y - X\beta - \bar{Z}\delta)'(Y - X\beta - \bar{Z}\delta) = \sum_{t=1}^{m+1} \sum_{t=T_{i-1}+1}^{T_i} [y_t - x_t'\beta - z_t'\delta_i]^2 \quad (3)$$

The global sum of squares $S_T(T_1, T_2, \dots, T_m)$ for the breaks dates is determined by first calculating the sum of squares of residuals for each regime and letting $\hat{\beta}\{T_j\}$ and $\hat{\delta}\{T_j\}$ be the estimates of m regimes. The break dates such that $(\hat{T}_1, \hat{T}_2, \dots \hat{T}_m) = argmin_{T_1, T_2, \dots, T_m} S_T(T_1, T_2, \dots, T_m)$. Bai and Perron (1998, 2003) proposed a sup-F type test to the null of $m = 0$ breaks and alternative of arbitrary number of changes $m = k$ to test for the maximum number of break dates. The fraction $\frac{T_i}{T} = \lambda_i$ is used to indirectly search for the break dates where $i = 1, 2, \dots, k$:

$$F_T(\lambda_1, \lambda_2, \lambda_3, \dots \lambda_k; q) = \left(\frac{T-(k+1)q-p}{kq}\right) \frac{\hat{\delta}' R'(R(\bar{Z} M_x \bar{Z})^{-1} R\hat{\delta}}{SSR_k} \quad (4)$$

Where R is the matrix of $(\hat{\delta}R) = (\hat{\delta}_1 - \hat{\delta}_2, \hat{\delta}_2 - \hat{\delta}_3 \dots, \hat{\delta}_k - \hat{\delta}_{k+1})$, $M_x = I - X(X'X)^{-1}X'$ and SSR_k is the sum of squares of the residuals under the alternative hypothesis. However, the following definition of the sup-F statistics follows:

$$F(k; q) = sup_{(\lambda_1, \lambda_2, \dots, \lambda_k) \in \Lambda_\varepsilon} F_T(\lambda_1, \lambda_2, \dots, \lambda_k; q) \quad (5)$$

To enable endogenous break estimation, the double-maximum test, also known as the Dmax test, was suggested by Bai and Perron (1998, 2003). It later upgraded to two variants, the UDmax and WDmax, and can be written as follows:

$$UDmaxF_T(M, q, a_1, a_2, \dots, a_M) = max_{1 \le m \le M}\, sup_{(\lambda_1, \lambda_2, \dots, \lambda_k) \in \Lambda_\varepsilon} F_T(\lambda_1, \lambda_2, \dots, \lambda_k; q) \quad (6)$$

$$WDmaxF_T(M, q, a_1, a_2, \dots, a_M) = max_{1 \le m \le M} \frac{c(q, \alpha, 1)}{c(q, \alpha, m)}\, sup_{(\lambda_1, \lambda_2, \dots, \lambda_k) \in \Lambda_\varepsilon} F_T(\lambda_1, \lambda_2, \dots, \lambda_k; q) \quad (7)$$

Where α is the significance level, m is the number of breaks, q is the number of time-varying parameters, and $c(q, \alpha, m)$ is the asymptotic critical for the test $sup_{(\lambda_1, \lambda_2, \dots, \lambda_k) \in \Lambda_\varepsilon} F_T(\lambda_1, \lambda_2, \dots, \lambda_k; q)$. Some fixed weights connected to breaks are contained in the specifications $(a_1, a_2, \dots, a_M)$.

An F-type test with the null of $H_0: m = l$ and the alternative of $H_1: m = l + 1$ was suggested by Bai and Perron (1998, 2003) to test for the number of break dates and isolate them. Therefore, this test is performed $l + 1$ times to determine the number of breaks until the null hypothesis cannot be rejected. The following is a definition of the F test statistics:

$$F_T(l + 1, l) = \frac{\{S_T(\hat{T}_1, \hat{T}_2, \dots \hat{T}_m) - min_{1 \le i \le l} \inf_{\tau \in \Lambda_i, \eta} S_T(\hat{T}_1, \hat{T}_2, \dots, \hat{T}_{i-1}, \tau, \hat{T}_i, \hat{T}_{i+1}, \dots, \hat{T}_l)\}}{\hat{\sigma}^2} \quad (8)$$

Where $\Lambda_i, \eta = \{\tau; T_{i-1} + (T_i - T_{i-1})\eta \le \tau \le T_i - (T_i - T_{i-1})\eta\}$, and $\hat{\sigma}^2$ is an accurate estimate of the residual variance. This method is extended by Bai and Perron (1998, 2003) to F tests for $0\ vs.\ l$ breaks and $l\ vs.\ l + 1$ breaks, respectively, with arbitrary but fixed l.

INDICATOR SATURATION APPROACH

We use two types of indicator saturation (IS) approach. The impulse indicator saturation (IIS) for outliers detection introduced by Hendry (1999) and Santos et al. (2008). Then, the step indicator saturation (SIS) for location shifts proposed by Castle et al. (2015). Impulse indicator saturation (IIS) generates a full set of indicator variables. For each indicator, a single observation yields a value of 1, whereas all other observations yield a value of 0. There are produced as many indicators as there are observations, each with a unique observation corresponding to the value 1. That means that IIS creates T additional variables when there are T observations (Castle & Hendry 2019). So, IIS can be stated mathematically as below:

$$\text{IIS} \quad \{1_{\{j=t\}}\} \text{ where } \{1_{\{j=t\}}\} = \begin{cases} 1, & \text{when } j = t \\ 0, & \text{Otherwise for } j = 1, \dots, T \end{cases} \quad (9)$$

In addition, IIS for an outlier can be illustrated by creating an outlier of size λ at observation k by:

$$y_t = \mu + \lambda I_{\{j=t\}} + \varepsilon_t, \text{ where } \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \text{ and } \lambda \neq 0 \quad (10)$$

In GETS modeling, this strategy would not use all indicators as regressors. If every indicator is considered, the model will have more regressors than there are observations, T (Castle et al. 2021). Santos et al. (2008) considered a linear regression with just an intercept; then, add the first $T/2$ impulse indicators to it to comprehend the "split-half" method. The first half will produce selected indicators for any observations that differ from those estimates by at least the set threshold of significance. Once any significant indicators have been located, their locations are noted. The first T/2 indicators are then swapped out for the second T/2, and the process is then repeated. To choose the final significant indicators, the two sets of sub-sample significant indicators are included to the model. Let y_t, an observed random variable, have an independent normal distribution as $y_t \sim N[\mu, \sigma_\varepsilon^2]$, with $t = 1, 2, \ldots, T$, where μ and σ_ε^2 are the relevant parameters. However, a researcher is unsure of where outliers (if any) may be hiding. Moreover, for split half approach consider starting by just adding the intercept and the other half of the indicators (for example, $1_{\{t=j\}}$ for $j = 1, \ldots, T/2$, assuming T is even). Determining the GUM of the first step, then

$$\text{IIS } y_t = \mu + \sum_{j=1}^{T/2} \delta_j 1_{\{t=j\}} + \varepsilon_t \quad (11)$$

