# The Effect of Isotropic Roughness on the Thermohydrodynamic Solution of Finite Journal Bearings

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#### ABSTRACT

Thermohydrodynamic (THD) analysis of finite journal bearings with isotropic roughness is presented. A modified version of the Reynolds equation which includes surface roughness effects is solved using the Successive-over-Relaxation method (SOR). The Swift-Stieber cavitation boundary condition was implemented in solving the Reynolds equation. The effects of various operating parameters as well as roughness parameters on the thermohydrod-ynamic finite journal bearing are investigated and the results are compared with the smooth surface solutions. Along with the bearings geometries, the roughness characteristics, such as the variance ratio, should be included as design parameters in order to accurately predict the performance of hydrodynamic bearings.

#### ABSTRAK

Analysis termohidrodinamik (THD) pada galas jurnal terhad dengan kekasaran isotropik dibentangkan. Persamaan Reynolds yang telah diubahsuaikan dengan memasukkan kesan kekasaran permukaan diselesaikan dengan menggunakan kaedah "Successive over Relaxation" (SOR). Keadaan sempadan Swift-Stieber telah digunakan bagi menyelesaikan Persamaan Reynolds' tadi. Kesan-kesan parameter-parameter pengoperasi dan parameter kekasaran ke atas galas jurnal terhad thermohidrodinamik disiasat dan keputusannya dibandingkan dengan hasil selesaian permukaan yang licin. Bersama geometri-geometri galas, ciri-ciri kekasaran, seperti nisbah variasi, seharusnya dijadikan parameter-parameter rekabentuk bagi meramalkan dengan tepat prestasi galas hidrodinamik.

#### INTRODUCTION

The study of surface roughness effects and thermal effects on lubrication has gained considerable attention in recent years. Surface roughness and thermal effects influenced the solution for pressure and leakage in hydrodynamic bearings. Many researchers have reported their prediction on the effect of surface roughness in hydrodynamic bearings. Tzeng and Saibel (1967) have introduced stochastic concept and applied it to a slider bearing with one-dimensional transverse roughness. Christensen and Tonder (1970, 1971, 1973) used the stochastic methods to obtain an average Reynolds' equation. Patir and Cheng (1978) introduced the "average flow model" and obtained an average Reynolds equation in terms of the pressure and shear flow factors.

However, in the previously mentioned literature, the lubricant viscosity is assumed constant. Thermal inhomogenity in hydrodynamic lubricating film has serious effects on bearing perfomance. A theoretical model which includes temperature and viscosity variation in the lubricant film is generally referred to as the thermohydrodynamic model (THD). Among the first to realize the importance of thermal effects was Cope (1949). However, Cope's assumption that the lubricant viscosity across the film is constant is not right. An experimental investigation of the thermal effects on bearing perfomance parameters in the journal bearings was reported by Dowson, Hudson, Hunter, and March (1966). The results showed that the shaft can be treated as an isothermal component and the axial temperature gradients within the bush were negligible. The results revealed by this major experimental study were used by other researchers. A two dimensional temperature and viscosity profile (along and across the lubricant film) is adequate for a successful THD model of journal bearing. Eventhough a full THD solution with the heat conduction to the bush would give more practical solutions, this case consumed a lot of computer time. A simplified boundary condition referred to as the ISOADI (Isothermal Shaft and Adiabatic bush inner surface) used by Khonsari et al. (1986) is used in this analysis.

The previous THD analyses were made by assuming that the bearings surfaces are smooth. In order to accurately predict the bearing performance, the combination of both surface roughness and thermal effects are essential. In this paper, the effects of isotropic surface roughness on THD solution of finite journal bearings are presented and results are compared with the experimental findings. In this analysis, the ISOADI boundary condition is used.

### **GOVERNING EQUATIONS**

The average flow model developed by Patir et al. (1978) is used in analyzing the roughness effects. Referring to Figure 1 and assuming steady-state flow with incompressible Newtonian fluid, the governing equations can be written as follows.

