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Dual Boundary Element Method in Modeling of Fatigue Crack Propagation

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ABSTRACT

This paper deals with the modelling of fatigue crack propagation on a centre member bar using a dual boundary element method. The effects of life cycle to the multiple site fatigue crack propagation were studied. Analysis of stress intensity factor was performed by the deterministic approach using a dual boundary element method. The dual boundary element method was used to simplify the crack model through the numerical approach. The complex problems have been solved using the information from a boundary condition only. Next, the initial crack and life cycle of the structure have been predicted using probabilistic method which is Monte Carlo. The crack size and fatigue life were computed until failure of the structure. The failure analysis was performed by a linear elastic fracture mechanics. The scenarios of the fatigue crack propagation were given by an integration of both dual boundary element and Monte Carlo method. Therefore, fatigue life of multiple site crack structure can be predicted.

Keywords: Crack propagation, fatigue, Monte Carlo, boundary element method.

ABSTRAK

Kertas kerja ini mempersembahkan permodelan perambatan retak lesu terhadap komponen rasuk tengah menggunakan kaedah dwi-unsur sempadan. Kesan kitaran hayat bagi perambatan retak lesu berbilang tempat dikaji. Analisis faktor penumpuan tegasan dilakukan berdasarkan pendekatan ketentuan iaitu menggunakan kaedah dwi unsur sempadan. Kaedah dwi unsur sempadan berperanan untuk memodelkan retak secara pengiraan berangka dengan permasalahan kompleks diselesaikan secara pengiraan di sempadan model. Seterusnya, penjanaan retak awal dan kitar hayat sesuatu struktur dianggarkan melalui kaedah kebarangkalian iaitu Monte Carlo. Saiz retak dan tempoh hayat lesu struktur dikira sehingga kegagalan struktur berlaku. Analisis kegagalan dilakukan berdasarkan sifat mekanik patah elastik linear. Senario *perambatan retak lesu diperolehi hasil integrasi antara kaedah dwi elemen sempadan dengan Monte Carlo. Oleh itu, hayat lesu retak berbilang tempat bagi struktur yang dikaji dapat dianggarkan.*

Kata Kunci: Perambatan retak, lesu, Monte Carlo, kaedah unsur sempadan.

INTRODUCTION

In the area of fatigue reliability, an estimation of probability of failure is required. This is caused by uncertainties in initial crack, surface roughness, material property, applied load, flaw, defects such as scratches or weld defects from manufacturing process (Yang et. al, 1988). In other words, as the crack grows, the crack size has a variation according to those uncertainties and the residual life of the structure is not deterministic but stochastic. Fatigue crack propagation is inherently a random process because of the inhomogeneous of material, connected with its crystal structure and with variations of convective film coefficient at the structures surface due to it non-smoothness and other similar reasons (Cherniavsky, 1995). The experimental results for the fatigue crack growth under constant amplitude loading show that the material resistance against crack propagation has the inter-specimen as well as the intra-specimen variability. A stochastic model considering both types of variability is thus needed for the rational assessment of fatigue crack propagation. Therefore, the analysis of fatigue crack propagation should be based on the probabilistic approach and the inspection interval or the repair method must be determined according to the possibility of structural failure considering the uncertainties mentioned above.

This paper presents the development of an inspection programme for the fatigue crack propagation which is an enhancement of an earlier programme (Kebir et. al., 2001), and the major differences between these two programmes are summarized below:

1. The crack propagation has been modelled using BEM principal of Beasy software with the combination of random function of Matlab software.
2. The life cycle of a centre member bar with more than one notch has been analyzed using dual boundary element and Monte Carlo.

Boundary Element Method (BEM)

Two-dimensional numerical stress analysis was carried out using the boundary element method. BEM is well-suited for the complex problem like crack by modeling only the boundaries of the part. In order to derive element stiffness matrix for a cracked domain, a method based on dual boundary element has been adopted in which requires two equation. Boundary displacement and traction are the fundamental of crack characteristic and the relation between the characteristics are shown below:

$$u_i\left(x'\right) = \int_{\Gamma} U_{ij}\left(x',x\right) t_j\left(x\right) d\Gamma\left(x\right) - \int_{\Gamma} T_{ij}\left(x',x\right) u_j\left(x\right) d\Gamma\left(x\right) \quad (1)$$

where $T_{ij}(x',x)$ and $U_{ij}(x',x)$ in Equation (1) is a Kelvin traction and displacement for x point field in Γ domain. A distance between the source point and field point is presented by $|x - x'|$.

The internal or edge surfaces which are included no area or volume and across with the displacement field have been defined as mathematical discontinuous cracks. For symmetric crack problems, only one side of the crack need to be modelled and a single-region boundary element analysis may be used. However, a solution of general crack problems cannot be achieved in a single-region analysis with the direct application of the boundary element method because coincidence of the crack boundaries that give rise to a singular system of algebraic equations. The equation for a point located at one of the boundaries of the crack is identical to those equations for the point with the same coordinates but on the opposite surface. This is because the same integral equation was collocated with the same integration path, at both coincident points.