Thus, the above equation also incorporates $T/2$ parameters for $T/2$ impulse indicators for the first observations, in addition to the mean and variance. Step-indicator saturation (SIS), is a block of consecutive impulses with the same signs and magnitudes constitutes a step shift. Although IIS can be used to detect these, the retained indicators might be aggregated into a single dummy variable that took the average value of the shift for the break period and 0 elsewhere (Castle & Hendry 2019). A saturating set of $T-1$ step-shift indicators that change from 1 to 0 at a different observation at each step can be created. These indicators take the value 1 from the start of the sample up to a specific observation and then take the value 0 after that. Step indicators are the accumulation of impulse indicators up to each subsequent observation. The only step following step 'T' would be the intercept. The group of potential predictors includes the $T-1$ stages. So, SIS can be stated mathematically as below:

$$\text{SIS} \quad \{1_{\{t\leq j\}}\}, \text{ where } \{1_{\{t\leq j\}}\} = \begin{cases} 1, & \text{for observations up to j} \\ 0, & \text{Otherwise} \end{cases} \text{ and } j = 1, \ldots, T \quad (12)$$

In addition, SIS for a Location shift can be illustrated by creating a location shift of size λ at observation by:

$$y_t = \mu + \lambda I_{\{j\geq t\}} + \varepsilon_t, \text{ where } \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \text{ and } \lambda \neq 0 \quad (13)$$

Castle and Hendry (2019) demonstrated 'half-sample' SIS instead of using the first and second halves altogether. However, The IIS/SIS first adds an impulsive dummy for each observation, divides the dummies into blocks, and then performs a regression on each block of dummies against the data while keeping the significant dummies. The process continues with the following block, and so on, until each period is finished. Only the significant dummies are left after combining and regressing all the kept dummies in each block again (Mariscal & Powell 2013). When employing IIS and SIS to create a simple model of the mean of y_t, the appropriate general unrestrictive models (GUMs) are as follows (Pretis et al. 2018):

$$\text{IIS } y_t = \mu + \sum_{j=1}^{n} \delta_j 1_{\{t=j\}} + \varepsilon_t \quad (14)$$

$$\text{SIS } y_t = \mu + \sum_{j=2}^{n} \delta_j 1_{\{t\geq j\}} + \varepsilon_t \quad (15)$$

$$\text{TIS } y_t = \mu + \sum_{j=1}^{n} \delta_j 1_{\{t>j\}}(t-j) + \varepsilon_t \quad (16)$$

The GUM supplies the initial information set and serves as the starting point for the model reduction process. The GUM provides enough details on the modelled process, and statistically accurate (Castle & Shephard 2009).

DATA SET

Our data comprises the weekly and monthly close prices of two cryptocurrencies: Bitcoin (BTC) and Ethereum (ETH), which we get from Yahoo Finance https://finance.yahoo.com/. While the data period for Ethereum begins on November 6, 2017, and for Bitcoin begins on January 1, 2015, both data periods finish on May 1, 2023. The series has 100 (BTC) and 65 (ETH) monthly observations and for a total of 435 (BTC) and 287 (ETH) weekly observations. From Table 1, both Bitcoin and Ethereum cryptocurrencies are further introduced.

TABLE 1. Data summary

Symbol	Frequency
BTC USD	100 months, 435 weeks
Market Cap	527.7B
Circulating Supply	19.39M
ETH USD	65 months, 287 weeks
Market Cap	229.14B
Circulating Supply	120.24M

DATA ANALYSIS FRAMEWORK

The data analysis begins by examining descriptive statistics. After that, we applied a constant to the data to evaluate the empirical fluctuation test using Ploberger and Krämer (1992) OLS-based CUSUM technique, which is based on the cumulative sums of typical OLS residuals, to check for structural changes in the model.

$$W_n^0(t) = \frac{1}{\hat{\sigma}\sqrt{n}} \sum_{i=1}^{\lfloor nt \rfloor} \hat{u}_i \quad (0 \le t \le 1) \tag{17}$$

The limiting process for $W_n^0(t)$ is the standard Brownian bridge $W^0(t) = W(t) - tW(1)$, where $W(\cdot)$ stands for standard Brownian motion; in the case of a single-shift alternative, the process should peak near the breaking point. Then, using a regression model with a constant as a regressor, the BP and IS tests were applied to find several breaks and outliers in each series with different levels of significance. For the BP test, we used a trimming percentage of 15% across the series and allowed no more than five breaks, but there was no restriction for the IS approach. We employ Eviews software to execute both the IS and BP tests.

RESULTS AND DISCUSSION

The empirical performance of IS and BP approaches are discussed here. We first visualize the series plots and the descriptive statistics.

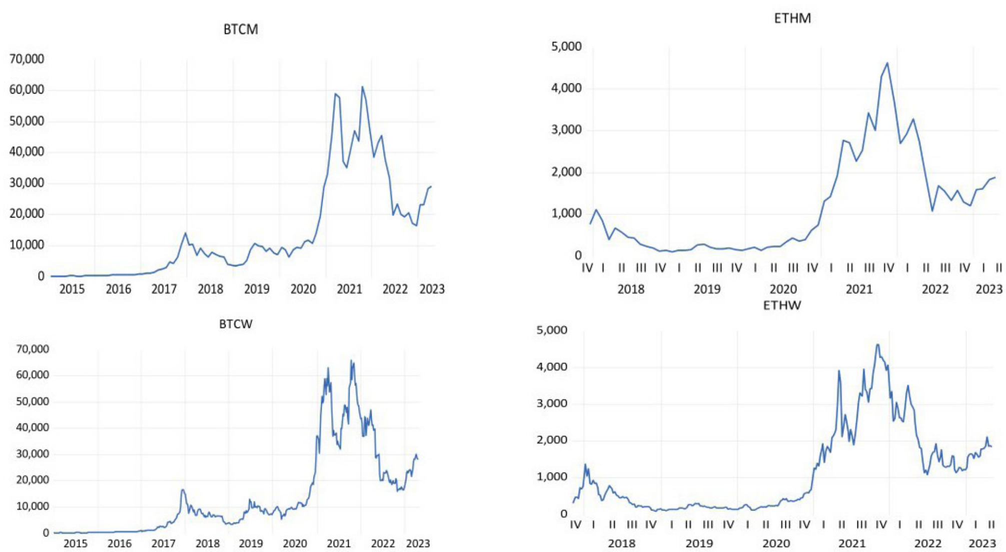


FIGURE 1. Monthly and weekly original-price of BTC and ETH

Figure 1 supports the existence of structural breaks/ outliers. There is a significant increase between 2017 and 2018 and again between 2020 and 2022. The Covid-19 dilemma and the Ukraine war is most likely to blame for these jumps.