THE NON-DIMENSIONAL AVERAGE REYNOLDS EQUATION

The non-dimensional average Reynolds equation is given by:

$$\begin{split} \varphi_{x} \frac{\partial^{2} \bar{P}}{\partial \theta^{2}} + \left(\frac{R}{L}\right)^{2} \theta_{y} \frac{\partial^{2} \bar{P}}{\partial \bar{Y}^{2}} + \frac{\partial \bar{P}}{\partial \theta} \left[ \varphi_{x} + \frac{3\bar{h}'}{\bar{h}} \varphi_{x} - \frac{\varphi_{x}}{\bar{\mu}^{2}} \frac{\partial \bar{\mu}}{\partial \theta} \right] \\ &= \frac{3\bar{h}}{\bar{h}} \bar{\mu} \left\{ 1 + \text{erf} \left[ \frac{\Lambda \bar{\mu}}{\sqrt{2}} \right] \right\} + \frac{6\bar{\mu}}{\bar{h}} \Lambda \varphi_{s} \quad (1) \end{split}$$

where  $\theta$  and Y are non-dimensional coordinates, h is the dimensionless film thickness,  $\phi_x$  and  $\phi_y$  pressure flow factors, and  $\phi_s$  is the shear flow factor. The flow factors are introduced by Patir et al. (1978) and is given by:

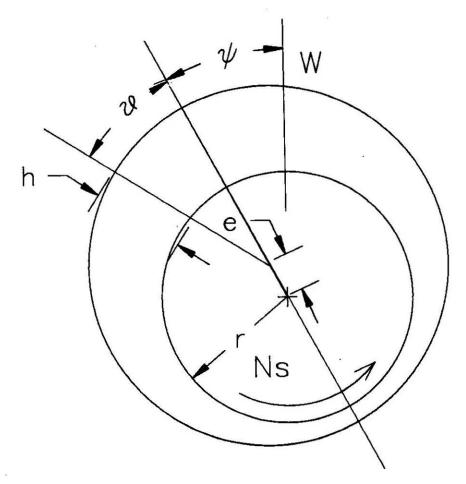


FIGURE 1. Diagram of journal bearing with exaggerated clearance

$$\phi_x = 1 - C_1 e^{-r(h/\sigma)} \text{ for } \gamma \le 1$$
 (2)

$$\phi_{x} = 1 - C_{1}(h/\sigma)^{-r} \text{ for } \gamma > 1$$
 (3)

$$\phi_{y}\left(\frac{h}{\sigma},\gamma\right) = \phi_{x}\left(\frac{h}{\sigma},\frac{1}{\gamma}\right) \tag{4}$$

$$\phi_s = A_1(h/\sigma)^{\alpha_1} e^{-\alpha_2(h/\sigma) + \alpha_3(h/\sigma)_2} \text{ for } h/\sigma \le 5$$
 (5)

$$\phi_s = A_2 e^{-0.25(h/e)} \text{ for } h/\sigma > 5$$
 (6)

where the coefficients  $C_1$ , r,  $A_1$ ,  $A_2$ ,  $\alpha_1$ ,  $\alpha_2$ ,  $\alpha_3$  are given by Patir et al. (1978)

#### **BOUNDARY CONDITIONS I**

The Reynolds or Swift-Steiber boundary conditions are:

$$\left. \vec{P} \right|_{x = \theta_{inlet}} = \left. \vec{P} \right|_{x = \theta_{inlet}} + z_z = \left. \vec{P}_s \right.$$

Cavation boundary conditions are:

at 
$$\theta = \pi + \theta_2$$
  $\frac{\partial \bar{P}}{\partial x} = 0$   
and  $\bar{P}_{cav} = \bar{P}_A = 0$ 

where  $\theta_2$  is the angle measured from  $h_{min}$  to the point of film rapture and  $P_{cav}$  is the non-dimensional caviation pressure.

#### NON-DIMENSIONAL ENERGY EQUATION

The energy equation in non-dimensional form is given by:

$$\begin{split} \overline{h}\overline{u}\,\frac{\partial\overline{T}}{\partial\varphi}\,+\,\overline{h}\left(\overline{w}\,-\,\overline{u}\,\overline{z}\,\frac{\partial\overline{h}}{\partial\theta}\right)&\frac{\partial\overline{T}}{\partial\bar{z}}\,=\,G_1\,\frac{\partial^2\overline{T}}{\partial\bar{z}^2}\,+\\ G_2\overline{\mu}\left[\left(\frac{\partial\overline{u}}{\partial\bar{z}}\right)^2\,+\left(\frac{\partial\overline{v}}{\partial\bar{z}}\right)^2\right] \end{split} \tag{7}$$

where

$$G_1 = \frac{k_0 P}{\rho C_p U C^2}$$

$$G_2 = \frac{RU\mu_{in}}{JT.C.\rho C_2}$$

The circumferencial heat conduction is small and can be neglected compared to the heat conducted normal to the direction of rotation,

$$k \frac{\partial^2 \bar{T}}{\partial \theta^2} \ll k \frac{\partial^2 \bar{T}}{\partial \bar{z}^2}$$