The Langrarian continuous or discontinuous boundary elements are used to satisfy Cauchy principle value integral which is defined as a displacement equation. The Hadamard principle value integral transforms the discontinuous element to the continuity requirement for the finite-part integral. The discontinuous element is defined from all nodes which is an internal point. Traction equation is then defined from the Hadamard principal value integral. The principal value integral has been performed to impose the dual boundary integral equation. By the changing of the discontinuous quadratic elements, crack modelling is presented in the J-integral function as given by:

$$J = \left(Wn_1 - t_j u_{j,1}\right) ds \tag{2}$$

where s is an arbitrary contour surrounding the crack tip; W is the strain energy density, given by $1/2\sigma_{ij}\varepsilon_{ij}$, where σ_{ij} and ε_{ij} are the stress and strain tensors, respectively; t_j are traction components, given by $\sigma_{ij}n_i$, where ni are the components of the unit outward normal to the contour path. The relationship between the J-integral and the first and second level stress intensity factor, K_I and K_{II} is given by:

$$J = \frac{K_I^2 + K_{II}^2}{E'} \tag{3}$$

where E' is the elasticity modulus for plane stress conditions and $E' = E/(1-\nu v^2)$ for plane strain conditions. The algorithm of the boundary element method was shown below:

- Carry out a dual boundary element method, stress analysis of the structure has been done.
  - o Stress intensity factors have been computed by the J-integral technique.
    - - Compute the direction of the crack-extension increment.
  - o Extend the crack one increment along the direction computed in the previous step.
- Repeat all the above steps sequentially until a specified number of crack-extension increments were achieved.

LAW OF FATIGUE CRACK PROPAGATION

In 1963, Paris and Erdogan created a Paris law equation for calculating fatigue crack propagation rate, da/dN as given in Equation (4).

$$\frac{da}{dN} = C(\Delta K^m) \tag{4}$$

$\Delta K = K_{max} - K_{min}$ is the range of stress concentration factor and C and m are the material properties.

The stress concentration factor is one of the parameters that are considered in linear elastic fracture mechanic. The theory only acceptable for the situation when there is no yield occurs at the crack tip. Therefore, Equation (4) can be used for high cyclic fatigue cases. Forman et al (1967) tried to modify the Equation (4) as it is not include the stress concentration ratio, $R = K_{min}/K_{max}$ and the fracture strength, K_C. From the definition of $\Delta K = K_{max}(1 - R)$ and $K_{max} = K_c$, the boundary condition for the crack propagation rate is:

$$\lim_{\Delta K \to (1-R)K_c} \frac{d}{dN} = \infty \tag{5}$$

Substituting Equation (5) in Equation (4) gives:

$$\frac{da}{dN} = \frac{C(\Delta K)^m}{(1-R)K_c - \Delta K} \tag{6}$$

From Equation (6), Forman (1967) found that the m value for aluminium alloy *7075-T6* and 2024-T3 was 3. Equation (6) is known as Forman equation. Starting from the Forman equation and considering the crack will not propagate if the ΔK value below ΔK_{th} (Figure. 1), a growth law was introduced as in Equation (7) to calculate the fatigue crack propagation rate.

$$\frac{da}{dN} = C\left(\frac{\Delta K - \Delta K_{th}}{K_c - K_{\max}}\right)^2 + C' \tag{7}$$

This model is valid for a soft metal that under both the fix and random loading amplitude, which C' is approaching to $2.4\text{x}10^{-7}$mm/cyclic.

Crack behaviour is diagnosed by the crack growth process, which are the function of an applied load and geometry of the crack structure. The crack growth process is formed by crack extension behaviour.

Crack behaviour is predicted based on stress intensity factor. This analysis also call damage tolerance analysis is developed based on linear elastic fracture mechanics theory.

LINEAR ELASTIC FRACTURE MECHANICS

Fracture mechanics seek to establish the local stress and the strain fields around a crack tip in term of global parameters such as the loading and the geometry of the structure. Linear elastic characteristic is used in crack path model. The long cracks have been modelled subjected to constant amplitude cyclic loading. For linear elastic solution, the stress in the vicinity of the crack is evaluated by stress intensity factors.