TABLE 2: Descriptive statistics

Original-Series					
Series	Mean	Median	Maximum	Minimum	Std. Dev.
BTCM	13868.38	8037.154	61318.96	217.4640	16114.66
BTCW	13817.14	7824.231	65992.84	178.1030	16187.00
ETHM	1178.144	669.9240	4631.479	107.0610	1170.865
ETHW	1166.664	618.3290	4626.359	85.26210	1159.702

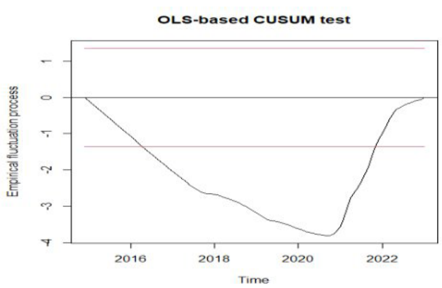
In Table 2, for original price, BTC costs a minimum of \$217.5, a maximum of \$61319 monthly, a minimum of \$178, and a maximum of \$65992.8 weekly. ETH costs a minimum of \$107, a maximum of \$4631.5 monthly, a minimum of \$85.3, and a maximum of \$4626.4 weekly. The mean for BTC and ETH is almost five digits, which makes us doubt the presence of structural change. Table 3 and Figure 2 show clear evidence.

TABLE 3. Existence of outliers in monthly data

	BTCM	BTCW	ETHM	ETHW
Q_1	1046.443	1023.725	219.8485	219.7798
Q_3	19850.95	20018.4	1822.022	1791.968
IQR	18804.54	18994.67	1602.174	1572.183
Upper Bound	48058	48510	4225	4150
Lower Bound	-27160	-27468	-2183	-2138

Table 3 demonstrates that BTC and ETH contain outliers and any value outside the boundaries should be regarded as an outlier.

EMPIRICAL FLUCTUATION TEST



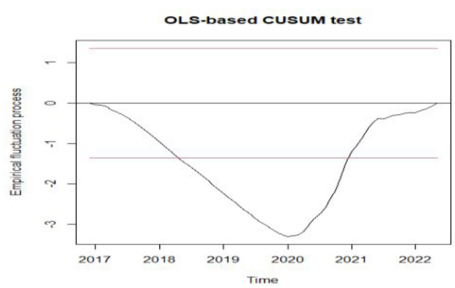


FIGURE 2: OLS-based CUSUM process for the BTC and ETH

Figure 2 shows fitted fluctuation process and its boundaries at a 5% significance level. The process has a peak around 2020 in both series that exceeds the boundaries, indicating a distinct structural shift at that time.

BAI AND PERRON TEST

The BP test is used to detect multiple breaks in each series using a regression model that includes a constant as a regressor and we allowed different significance levels. The following two tables present the performance of the BP test including the number of breaks discovered and their dates at various levels of significance.

TABLE 4. BP in original-series

Original-Series (BP)			
Series	α	Sequential	Repartition
BTCM	0.01	2017M10,2020M12	2017M10,2020M12
	0.025	2017M10,2020M12	2017M10,2020M12
	0.05	2017M10,2020M12	2017M10,2020M12
	0.10	2017M10,2020M12	2017M10,2020M12
BTCW	0.01	10/05/2017,12/10/2020	10/05/2017,12/17/2020
	0.025	10/05/2017,12/10/2020	10/05/2017,12/17/2020
	0.05	10/05/2017,12/10/2020	10/05/2017,12/17/2020
	0.10	10/05/2017,12/10/2020	10/05/2017,12/17/2020
ETHM	0.01	2021M02,2022M05	2021M03,2022M05
	0.025	2021M02,2022M05	2021M03,2022M05
	0.05	2021M02,2022M05	2021M03,2022M05
	0.10	2021M02,2022M05	2021M03,2022M05
ETHW	0.01	9/03/2018,1/18/2021,5/09/2022	9/03/2018,2/01/2021,5/09/2022
	0.025	9/03/2018,1/18/2021,5/09/2022	9/03/2018,2/01/2021,5/09/2022
	0.05	9/03/2018,1/18/2021,5/09/2022	9/03/2018,2/01/2021,5/09/2022
	0.10	9/03/2018,1/18/2021,5/09/2022	9/03/2018,2/01/2021,5/09/2022

After applying the BP test to the original prices, the results in Tables 4 and 5 revealed that: first, if we vary the significance levels but not the frequency, the F-stat value for each break remains constant. Second, the critical value of F statistics for each break remains the same if the frequency changes, but the values will vary if the significance level changes. Third, if the significance levels change, the BP test does not detect more breaks. Additionally, after applying the BP test on the log prices, we found the following: first, the number of breaks discovered by the BP test has increased. Second, the BP test consistently identified the same number of breaks across alpha levels and frequencies. Third, the BP test produced similar results in sequential and repartition approaches. Lastly, the BP test does not give an indication, such as a positive sign for an upward shock or a negative sign for a downward shock.

TABLE 5. BP in log-series

Log-Series (BP)			
Series	α	Sequential	Repartition
LBTCM	0.01	2016M04,2017M07,2020M11	2016M04, 2017M07, 2020M11
	0.025	2016M04,2017M07,2020M11	2016M04, 2017M07, 2020M11
	0.05	2016M04,2017M07,2019M05,2020M11,2022M02	2016M04,2017M07,2019M05,2020M11, 2022M02
	0.10	2016M04,2017M07,2019M05,2020M11,2022M02	2016M04,2017M07,2019M05,2020M11, 2022M02
LBTCW	0.01	4/28/2016,7/27/2017,5/09/2019,11/05/2020, 2/03/2022	4/28/2016,7/27/2017,5/09/2019,11/05/2020, 2/03/2022
	0.025	4/28/2016,7/27/2017,5/09/2019,11/05/2020, 2/03/2022	4/28/2016,7/27/2017,5/09/2019,11/05/2020, 2/03/2022
	0.05	4/28/2016,7/27/2017,5/09/2019,11/05/2020, 2/03/2022	4/28/2016,7/27/2017,5/09/2019,11/05/2020, 2/03/2022
	0.10	4/28/2016,7/27/2017,5/09/2019,11/05/2020, 2/03/2022	4/28/2016,7/27/2017,5/09/2019,11/05/2020, 2/03/2022
LETHM	0.01	2018M09, 2020M04, 2020M05, 2021M01	2018M09, 2020M04, 2020M05, 2021M01
	0.025	2018M09, 2020M04, 2020M05, 2021M01	2018M09, 2020M04, 2020M05, 2021M01
	0.05	2018M09, 2020M04, 2020M05, 2021M01	2018M09, 2020M04, 2020M05, 2021M01
	0.10	2018M09, 2020M04, 2020M05, 2021M01	2018M09, 2020M04, 2020M05, 2021M01

continue ...