For journal bearing, the film thickness, h, is very small if compared to the radius of the journal r. Hence, the curvature effect in the fluid film is neglected. In order to incorporate the effect of transforming the film thickness into a rectangular shape, the following operator is used.

$$\frac{\partial \bar{T}}{\partial \theta^*} = \frac{\partial \bar{T}}{\partial \theta} - \frac{\bar{z}}{\bar{h}} \left( \frac{\partial \bar{h}}{\partial \theta} \, \frac{\partial \bar{T}}{\partial \bar{z}} \right]$$

#### BOUNDARY CONDITIONS II

The so-called ISOADI boundary condition is used in solving the energy equation. Since the shaft is assumed to be an isothermal component, therefore at the oil-shaft interface the no "net-heat-flow" to the shaft is required, that is:

$$\bar{q} = \int_0^{2\pi} \frac{1}{\bar{h}} \frac{d\bar{T}}{d\bar{z}} d\bar{x} = 0$$

For an insulated bush, the temperature gradients at the oil-bush interface is zero.

The caviations boundary condition for lubricant thermal conductivity and viscosity are as follows:

$$k_0 = k_{air} - \frac{h_{icav}}{h} (k_{air} - k_{oil})$$

$$\mu' = \mu_{air} - \frac{h_{icav}}{h} (\mu_{air} - \mu_{oil})$$

The homogeneous mixing of the recirculating oil,  $Q_{rec}$ , and the supply oil,  $Q_{supply}$ , can be represented by

$$\bar{T}_{mix} = \frac{Q_{rec} \, \bar{T}_{rec} + Q_{leak} \, \bar{T}_s}{Q_{leak} + Q_{rec}} \tag{8}$$

#### NUMERICAL SOLUTION PROCEDURE

The governing equations were discretized and solved simultaneously using a finite difference method. A forward difference formula is used for  $dT/d\theta$  and a central difference formula is applied to all other derivatives. The global solution scheme is as follows. The arbitrary pressure, and viscosity, and temperature profiles are assumed to be 1.0. Next, the attitude angle,  $\psi$ , is estimated from the short journal bearing theory. The program proceeds to calculate the mesh sizes in all x, y, and z directions of the lubricant film. Next, the film thickness in the bearing is evaluated. Then the pressure and shear flow factors can be evaluated for isotropic roughness.

Next, the program proceeds to solve the Reynolds equation for the pressure distribution in the lubricant film. For a rapid convergence, the successive over-relaxation (SOR) method is used in solving the Reynolds equation. The cavitation boundary condition is implemented by setting all negative pressures equal to zero. Having calculated the converged pressure distribution, the velocity, velocity gradients, pressure gradients, load carrying capacity, and side leakage are then calculated. The intergrations of equations for solving the velocity (w), load carrying capacity, and side leakage were performed numerically by Simpson's rule. Next, the energy equation is solved for nodal temperature by an implicit marching technique starting from the oil inlet groove and travelling in the direction of rotation. The lubricant temperature field is obtained by solving the tridiagonal matrix by the two dimensional energy. A new mixing temperature at the inlet, T<sub>mix</sub>, is computed from Equation (8). This temperature is used as an initial temperature for the next iteration of the energy equation. The iteration process is repeated until the relative diferrence between two successive iterations falls below a specified tolerance of 0.001. Once the convergence is obtained, the program proceeds to calculate the shaft temperature which satisfies the no "net-heat" flow condition given by the equation. Once the shaft temperature is found, the converged ISOADI solutions are obtained. This new temperature distribution is used to calculate a new set of viscosity. Next, a new set of pressure distribution is

computed by solving the Reynolds equation and once the converged pressure distributions are obtained, the new velocity, velocity gradients, pressure gradients, hydrodynamic load, and side leakage are calculated. Knowing these new values, the new temperature distributions are obtained by solving the energy equation. The program cycles between the energy and Reynolds equations several times until the relative difference between two successive iterations falls below a specified tolerance. Once the convergence is achieved, the final converged ISOADI solutions for an isotropic rough surface THD journal bearing problem are printed out.