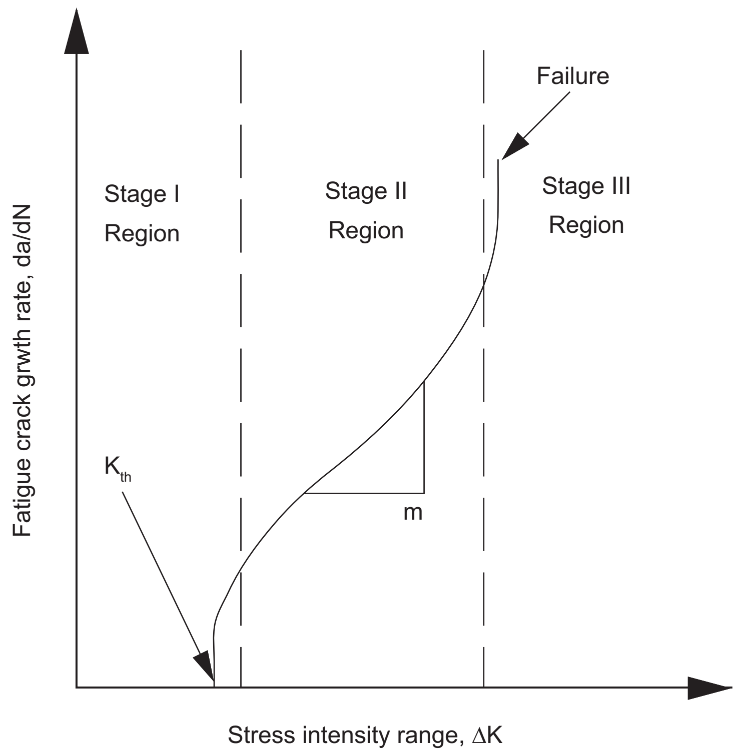

FIGURE 1. Scheme diagram of short and long fatigue crack propagation (Dharani 2001)

S-N curve assumed the fatigue life average; N_i at certain point for 2024-T3 aluminium alloy is illustrated as below:

$$Ni = 10^5 \left(\frac{S_m - S_{lim}}{IQF - S_{lim}} \right)^p \quad (8)$$

where $P = 2.28$, IQF =176 MPa, S_{lim} = 59 MPa and S_m = the average stress.

In linear elastic fracture mechanics, there are several mixed-mode propagation criteria. One of them is the stress intensity factor, K_i which controls the near tip stress field. Magnitude of the crack tip stresses is governed by the stress intensity factors, K_I and K_{II} as shown in Equation (9). It is also observed that the displacements are controlled by the stress intensity factors as shown in Equation (10). The distribution of the stresses is governed by the position relative to the crack tip given by r and θ. Here μ is the shear modulus of metal alloy material.

The two-dimensional numerical stress analysis has been carried out using the boundary element method. BEM is well suited for complex modelling case such as crack problems by modelling only the boundaries.

CRACK MODELING STRATEGY

The domain region has been treated as dual boundary element by Boundary Element System (BEASY) software. It is necessary to calculate the related stiffness matrix and effective stress intensity factor, Keff by means of Dual Boundary Elements Method (DBEM). The crack modelling strategy shown by algorithm below:

- Carry out a dual boundary element method for stress analysis of the structure.
- Compute the effective stress intensity factors, K_{eff} with the *J*-integral technique.
- Compute the direction of the crack-

$$\sigma_{11} = \frac{K_I}{\sqrt{2\pi r}} cos\frac{\theta}{2}\left(1 - 2\,sin\frac{\theta}{2}\,sin\frac{3\theta}{2}\right) - \frac{K_{II}}{\sqrt{2\rho r}}\,sin\frac{\theta}{2}\left(2 + cos\frac{\theta}{2}\,cos\frac{3\theta}{2}\right) + O(r^{1/2}) \quad (9)$$

$$\mu = \frac{1}{4u}\sqrt{\frac{r}{2\pi}}\left[K_I\left(\left(2k-1\right)cos\frac{\theta}{2} - cos\frac{3\theta}{2}\right) - K_\Pi\left(\left(2k+3\right)sin\frac{\theta}{2} + sin\right)\frac{3\theta}{2}\right] \quad (10)$$

extension increment.

- Extent the crack one increment along the direction computed in the previous step.
- Repeat all the above steps sequentially until a specified number of crack-extension increments have been achieved.

The boundary stiffness matrix and K_{eff}, after condensation, have been inserted into crack initial and propagation routine using Monte Carlo method which is provided by MATLAB source code. Using the *S-N* curve at 50% includes the deterministic approach.

By running a Monte Carlo method using MATLAB program, it possible to calculate the cycle number for each of the propagation and the crack length. The initial point also indicated by random process as shown in Figure 2. The modified data files in BEASY has been run to update the crack parameter.

NUMERICAL RESULTS OF PLATE 14 HOLES

In order to validate the global probabilistic approach, the results have been compared with the fatigue test on a plane plate with 14 free holes that was conducted by Kebir H. et. al. (2001) at Aerospatiale-Matra laboratory in Suresnes, France as shown in Figure 3.

The testing was conducted using aluminum alloy 2024-T3 sheets with a thickness of 1.6 mm. The load has been applied on transversal direction. The Young modulus of the sample was 72.7 GPa. The initial structure has been discreet with 262 elements, in one zone with 1202 degrees of freedom. It has 897 internal points