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LETHW	0.01	9/03/2018,2/03/2020, 12/28/2020, 5/23/2022	9/03/2018, 2/03/2020,12/28/2020,5/23/2022
	0.025	9/03/2018,2/03/2020, 12/28/2020, 5/23/2022	9/03/2018,2/03/2020, 12/28/2020,5/23/2022
	0.05	9/03/2018,2/03/2020, 12/28/2020, 5/23/2022	9/03/2018, 2/03/2020, 12/28/2020,5/23/2022
	0.10	9/03/2018,2/03/2020, 12/28/2020, 5/23/2022	9/03/2018, 2/03/2020, 12/28/2020,5/23/2022

INDICATOR SATURATION APPROACH

The IS approach is then employed in each series using a regression model that includes a constant as a regressor in the same data with different significance levels. The results of the SIS test are presented in table 6 and 7 including series, blocks utilized, the SIC selection criteria, breaks and their dates. For original and log monthly data, SIS identified the same date of breaks where breaks with a one-month difference are considered the same. For Bitcoin, SIS similarly identified 2017M02 8 times and 2020M11 9 times across alpha levels in all settings. BTC and ETH share three breaks 2021M01, 2021M09, and 2022M05. For original and log weekly data, SIS similarly detected 9 weeks for BTC and 7 weeks for ETH across all settings.

TABLE 6. SIS in original-series

Series	α	Original-Series (SIS)
BTCM	0.001	Obs:100,Blocks:4,SIS:3,SIC:9.98 2017M02(+),2020M12(+),2022M05(-)
	0.01	Obs:100,Blocks:4,SIS:6,SIC:9.67 2017M02(+),2019M05(+),2020M11(+),2021M02(+),2021M05(-),2022M05(-)
	0.025	Obs:100,Blocks:4,SIS:6,SIC:9.67 2017M02(+),2019M05(+),2020M11(+),2021M02(+),2021M05(-),2022M05(-)
	0.05	Obs:100,Blocks:4,SIS:6,SIC:9.67 2017M02(+),2019M05(+),2020M11(+),2021M02(+),2021M05(-),2022M05(-)
	0.10	Obs:100,Blocks:4,SIS:9,SIC:9.25 2017M02(+),2017M10(+),2019M05(+),2020M11(+),2021M02(+),2021M05(-),2021M10(+),2021M12(-),2022M05(-)
BTCW	0.001	Obs:435,Blocks:15,SIS:9,SIC:9.52 10/12/2017(+),5/03/2018(-),6/13/2019(+),7/30/2020(+),12/24/2020(+),2/18/2021(+),5/13/2021(-),9/09/2021(+),3/31/2022(-)
	0.01	Obs:435,Blocks:15,SIS:11,SIC:9.35 3/23/2017(+),10/19/2017(+),5/03/2018(-),6/13/2019(+),7/30/2020(+),12/24/2020(+),2/18/2021(+),5/13/2021(-),9/09/2021(+),12/09/2021(-),3/31/2022(-)
	0.025	Obs:435,Blocks:5,SIS:14,SIC:8.97 3/23/2017(+),10/19/2017(+),5/03/2018(-),11/22/2018(-),6/13/2019(+),7/30/2020(+),12/24/2020(+),2/18/2021(+),5/13/2021(-),9/02/2021(+),9/09/2021(+),12/09/2021(-),5/05/2022(-),1/12/2023(+)
	0.05	Obs:435,Blocks:5,SIS:15,SIC:8.92 3/23/2017(+),10/19/2017(+),5/03/2018(-),11/22/2018(-),6/13/2019(+),7/23/2020(+),11/05/2020(+),12/24/2020(+),2/18/2021(+),5/06/2021(-),9/02/2021(+),9/09/2021(+),12/09/2021(-),5/05/2022(-),1/12/2023(+)
	0.10	Obs:435,Blocks:15,SIS:16,SIC:8.61 3/23/2017(+),10/19/2017(+),5/03/2018(-),11/22/2018(-),6/13/2019(+),7/23/2020(+),11/05/2020(+),12/24/2020(+),2/18/2021(+),5/13/2021(-),8/05/2021(+),9/30/2021(+),12/02/2021(-),3/31/2022(-),5/05/2022(-),1/12/2023(+)
ETHM	0.001	Obs:65,Blocks:3,SIS:3,SIC:15.07 2021M01(+),2021M09(+),2022M05(-)
	0.01	Obs:65,Blocks:3,SIS:3,SIC:15.07 2021M01(+),2021M09(+),2022M05(-)
	0.025	Obs:65,Blocks:3,SIS:3,SIC:15.07 2021M01(+),2021M09(+),2022M05(-)
	0.05	Obs:65,Blocks:3,SIS:4,SIC:14.83 2021M01(+),2021M04(+),2021M09(+),2022M05(-)
	0.10	Obs:65,Blocks:3,SIS:5,SIC:14.59 2021M01(+),2021M04(+),2021M09(+),2022M01(-),2022M05(-)

continue ...