#### RESULTS AND DISCUSSIONS

The influence of the isotropic surface roughness together with thermal effect on the bearing performance parameters are studied. The types of roughness used is isotropic. The roughness effects is included in the analysis by introducing the pressure and shear flow in the Reynolds equation. The pressure and shear flow factors developed by Patir et al. (1978) were used. The list of the input data used in the computer simulations are shown in Table 1.

TABLE 1. List of input data used in the computer simulation

Bearing Parameters	Units (SI)
Journal Radius, r	50.8 mm
Radial Clearance, C	$6.35 \times 10^{-2}  \text{mm}$
Bearing length, L	76.2 mm
Load range, W	1800 < W < 16,000 N
Speed Range, N <sub>s</sub>	1500 RPM
Lubricant Parameters	Units (SI)
Inlet viscosity, µ,	0.3 poise
Specific heat, Cp	2000 Joules/kg°C
Density, p	$860 \text{ kg/m}^3$
Thermal conductivity, k <sub>0</sub>	0.13 W/m°C
Viscosity coefficient, β	0.023
Operating Parameter	Units (SI)
Oil inlet temperature, Talet	36.8°C
Oil supply pressure, P	$280 \times 10^3 Pa$

## OIL FILM TEMPERATURE DISTRIBUTION

The isotherm contour of ISOADI case of the oil film temperatures for the isotropic, roughness with roughness parameter  $\Lambda=2$  are shown in Figure 2. The heat flow pattern of those contour plots shows close resemblance. In

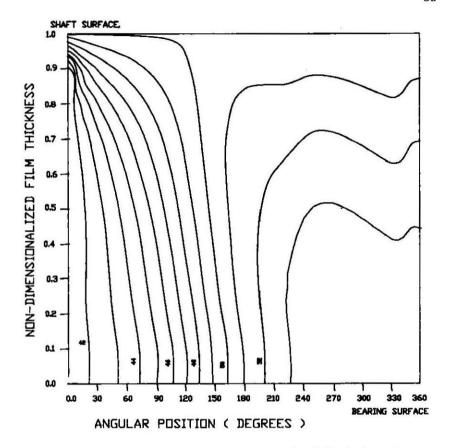


FIGURE 2. The contour of temperature distributions in oil film for isotropic roughness with roughness parameter of 2

the cavitation region, the oil pressure falls to that of the ambient pressure therefore the pressure gradient is zero.

The maximum oil-film temperature occurs near the surface of the stationary bush and close to the minimum film thickness. The same behaviour is also reported by McCallion et al. (1970) and Mitsui et al. (1986). The ISOADI contour plots also show that at the oil-bush interface the isotherms are perpendicular to the bush surface. These are due to the adiabatic condition used at the oil-bush interface.

# TEMPERATURE VARIATION AS A FUNCTION OF THE LOAD CAPACITY

The computed shaft temperature for isotropic, roughness with  $\Lambda=4$ , and the experimental results of Dowson et. al as function of the bearing load capacity are shown in Figure 3. The computed maximum oil temperature for isotropic, roughness with  $\Lambda=4$ , and the experimental results of Dowson et. al as function of local capacity are shown in Figure 4.

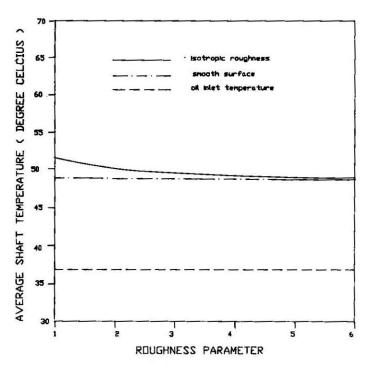


FIGURE 3. Average shaft temperature versus roughness parameter for eccentricity of 0.58

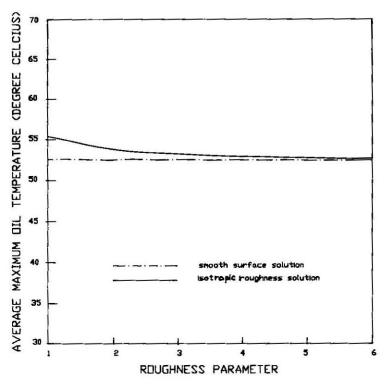


FIGURE 4. Average maximum oil temperature versus roughness parameter for eccentricity ratio of 0.58

The ISOADI solutions for isotropic roughness types underestimated the measured results when the load is less then 4000 Newtons; but as the load carrying capacity increases, the ISOADI solutions overestimated the measured results. The maximum discrepancy obtained for isotropic roughness is about 5°C.