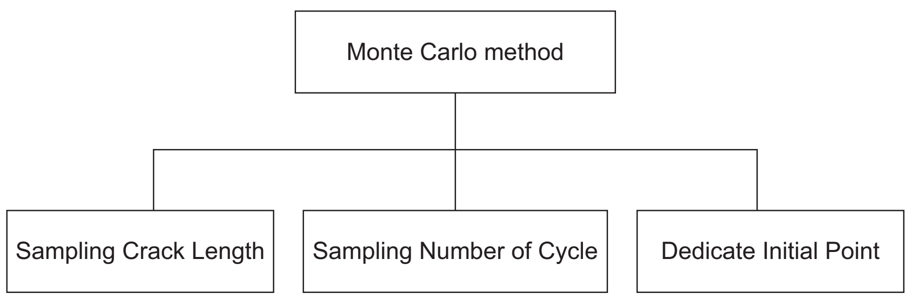

FIGURE 2. Random parameter for fatigue crack propagation

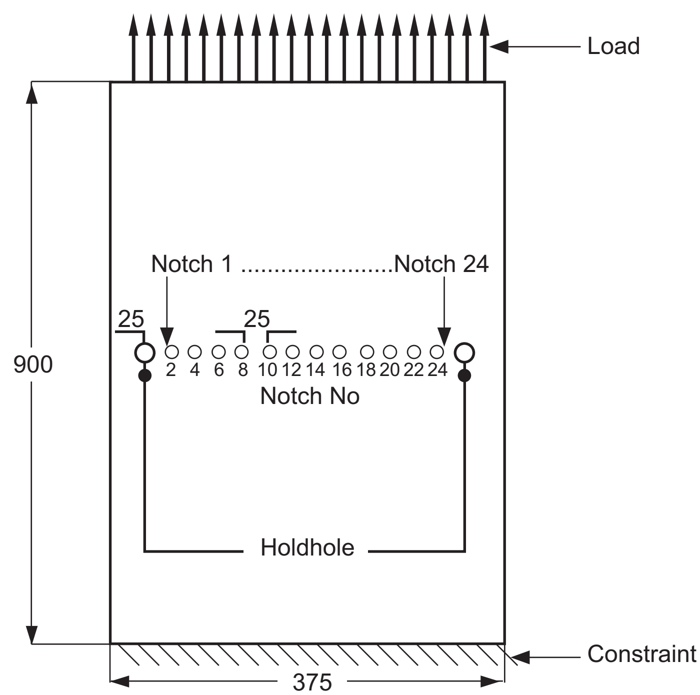

FIGURE 3. Schematic diagram of plate 14 holes

patch in the model. The numerical results have a good compromise between the test results. The total numbers of cycles with the probabilistic approach are closely similar to the test expressed in Figure. 4.

process. In a long duration time, the small cracks size may have the most dominant effect on the failure probability.

Table 1 shows a maximum crack length, 3.5293 mm before the component is failure. At this

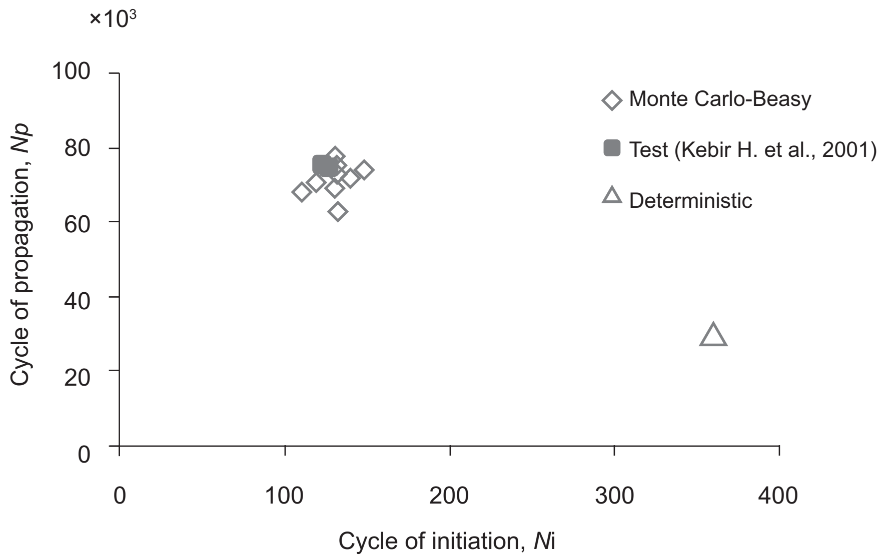

FIGURE 4. Fatigue prediction life

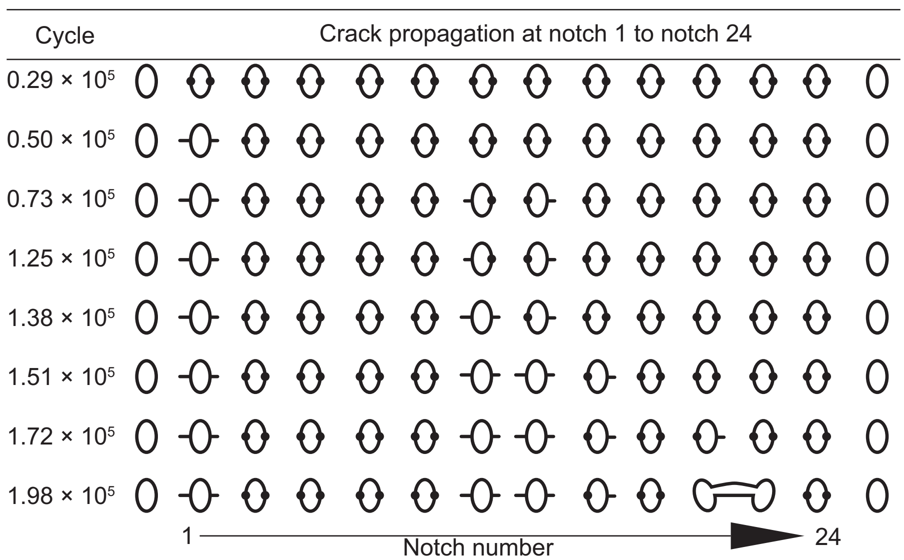

FIGURE 5. Life cycle of fatigue crack propagation by iterations

The synthesis of the probabilistic results is expressed in Figure 5. In the deterministic approach, the propagation phase was short with is 30×10^3 cycle. It was because all the cracks assumed begin at the same time, since all the sites were undergoing the same stress level. So, the probabilistic approach has an advantage of giving the view of initial crack propagation. A large crack size has been dominated the failure probability at the beginning of the failure