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ETHW	0.001	Obs:287,Blocks:10,SIS:8,SIC:14.35 12/17/2018(-),8/17/2020(+),12/28/2020(+),3/15/2021(+),8/02/2021(+),10/04/2021(+),1/03/2022(-),4/18/2022(-)
	0.01	Obs:287,Blocks:10,SIS:8,SIC:14.35 12/17/2018(-),8/17/2020(+),12/28/2020(+),3/15/2021(+),8/02/2021(+),10/04/2021(+),1/03/2022(-),4/18/2022(-)
	0.025	Obs:287,Blocks:10,SIS:8,SIC:14.35 12/17/2018(-),8/17/2020(+),12/28/2020(+),3/15/2021(+),8/02/2021(+),10/04/2021(+),1/03/2022(-),4/25/2022(-)
	0.05	Obs:287,Blocks:10,SIS:12,SIC:13.94 5/28/2018(-),12/17/2018(-),8/17/2020(+),12/28/2020(+),3/08/2021(+),4/26/2021(+),5/17/2021(-),8/02/2021(+),10/04/2021(+),1/03/2022(-),4/25/2022(-),1/09/2023(+)
	0.10	Obs:287,Blocks:10,SIS:12,SIC:13.94 5/28/2018(-),12/17/2018(-),8/17/2020(+),12/28/2020(+),3/08/2021(+),4/26/2021(+),5/17/2021(-),8/02/2021(+),10/04/2021(+),1/03/2022(-),4/25/2022(-),1/09/2023(+)

The IIS test is then utilized. Table 8 and 9 summarize IIS performance including series, blocks used, the SIC selection criteria, outliers, and their dates.In the original series for BTC IIS recognized most observations from 2021M01 to 2022M04 as upward outliers across significance levels and from 2015 and 2016 as downward outliers in the log series. The series start date, however, caused IIS in ETH to function differently.

TABLE 7. SIS in log-series

Series	α	Log-Series (SIS)
LBTCM	0.001	Obs:100,Blocks:4,SIS:3,SIC:1.20 2017M02 (+),2017M08 (+),2020M11(+)
	0.01	Obs:100,Blocks:4,SIS:4,SIC:1.17 2017M02(+),2017M08(+),2019M04(+),2020M11(+)
	0.025	Obs:100,Blocks:4,SIS:5,SIC:0.90 2016M05(+),2017M02(+),2017M08(+),2019M04(+),2020M11(+)
	0.05	Obs:100,Blocks:4,SIS:5, SIC:0.90 2016M05(+),2017M02(+),2017M08(+),2019M04(+),2020M11(+)
	0.10	Obs:100,Blocks:4,SIS:6,SIC:0.78 2016M05(+),2017M02(+),2017M08(+),2018M11(-),2019M04(+),2020M11(+)
LBTCW	0.001	Obs:435, Blocks:15,SIS:11,SIC:0.34 7/23/2015(+),2/11/2016(+),9/08/2016(+),3/30/2017(+),10/12/2017(+),5/10/2018(-),11/22/2018(-), 6/13/2019(+), 7/30/2020(+),2/18/2021(+),3/31/2022(-)
	0.01	Obs:435,Blocks:15,SIS:13,SIC:-0.01 7/23/2015(+),2/11/2016(+),9/08/2016(+),3/30/2017(+),5/18/2017(+),10/19/2017(+),5/10/2018(-),11/22/2018 (-), 6/13/2019(+),7/23/2020(+),12/17/2020(+),2/18/2021(+),3/31/2022(-)
	0.025	Obs:435,Blocks:15, SIS:15,SIC:-0.01 7/23/2015(+),2/11/2016(+),9/08/2016(+),3/30/2017(+),5/18/2017(+),10/12/2017(+),10/19/2017(+),5/10/2018 (-), 11/22/2018(-),6/13/2019(+),7/23/2020(+),12/17/2020(+),2/18/2021(+),3/31/2022(-),10/20/2022(-)
	0.05	Obs:435,Blocks:15,SIS:15,SIC:-0.01 7/23/2015(+),2/11/2016(+),9/08/2016(+),3/30/2017(+),5/18/2017(+),10/12/2017(+),10/19/2017(+),5/10/2018 (-), 11/22/2018(-),6/13/2019(+),7/23/2020(+),12/17/2020(+),2/18/2021(+),3/31/2022(-),10/20/2022 (-)
	0.10	Obs:435,Blocks:15,SIS:15,SIC:-0.01 7/23/2015(+),2/11/2016(+),9/08/2016(+),3/30/2017(+),5/18/2017(+),10/12/2017(+),10/19/2017(+),5/10/2018 (-), 11/22/2018(-),6/13/2019(+),7/23/2020(+),12/17/2020(+),2/18/2021(+),3/31/2022(-),10/20/2022(-)

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LETHM	0.001	Obs: 65,Blocks:3,SIS:1, SIC:1.72 2021M01(+)
	0.01	Obs:65,Blocks:3, SIS:3,SIC:0.96 2018M08(-),2020M07(+),2021M01(+)
	0.025	Obs:65,Blocks:3,SIS:3,SIC:0.96 2018M08(-), 2020M07(+), 2021M01(+)
	0.05	Obs:65,Blocks:3,SIS:5,SIC:0.60 2018M08(-),2020M07(+),2021M01(+),2021M09(+),2022M05(-)
	0.10	Obs:65,Blocks:3,SIS:5,SIC:0.60 2018M08(-),2020M07(+),2021M01(+),2021M09(+),2022M05(-)
LETHW	0.001	Obs:287,Blocks:10,SIS:7,SIC:0.38 8/06/2018(-),1/27/2020(+),8/17/2020(+),1/04/2021(+),3/15/2021(+),10/04/2021(+),4/25/2022(-)
	0.01	Obs:287,Blocks:10,SIS:7,SIC:0.38 8/06/2018(-),1/27/2020(+),8/17/2020(+),1/04/2021(+),3/15/2021(+),10/04/2021(+),4/25/2022(-)
	0.025	Obs:287,Blocks:10,SIS:7,SIC:0.38 8/06/2018(-),1/27/2020(+),8/17/2020(+),1/04/2021(+),3/15/2021(+),10/04/2021(+),4/25/2022(-)
	0.05	Obs:287,Blocks:10,SIS:7,SIC:0.38 8/06/2018(-),1/27/2020(+),8/17/2020(+),1/04/2021(+),3/15/2021(+),10/04/2021(+),4/25/2022(-)
	0.10	Obs:287,Blocks:10,SIS:7,SIC:0.38 8/06/2018(-),1/27/2020(+),8/17/2020(+),1/04/2021(+),3/15/2021(+),10/04/2021(+),4/25/2022(-)

PERFORMANCE OF IIS TEST

A test for the general presence of outliers can be performed using the proportion or count of outliers found in regression models. Jiao and Pretis (2022) suggested two sets of tests for the overall presence of outliers under the null hypothesis of no outliers. Jiao-Pretis test tests if the proportion (or number) of outliers found using IIS differs from the proportion (or number) expected by the null hypothesis of no outliers. H_0: The proportion of outliers expected to be discovered under H_0 of no outliers is the same as the proportion of outliers spotted by IIS. Table 10 presents Jiao-Pretis Test p-values and the number of outliers by IIS. Based on the p-values given in table 10, we can assess the efficiency of IIS. Of 40 cases of detected outliers, 31 cases rejected the null of Jiao-Pretis. In 31 cases, IIS recognized the presence of an outlier but failed to reject it in only 9 cases. Of the 9 cases, 7 were identified when IIS was applied to log series, mostly at 0.001 significance level, where IIS recognized observations in 2015 and 2016 as downward outliers. However, IIS 78% discovered the presence of outliers. Rejecting the null hypothesis at a given significance level indicates the presence of outliers. However, non-rejection does not rule out that specific outliers denote actual outlying observations because the outlier magnitude is not considered when using this test (Jiao & Pretis 2022).