Figure 3 and 4 show that at the roughness parameter  $\Lambda = 4$ , the results obtained are slightly higher compared to the smooth surface solutions.

# ISOTROPIC ROUGHNESS WITH AND WITHOUT THERMAL EFFECT

As discussed in the earlier sections, both surface roughness and temperature affected the bearing performance parameters appreciably. The solutions obtained for the bearing performance parameters in the finite journal bearing with and without thermal effects are compared.

The plots for the side leakage with eccentricity ratios equal to 0.2 and 0.58 versus roughness parameter with and without thermal effects are shown in Figures 5 and 6. In both plots, the side leakage with the thermal effects are higher compared to the side leakage without the thermal effects. This is because the viscosity decreases when the lubricant temperature increases. At eccentricity ratio of 0.2, the difference between the two sets of curves (with and without thermal effects) are noticeable when roughness parameters are between 2 and 6. As roughness parameter decreases, the difference also decreases (Figure 7). In another word, the thermal effect does not play a major role in the side leakage when the mean film thickness is very small, where the asperity contacts occurred. For a higher eccentricity ration ( $\epsilon = 0.58$ ) the difference between those two sets of side leakage curves are very small and almost negligible. In this case, the bearing is operating in a thinner film. Once again the results shown in Figure 5 proved that the temperature does not give a major effect to the leakage when the film thickness is very small.

However, different results are obtained in the hydrodynamic load cases. In both eccentricity ratios ( $\epsilon = 0.2$  and 0.58), the loads for isotropic types of roughness structure with the thermal effects are low compared to the one without the thermal effect. Figure 7 shows that at eccentricity ratio equal to 0.2, the isotropic roughness wothout thermal effect. The stiffness of the bearing reduced appreciably when the thermal effects is included in the analysis. The difference of load between those two sets of data are between 200-450 Newtons depending on the roughness parameter. At the eccentricity ratio of 0.58, the highest load is obtained for isotropic roughness without thermal effect (Figure 8). The difference of load between those two sets of data are between 6000-10000 Newtons depending on the roughness parameter. At roughness parameter lower than 2, the load for isotropic roughness increased sharply both with or without thermal effect. This is due to the pressure generated in the lubricant oil. Since the isotropic roughness restricts more flow both in the x and y directions therefore the pressure is maximum compared to smooth surface.

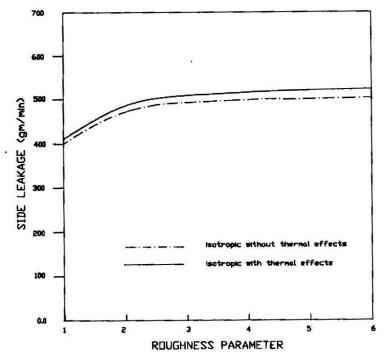


FIGURE 5. Comparing side leakage with and without thermal effects for eccentricity ratio of 0.2

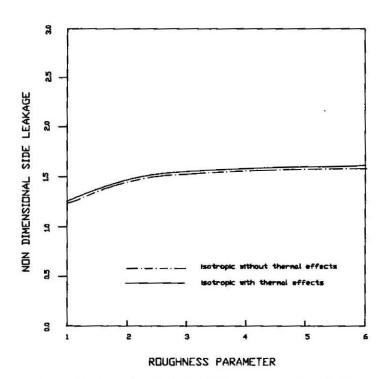


FIGURE 6. Comparing side leakage with and without thermal effects for eccentricity ratio of 0.58

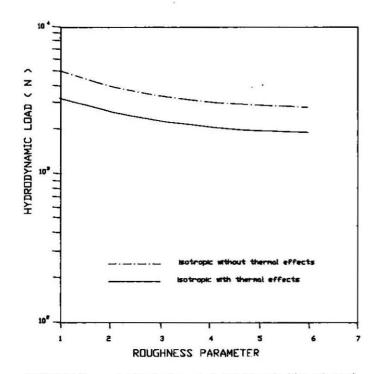


FIGURE 7. Comparing Hydrodynamic load with and without thermal effects for eccentricity ratio of 0.2