moment, the life cycle only $0.7354\text{x}10^5$ cycles as presents in 7th iteration. This happened because a crack notch 14 had enough energy to propagate an initial crack. The crack growth very fast and verifies the high propagation rate. However, the failure of the sample has been not happened yet until the life cycle reached at $1.9825\text{x}10^5$ cycles. The notch is randomly propagated the initial crack appropriate with the increase of stress intensity factor value. The stress intensity factor

TABLE 1. Results of fatigue crack propagation

Iteration	Crack Length	Cycle, N_{Total} (10^5)	Point No.
1	0.0056	0.2957	7
2	0.1702	0.3058	2
3	0.1034	0.3985	1
4	0.3706	0.4319	1
5	0.2498	0.5012	1
6	0.2077	0.6541	11
7	3.5293	0.7354	14
8	0.2219	1.2595	14
9	0.2557	1.3871	12
10	0.6363	1.4735	13
11	0.1043	1.5108	16
12	0.1557	1.7244	21
13	Failure	1.9825	21

value is constantly increased for a few iterations until it is slowly trended to achieve a maximum value. For the fourteen holes plate, notches 1, 2, 11, 12 and 14 have been chosen for propagates an initial crack as shown in Figure 6. The increasing was continuing for certain iterations. After that, the crack has been randomly propagated at other notch, which had lower stress intensity factor. In this scenario, the notch 6, 15, 16, 17, 18 have been randomly propagated the crack. The crack has been continuing propagate for a certain iterations to get close with the maximum stress intensity factor at that time. The increased of the life cycle was continuing the crack propagation at high probability location randomly. At this moment, the crack propagation can make the sample fail. Notch 21 was having a catastrophic failure when its effective stress intensity factor reached the value of 276, 659.75 MPa m. The high potential energy has been assembles at a low stress intensity factor notch and catastrophic failure was occurred because of the high grow crack propagation rate.

The result of the crack propagation base on the component life cycle was shown that the crack propagates in three phases. The first phase started at 70 000 cycles and finished at 460, 000 cycles. At this phase, each of the iteration causes of 0.007 cm crack length which is considered as a minor crack.

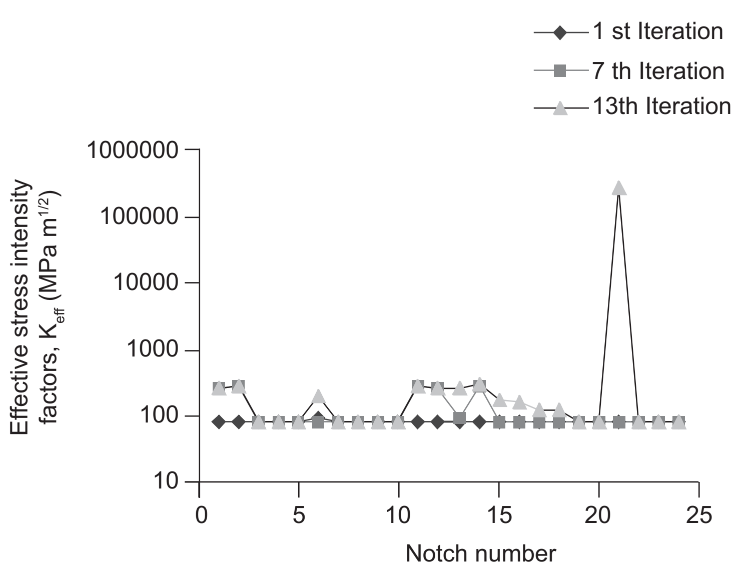

FIGURE 6. Graph of effective stress intensity factor versus notches number for 1st, 7th and 13th iteration