TABLE 8. IIS in original-series

Series	α	Original-Series (IIS)
BTCM	0.001	Obs:100, Blocks:4, IIS:25,SIC:22.10 2021(M4,5,6,7,8,9,10,11,12) , 2022 (M1,2,3,4,5,6,7,8,9,10,11,12), 2023 (M1,2,3,4) Note: All(+)
	0.01	Obs:100,Blocks:4,IIS:4,SIC:22.04 2021M03,2021M04,2021M10,2021M11All(+)
	0.025	Obs:100,Blocks:4, IIS:14,SIC:21.64 2021(M3,4,5,6,7,8,9,10,11,12),2022(M1,2,3,4) All(+)
	0.05	Obs:100, Blocks:4, IIS:16, SIC:21.59 2021(M3,4,5,6,7,8,9,10,11,12),2022 (M1,2,3,4,5),2023 (M04)All(+)
	0.10	Obs:100, Blocks:4, IIS:18, SIC:21.34 2021(M2,3,4,5,6,7,8,9,10,11,12),2022(M1,2,3,4,5),2023(M3,4)All(+)

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BTCW	0.001	Obs:435, Blocks:15,IIS:29, SIC:22.21 9/02/2021,9/09/2021,9/16/2021,9/23/2021,9/30/2021,10/07/2021,10/14/2021,10/21/2021,10/28/2021, 11/04/2021,11/11/2021,11/18/2021,11/25/2021,12/02/2021,12/09/2021,12/16/2021,12/23/2021,12/30/2021, 1/06/202, 1/13/2022,1/20/2022,1/27/2022,2/03/2022,2/10/2022,2/17/2022,2/24/2022,3/03/2022,3/10/2022, 3/17/2022All(+)
	0.01	Obs:435,Blocks:15,IIS:19,SIC:22.08 2/11/2021,3/04/2021,3/11/2021,3/18/2021,3/25/2021,4/01/2021,4/08/2021,4/15/2021,4/22/2021,4/29/2021, 9/30/2021,10/07/2021,10/14/2021,10/21/2021,10/28/2021,11/04/2021,11/11/2021,11/18/2021,11/25/2021All(+)
	0.025	Obs:435, Blocks:15,IIS:38, SIC:21.94 2/11/2021,2/18/2021,2/25/2021,3/04/2021,3/11/2021,3/18/2021,3/25/2021,4/01/2021,4/08/2021,4/15/2021, 4/22/2021,4/29/2021,5/06/2021,8/05/2021,8/12/2021,8/19/2021,8/26/2021,9/02/2021,9/09/2021,9/16/2021, 9/30/2021,10/07/2021,10/14/2021,10/21/2021,10/28/2021,11/04/2021,11/11/2021,11/18/2021,11/25/2021, 12/02/2021,12/09/2021,12/16/2021,12/23/2021,12/30/2021,1/06/2022,2/03/2022,2/10/2022,2/24/2022All(+)
	0.05	Obs:435,Blocks:15,IIS:44,SIC:21.89 2/11/2021,2/18/2021,2/25/2021,3/04/2021,3/11/2021,3/18/2021,3/25/2021,4/01/2021,4/08/2021,4/15/2021, 4/22/2021,4/29/2021,5/06/2021,8/05/2021,8/12/2021,8/19/2021,8/26/2021,9/02/2021,9/09/2021,9/16/2021, 9/23/2021,9/30/2021,10/07/2021,10/14/2021,10/21/2021,10/28/2021,11/04/2021,11/11/2021,11/18/2021, 11/25/2021,12/02/2021,12/09/2021,12/16/2021,12/23/2021,12/30/2021,1/06/2022,1/13/2022,2/03/2022, 2/10/2022,2/24/2022,3/03/2022,3/10/2022, 3/17/2022,3/24/2022All(+)
	0.10	Obs:435,Blocks:15,IIS:59,SIC:21.72 2/04/2021,2/11/2021,2/18/2021,2/25/2021,3/04/2021,3/11/2021,3/18/2021,3/25/2021,4/01/2021,4/08/2021, 4/15/2021,4/22/2021,4/29/2021,5/06/2021,5/13/2021,5/20/2021,5/27/2021,6/03/2021,6/10/2021,6/24/2021, 7/22/2021,7/29/2021,8/05/2021,8/12/2021,8/19/2021,8/26/2021,9/02/2021,9/09/2021,9/16/2021,9/23/2021, 9/30/2021,10/07/2021,10/14/2021,10/21/2021,10/28/2021,11/04/2021,11/11/2021,11/18/2021,11/25/2021, 12/02/2021,12/09/2021,12/16/2021,12/23/2021,12/30/2021,1/06/2022,1/13/2022,1/20/2022,1/27/2022,2/03/2022, 2/10/2022,2/17/2022,2/24/2022,3/03/2022, 3/10/2022,3/17/2022,3/24/2022,3/31/2022,4/07/2022,4/14/2022All(+)
ETHM	0.001	Obs:65,Blocks:3, IIS:0, SIC:17.02
	0.01	Obs:65,Blocks:3, IIS:3, SIC:16.81 2021M10,2021M11,2021M12All(+)
	0.025	Obs:65, Blocks:3, IIS:9, SIC:16.59 2021M08,2021M09,2021M10,2021M11,2021M12,2022M01,2022M02,2022M03,2022M04All(+)
	0.05	Obs:65, Blocks:3, IIS:9, SIC:16.59 2021M08,2021M09,2021M10,2021M11,2021M12,2022M01,2022M02,2022M03,2022M04All(+)
	0.10	Obs:65,Blocks:3,IIS:9,SIC:16.59 2021M08,2021M09,2021M10,2021M11,2021M12,2022M01,2022M02,2022M03,2022M04All(+)
ETHW	0.001	Obs:287,Blocks:10, IIS:29,SIC:16.88 9/27/2021,10/04/2021,10/11/2021,10/18/2021,10/25/2021,11/01/2021,11/08/2021,11/15/2021,11/22/2021, 11/29/2021,12/06/2021,12/13/2021,12/20/2021,12/27/2021,1/03/2022,1/10/2022,1/17/2022,1/24/2022,1/31/2022, 2/07/2022, 2/14/2022,2/21/2022,2/28/2022,3/07/2022,3/14/2022,3/21/2022,3/28/2022,4/04/2022,4/11/2022All(+)
	0.01	Obs:287,Blocks:10,IIS:12,SIC:16.85 10/11/2021,10/18/2021,10/25/2021,11/01/2021,11/08/2021,11/15/2021,11/22/2021,11/29/2021,12/06/2021, 12/13/2021, 12/20/2021,12/27/2021All(+)
	0.025	Obs:287,Blocks:10,IIS:22,SIC:16.79 5/03/2021,8/30/2021,9/27/2021,10/04/2021,10/11/2021,10/18/2021,10/25/2021,11/01/2021,11/08/2021,11/15/2021,11/22/2021,11/29/2021,12/06/2021,12/13/2021,12/20/2021,12/27/2021,1/03/2022,1/10/2022,1/31/2022, 3/21/2022, 3/28/2022, 4/04/2022 All(+)
	0.05	Obs:287,Blocks:10, IIS:27,SIC:16.76 5/03/2021,5/10/2021,8/30/2021,9/06/2021,9/27/2021,10/04/2021,10/11/2021,10/18/2021,10/25/2021,11/01/2021, 11/08/2021,11/15/2021,11/22/2021,11/29/2021,12/06/2021,12/13/2021,12/20/2021,12/27/2021,1/03/2022, 1/10/2022, 1/31/2022,2/07/2022,3/14/2022,3/21/2022,3/28/2022,4/04/2022,4/11/2022All(+)
	0.10	Obs:287,Blocks:10,IIS:40,SIC:16.66 4/26/2021,5/03/2021,5/10/2021,8/02/2021,8/09/2021,8/16/2021,8/23/2021,8/30/2021,9/06/2021,9/13/2021, 9/20/2021,9/27/2021,10/04/2021,10/11/2021,10/18/2021,10/25/2021,11/01/2021,11/08/2021,11/15/2021, 11/22/2021,11/29/2021,12/06/2021,12/13/2021,12/20/2021,12/27/2021,1/03/2022,1/10/2022,1/17/2022, 1/24/2022,1/31/2022,2/07/2022,2/14/2022, 2/21/2022,2/28/2022,3/07/2022,3/14/2022,3/21/2022,3/28/2022, 4/04/2022,4/11/2022 All(+)