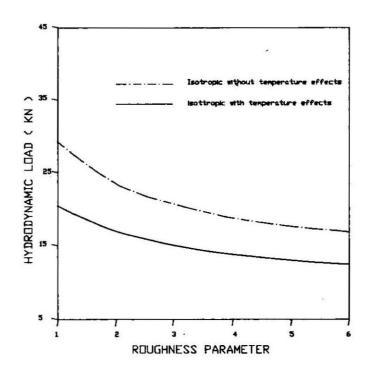


FIGURE 8. Comparing Hydrodynamic load with and without thermal effects for eccentricity ratio of 0.58

In both eccentricity ratios studied, the solutions obtained when the thermal effect is included give a significant change in most bearing's perfomance parameters. By including the thermal effect, a small increase in side leakage and an appreciable reduction in load are observed. The thermal effect also reduces the lubricant pressure significantly, hence reducing the bearing stiffness.

#### CONCLUSION

THD analysis of journal bearings with rough surface is studied. A computer program is developed that can generate the THD solutions for bearing with rough surface. The bearing performance parameter with and without thermal effect are reported. In general, the thermal effect will increase side leakage, reduce viscosity, pressure and load. Along with the bearing's geometries, the thermal effects, the roughness parameters, the surface pattern parameter c, and the variance ratio  $V_{\rm rj}$  should be included as design parameters. The results show that when both surface roughness and thermal effects are included in the analysis, a more accurate prediction of the bearing performance parameters are obtained.

#### NOMENCLATURE

Symbol	Explanation
С	radial clearance, m
C <sub>p</sub>	lubricant specific heat, Joules/kg°C
e	eccentricity, m
erf( )	error function
h	film thickness, m
ħ	nondimensional film thickness, h/C
h <sub>min</sub>	minimum film thickness, m
i, j, k	finite difference mesh indices in the circumferential, radial,
	and axial directions, respectively
k <sub>o</sub>	thermal conductivitiy of lubricant, W/m°C
k <sub>air</sub>	thermal conductivity of air, W/m°C
L	bearing length, m
N <sub>s</sub>	Speed range, rpm
Ρ .	pressure, N/m <sup>2</sup>
P	nondimensional pressure,
$P_A$	ambient pressure, N/m <sup>2</sup>
Pcav	pressure in the cavitation region, N/m <sup>2</sup>
Ps	supply pressure, N/m <sup>2</sup>
$q_x,q_y$	unit oil flow in x and y directions
Q <sub>leak</sub>	axial leakage flow rate, m <sup>3</sup> /s
Qrec	recirculating flow rate, m <sup>3</sup> /s
Q <sub>supply</sub>	supply flow rate, m <sup>3</sup> /s
R	bearing radius, m
r	journal radius, m
T	oil temperature, °C
T	nondimensional temperature, T/T <sub>inlet</sub>
$T_B$	bush temperature, °C

Tinlet	inlet oil temperature, °C
T <sub>max</sub>	maximum oil film temperature, °C
Tmix	inlet mixing temperature, °C
Trec	recirculating oil temperature, °C
Tshaft	shaft temperature, °C
T <sub>supply</sub>	supply oil temperature, °C
u	velocity vector
u,v,w	lubricant velocity in x, y, z directions, m/s
U	shaft speed, m/s
u	dimensionless velocity in x direction, u/U
V	dimensionless velocity in y direction, v/V
W	load, N
x, y, z	coordinate system
x	dimensionless, x/R
у	dimensionless, y/L
Z	dimensionless, z/Ch
β	viscosity-temperature coefficient
δ	combined roughness, $\delta = \delta_1 + \delta_2$ , m
$\delta_1, \delta_2$	roughness height, m
ε	eccentricity ratio, $\varepsilon = e/C$ , dimensionless
$\phi_{ extsf{S}}$	shear flow factor
$\phi_{X}, \phi_{Y}$	pressure flow factors
γ	surface pattern parameter, dimensionless
٨	roughness parameter, $\Lambda = C/\sigma$
$\mu$	lubricant viscosity, Pa-sec
$\mu_{in}$	inlet lubricant viscosity, Pa-sec
$\theta$	circumferencial coordinate, degrees
$\theta_2$	position of cavitation, degrees
$ heta_{ ext{inlet}}$	position of the inlet oil, degrees
ρ	lubricant density, kg/m <sup>3</sup>

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