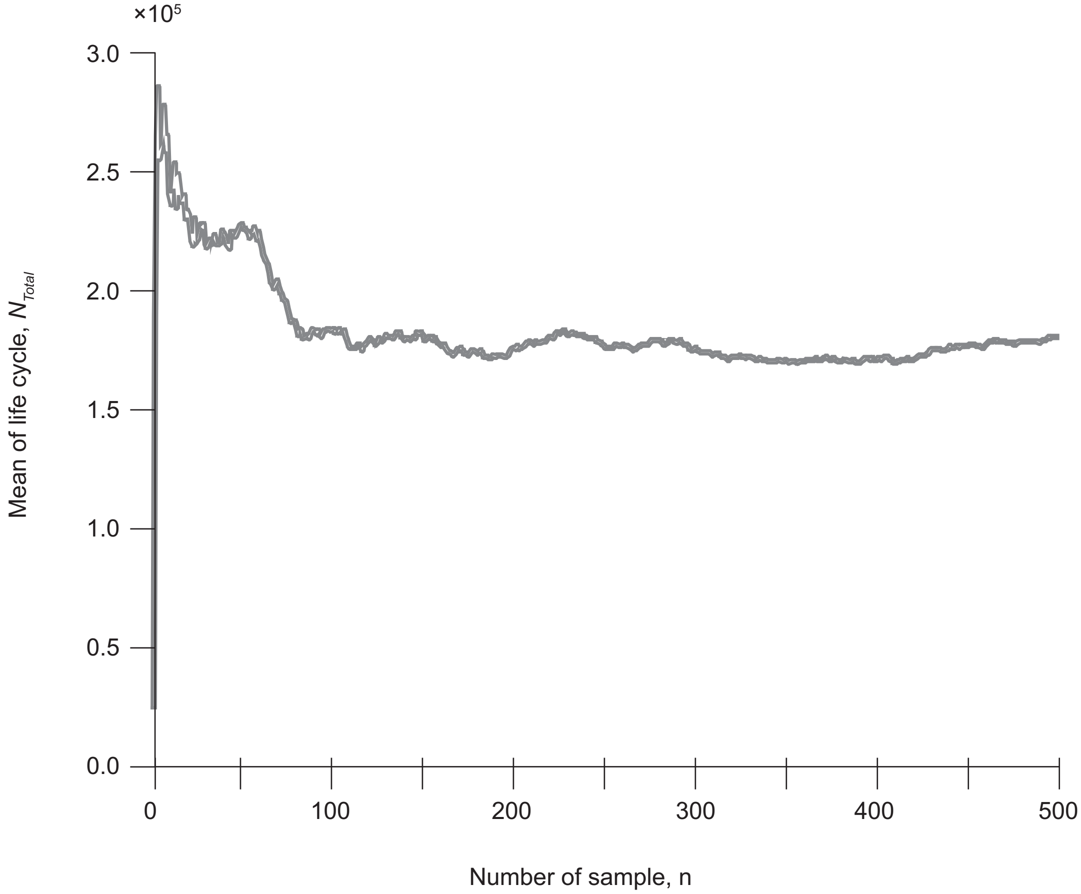

FIGURE 7. Mean life cycle versus number of sample for 13th iteration

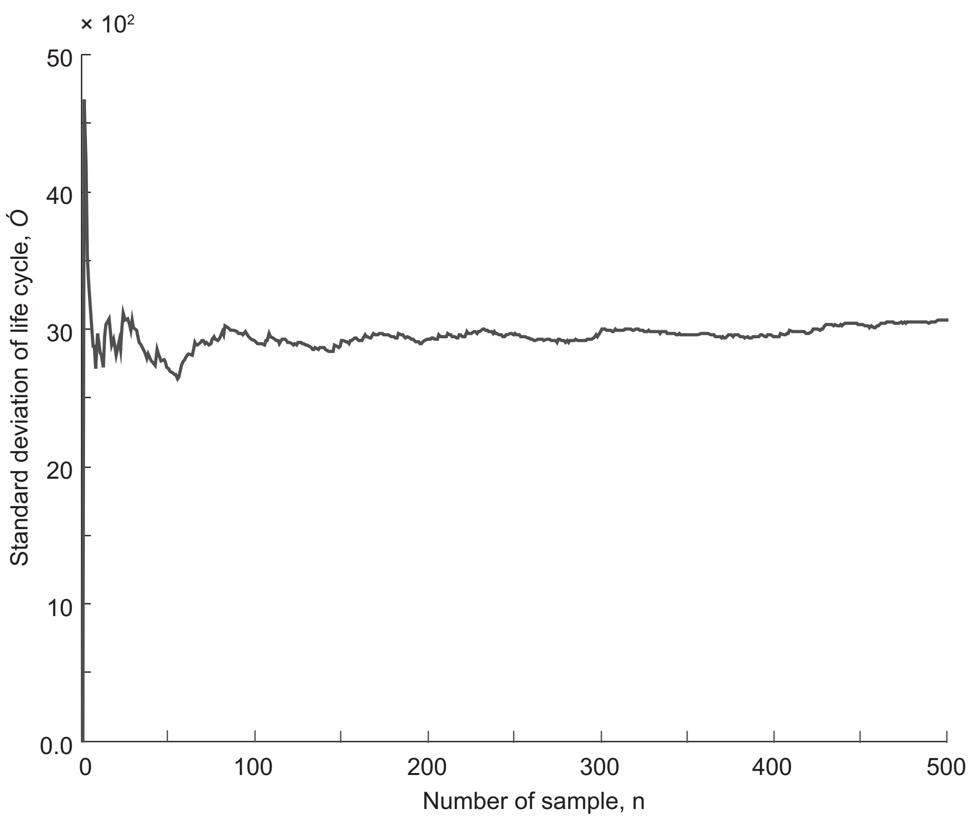

FIGURE 8. Standard deviation of life cycle versus number of sample for 13th iteration

The second phase was started at 460, 000 cycles. Between 460, 000 and 750, 000 cycles, a major crack was propagated. The size is 0.1 cm for each life cycle's iteration.