TABLE 9. IIS in log-Series

Series	α	Log-Series (IIS)
LBTCM	0.001	Obs: 100, Blocks: 4, IIS: 25, SIC: 3.70 2015 (M1,2,3,4,5,6,7,8,9,10,11,12), 2016(M1,2,3,4,5,6,7,8,9,10,11,12), 2017 (M1) All(-)
	0.01	Obs: 100, Blocks: 4, IIS: 25, SIC: 3.70 2015 (M1,2,3,4,5,6,7,8,9,10,11,12), 2016(M1,2,3,4,5,6,7,8,9,10,11,12), 2017 (M1) All(-)
	0.025	Obs: 100, Blocks: 4, IIS: 25, SIC: 3.70 2015 (M1,2,3,4,5,6,7,8,9,10,11,12), 2016(M1,2,3,4,5,6,7,8,9,10,11,12), 2017 (M1) All(-)
	0.05	Obs: 100, Blocks: 4, IIS: 25, SIC: 3.70 2015 (M1,2,3,4,5,6,7,8,9,10,11,12), 2016(M1,2,3,4,5,6,7,8,9,10,11,12), 2017 (M1) All(-)
	0.10	Obs:100, Blocks:4, IIS:25, SIC:3.70 2015 (M1,2,3,4,5,6,7,8,9,10,11,12), 2016(M1,2,3,4,5,6,7,8,9,10,11,12), 2017 (M1) All(-)
LBTCW	0.001	Obs:435, Blocks:15, IIS:0, SIC:3.90
	0.01	Obs:435, Blocks:15, IIS:0, SIC:3.90
	0.025	Obs:435, Blocks:15, IIS:0, SIC:3.90
	0.05	Obs:435, Blocks:15, IIS:23, SIC:4.05 1/08/2015, 1/15/2015, 1/22/2015, 1/29/2015, 2/05/2015, 2/12/2015, 2/19/2015, 3/12/2015, 3/19/2015, 3/26/2015, 4/02/2015, 4/09/2015, 4/16/2015, 4/23/2015, 4/30/2015, 5/07/2015, 5/14/2015, 5/21/2015, 5/28/2015, 6/04/2015, 6/11/2015, 6/18/2015, 6/25/2015 All (-)
	0.10	Obs: 435, Blocks: 15, IIS: 50, SIC: 4.13 1/01/2015, 1/08/2015, 1/15/2015, 1/22/2015, 1/29/2015, 2/05/2015, 2/12/2015, 2/19/2015, 2/26/2015, 3/05/2015, 3/12/2015, 3/19/2015, 3/26/2015, 4/02/2015, 4/09/2015, 4/16/2015, 4/23/2015, 4/30/2015, 5/07/2015, 5/14/2015, 5/21/2015, 5/28/2015, 6/04/2015, 6/11/2015, 6/18/2015, 6/25/2015, 7/02/2015, 7/09/2015, 7/16/2015, 7/23/2015, 7/30/2015, 8/06/2015, 8/13/2015, 8/20/2015, 8/27/2015, 9/03/2015, 9/10/2015, 9/17/2015, 9/24/2015, 10/01/2015, 10/08/2015, 10/15/2015, 10/22/2015, 11/05/2015, 11/12/2015, 11/19/2015, 11/26/2015, 1/21/2016, 1/28/2016, 2/04/2016 All (-)
LETHM	0.001	Obs:65, Blocks:3, IIS:0, SIC:3.17
	0.01	Obs:65, Blocks:3, IIS:0, SIC:3.17
	0.025	Obs:65, Blocks:3, IIS:0, SIC:3.17
	0.05	Obs:65, Blocks:3, IIS:0, SIC:3.17
	0.10	Obs:65, Blocks:3, IIS:2, SIC:3.21 2021M10 (+), 2021M11 (+)
LETHW	0.001	Obs:287, Blocks:10, IIS:0, SIC:3.11
	0.01	Obs:287, Blocks:10, IIS:0, SIC:3.11
	0.025	Obs:287, Blocks:10, IIS:0, SIC:3.11
	0.05	Obs:287, Blocks:10, IIS:0, SIC:3.11
	0.10	Obs:287, Blocks:10, IIS:14, SIC:3.24 12/03/2018 (-), 12/10/2018 (-), 10/11/2021 (+), 10/18/2021 (+), 10/25/2021 (+), 11/01/2021 (+), 11/08/2021 (+), 11/15/2021 (+), 11/22/2021 (+), 11/29/2021 (+), 12/06/2021 (+), 12/13/2021 (+), 12/20/2021 (+), 12/27/2021 (+)