For the third phase, minor crack propagation is occurred as in first phase. However, the third phase was too short which only 200, 000 cycles. The component totally failed after 970, 000 cycles.

Figure 7 and Figure 8 show a mean life and a standard deviation prediction for a 13th iteration by the effect of number of samples. It is seen that the number of samples influences the fatigue life cycle. The results are constant when the number of samples is over 300. So, the Monte Carlo-BEM statistical test is only valid with the high number of samples. The mean life and a standard deviation prediction have given the same trend result like the 13th iteration.

NUMERICAL RESULTS OF CENTER MEMBER BAR

The boundary element method has been applied to a centre member bar. Four holes with notches have been modelled as shown in Figure 9 and 10. The location of the notches based on the high force concentration and high probability of the

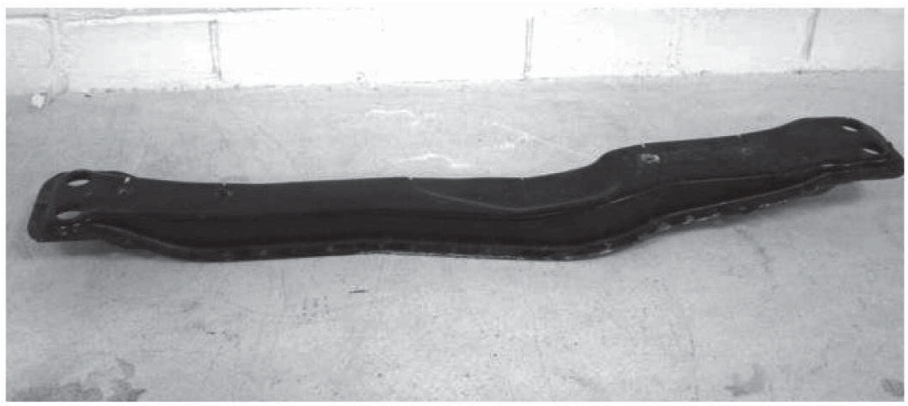

FIGURE 9. A photograph of a centre member bar

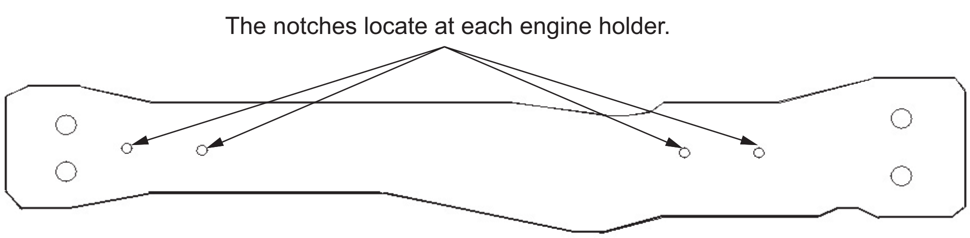

FIGURE 10. Top view of the centre member bar

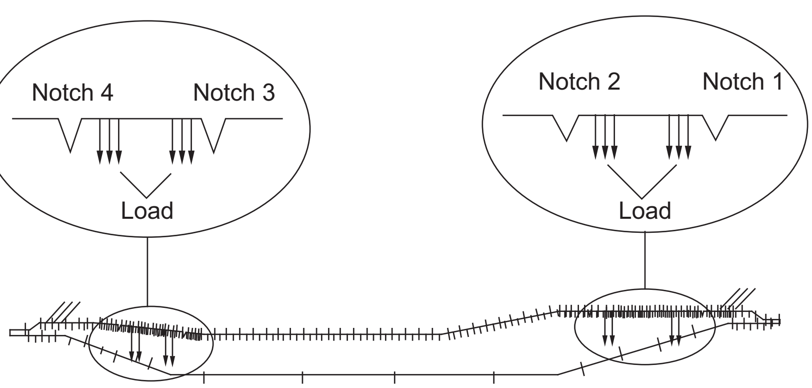

FIGURE 11. Boundary elements of the car centre member bar

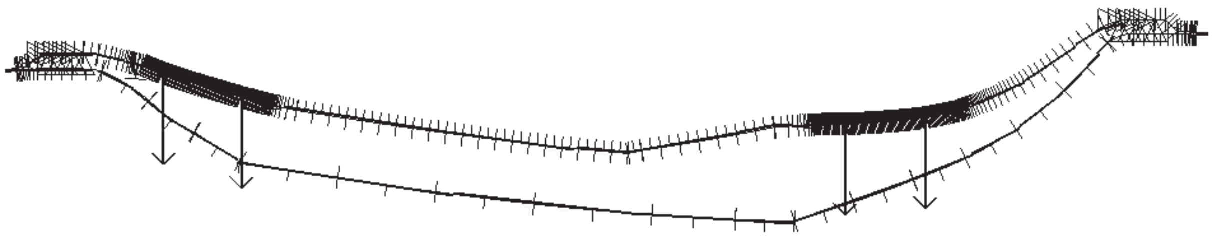

FIGURE 12. Component deformation at 18 iterations

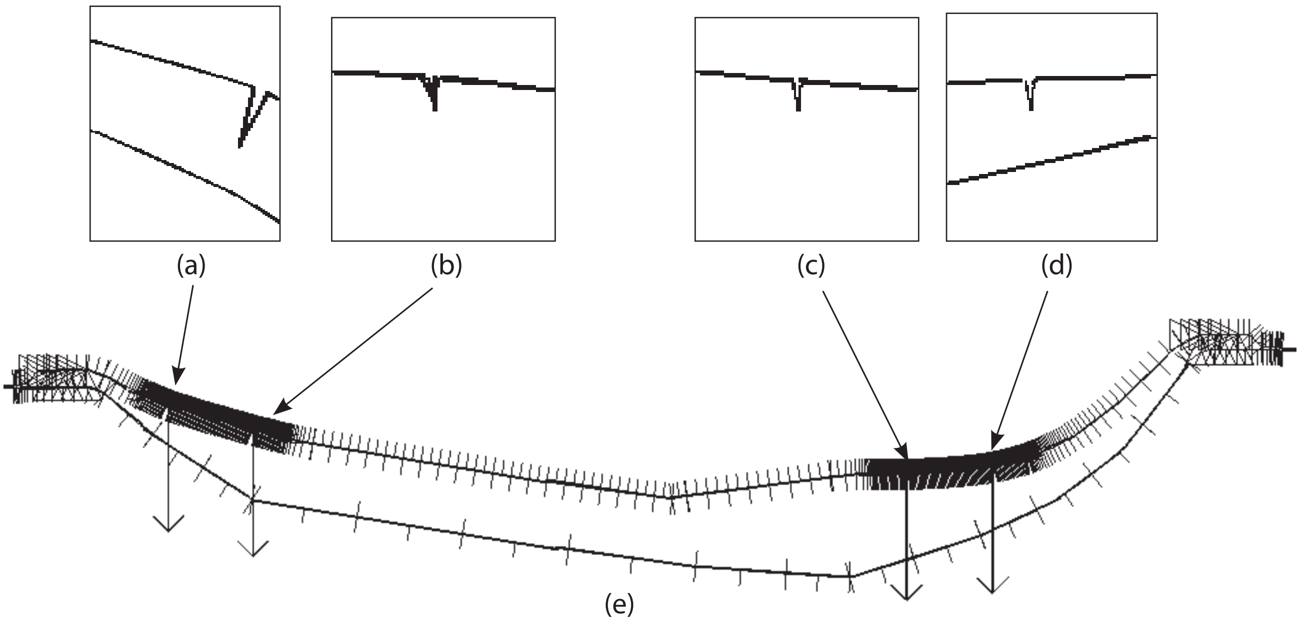