Multiple comparisons of IS and BP in all settings examined are shown and summarized in Table 11. Table 11 reveals that SIS test identified more breaks than the BP test from all perspectives. IS technique has the capacity to identify breaks and outliers utilizing SIS and IIS, respectively. The IS is sensitive to significant levels as its result increases with alpha and frequency. In contrast, the BP approach provides similar breaks across alpha and frequency but is sensitive to the original and log series. Detected outliers by IIS happen at every point in the sample, but BP test cannot detect shifts at the beginning or end of the sample. In all, the 10% significant level has the greatest impact on increasing the detectability of each test. Even though their overall performance varies, both tests provide dates for breaks corresponding to actual events.

From Table 11, there are 1,049 changes overall that were picked up by the two tests at various alpha levels, 945 of which were picked up by the IS approach. Of the 945 SIS detected 305 breaks in total across all settings, of which 212 were positive breaks in both the original and log series, while the remaining 93 were negative breaks. Of the 945 IIS detected 640 outliers in all settings, 440 of them were positive outliers in both the original and log series and the remaining 200 were negative outliers.

TABLE 10. Jiao-Pretis proportion test

Jiao-Pretis Proportion Test of IIS			
Series	α	Original-Series	Log-Series
BTCM	0.001	Ouliers:25,P-value:0.000	Ouliers:25,P-value: 0.000
	0.01	Ouliers:4,P-value:0.0004	Ouliers:25,P-value:0.000
	0.025	Ouliers:14,P-value:0.000	Ouliers:25,P-value:0.000
	0.05	Ouliers:16,P-value:0.000	Ouliers:25,P-value:0.000
	0.10	Ouliers:18,P-value:0.000	Ouliers:25,P-value:0.000
LBTCW	0.001	Ouliers:29,P-value:0.000	Ouliers:0,P-value:0.494
	0.01	Ouliers:19,P-value:0.000	Ouliers:0,P-value:0.01
	0.025	Ouliers:38,P-value:0.000	Ouliers:0,P-value:0.000
	0.05	Ouliers:44,P-value:0.000	Ouliers:23,P-value:0.68
	0.10	Ouliers:59,P-value:0.000	Ouliers:50,P-value:0.084
LETHM	0.001	Ouliers:0,P-value:0.7916	Ouliers:0,P-value:0.792
	0.01	Ouliers:3,P-value:0.0006	Ouliers:0,P-value:0.339
	0.025	Ouliers:9,P-value:0.000	Ouliers:0,P-value:0.087
	0.05	Ouliers:9,P-value:0.000	Ouliers:0,P-value:0.006
	0.10	Ouliers:9,P-value:0.0851	Ouliers:2,P-value:0.002
LETHW	0.001	Ouliers:29,P-value:0.000	Ouliers:0,P-value:0.578
	0.01	Ouliers:12,P-value:0.000	Ouliers:0,P-value:0.045
	0.025	Ouliers:22,P-value:0.000	Ouliers:0,P-value:0.000
	0.05	Ouliers:27,P-value:0.000	Ouliers:0,P-value:0.000
	0.10	Ouliers:40,P-value:0.000	Ouliers:14,P-value:0.000

TABLE 11. Comparison of IS and BP test

	Performance of BP and IS									
	Original-Series					Log-Series				
Alpha&Series										
BP-BTCM		2	2	2	2		3	3	5	5
BP-BTCW		2	2	2	2		5	5	5	5
BP-ETHM		2	2	2	2		4	4	4	4
BP-ETHW		3	3	3	3		4	4	4	4
SIS-BTCM	3	6	6	6	9	3	4	5	5	6
SIS-BTCW	9	11	14	15	16	11	13	15	15	15
SIS-ETHM	3	3	3	4	5	1	3	3	5	5
SIS-ETHW	8	8	8	12	12	7	7	7	7	7
IIS-BTCM	25	4	14	16	18	25	25	25	25	25
IIS-BTCW	29	19	38	44	59	0	0	0	23	50
IIS-ETHM	0	3	9	9	9	0	0	0	0	2
IIS-ETHW	29	12	22	27	40	0	0	0	0	14

CONCLUSION

The paper compared the performance of the BP and IS approaches in terms of detecting breaks and outliers in the prices of Bitcoin and Ethereum using multiple empirical comparisons. We considered different settings for the comparisons including two types of cryptocurrencies with different data frequencies, the log and original of each series, and 5 significant levels to compete the two approaches. The results show that, firstly, the BP test needs the user to determine the number of breaks; the maximum is five. However, the process will identify the breakpoint number in the data for the indicator saturation test. Secondly, the BP test will trim the data; thus, the test can fail in detecting any structural break at the beginning and end of the data. In contrast, the IS approach uses all the data to detect structural breaks and outliers. Thirdly, we observed that the IS approach successfully identified more breaks and outliers than the BP test, which only covered fewer breaks. Fourthly, the result of the IS approach increases with an increase in alpha and frequency while that of the BP test remains constant. Changes in the prices of crypto currencies can be attributed to a variety of factors, including market sentiment, the rate of inflation, rising supply and demand, and technological improvements. In addition, the 2017 landmark year for BTC, the 2020 and 2021 pandemic years, and the 2022 Ukraine war. Additionally, we demonstrated the effectiveness of IIS in identifying outliers using the Jao-Pretis outlier test. Hence, we showed the superiority of the IS approach based on the performance of the two tests, and we encourage using the IS approach to identify these features. The study is constrained by comparing only two tests and considering two types of cryptocurrency. Therefore, it is advised that future research attempt to generalize the findings of this study to other cryptocurrency types and consider other tests and higher frequency. Finally, empirical identification and analysis of data breaks and outliers are crucial for comprehending the dynamics of the cryptocurrency market. Breaks and outliers in the data can help analysts better understand what drives market movements, spot unusual market activity, control risk, and make more wise investment decisions. Comparing their findings is essential for identifying these traits, for quality control in industries, for setting price targets, and for confirming trading signals to reduce potential losses.

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