FIGURE 13. Crack at (a) notch 4 (b) notch 3 (c) notch 2 and (d) notch 1 (e) Component deformation at 29 iterations

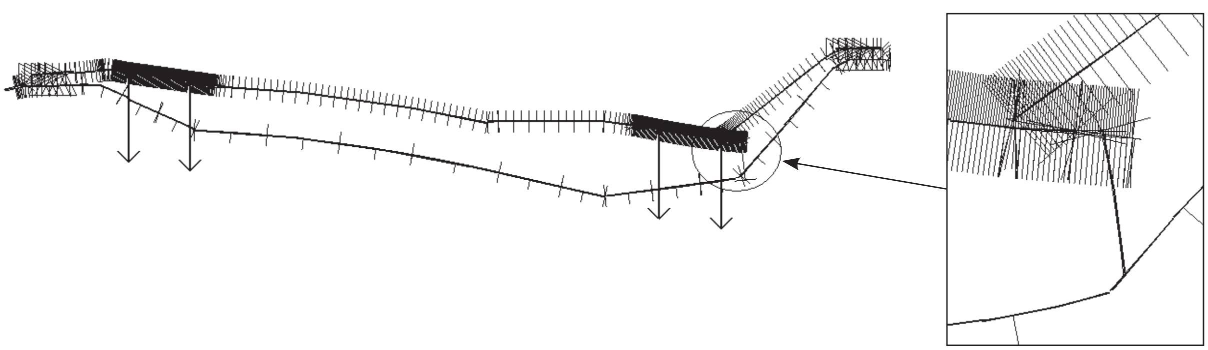

FIGURE 14. Failure of the component after 34 iterations

crack propagation. The centre member bar was made by steel with Young Modulus of 200 GPa. The fatigue load is 450-600 MPa based on the engine load.

The elements have been generated only at the boundary as shown in Figure 11. With the load given, the elements of the component have displaced as shown in Figure 12. The cracks have been propagated at all notches as shown in Figure 13. Finally the failure occurred at 970,000 cycles after 34 iterations as shown in Figure 14.

CONCLUSION

An overall assessment method proposed in this paper was developed in order to validate the fatigue crack propagation with the probabilistic method through the Monte-Carlo. The modeling process was using the dual boundary element method. The results of the boundary element method and Monte Carlo analysis show that the life cycle of structure can be predicted and obtained in good agreement with the experiment results. The results obtained prove that the computer simulation can be used to predict fatigue crack propagation. The proposed algorithm can be used for a guideline to have a risk and reliability analysis and life expectancy of the structure.

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