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Augmented Model Based Double Iterative Loop Techniques for Integrated System Optimisation and Parameter Estimation of Large Scale Industrial Processes

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ABSTRACT

Two novel hierarchical structures are presented which extend the applicability of previous model based double iterative loop techniques to non-convex problems. The methods incorporate integrated system optimisation and parameter estimation which utilizes process measurements to achieve real process optimality inspite of model reality differences. The double iterative loop structures of the proposed algorithms use the real process measurement within the outer loops while the inner loops involve model based computation only. This means that the algorithms use available information from the real process efficiently and a significant reduction in set-point alterations to real subprocesses is achieved. In order to cater for process complexity the inner loops are organised in the form of two level hierarchical structures. The paper presents the convergence conditions of the augmented techniques and simulation examples are provided to illustrate and compare the methods.

ABSTRAK

Dua algoritma terbaru berstruktur hierarki dibentangkan. Algoritma-algoritma ini berjaya memperluaskan penggunaan algoritma-algoritma yang ditemui sebelum ini, iaitu algoritma yang berdasarkan kepada teknik gelung lelaran dua berasas model yang menggunakan maklumat suapbalik masukan-keluaran bagi masalah tak cembung. Teknik yang dicadangkan ini hasil daripada gabungan di antara sistem pengoptimuman dan penganggaran parameter. Walaupun terdapat bezaan model dan proses sebenar yang besar, teknik ini berjaya mencapai optimaliti proses sebenar dengan menggunakan ukuran daripada proses sebenar. Algoritma yang berstruktur gelung lelaran dua menggunakan ukuran proses sebenar di dalam gelung luarannya. Manakala, gelung dalaman melakukan pengiraan berasaskan kepada model sahaja. Ini menunjukkan algoritma-algoritma ini dapat menggunakan maklumat yang tersedia ada daripada proses sebenar dengan lebih cekap dan algoritma ini juga dapat mengurangkan pertukaran titik-set kepada sub-proses sebenar dengan banyaknya. Gelung dalaman diatur dalam bentuk berstruktur hierarki dua aras. Contoh-contoh simulasi dipamirkan untuk menjelaskan kaedah-kaedah ini.

INTRODUCTION

In order to determine and maintain optimum steady-state operation of an industrial process a common method is to compute optimum values of

feedback controller set points using a steady-state mathematical model of the process (e.g., Lefkowitz 1977). Often values of parameters in the mathematical model are unknown or uncertain and it is necessary to estimate these parameters by matching real process and model output responses. In practice, uncertainties exist in the structure of the mathematical model giving rise to output responses. In practice, uncertainties exist in the structure of the mathematical model giving rise to interaction problems between system optimisation and model parameter estimation which require that the solution must be obtained by iterating between successive solutions of the system optimisation and parameter estimation problems. When applied to a single process, techniques have been developed for performing these iterations using a modified two-step procedure which provides the correct final steady-state optimum operating condition on the real process inspite of deficiencies in the mathematical model (Roberts, 1979). These ideas, which are based on Lagrangian optimality conditions, have been recently extended to large scale processes consisting of a collection of interconnected subprocesses (e.g., Brdys' and Roberts, 1986); have been developed which are of an iterative type and utilize information feedback from the real process and price decomposition to cater for model reality differences and process complexity, respectively. These structures show considerable practical significance because, unlike previous on-line hierarchical optimizing control techniques which are sub-optimal (e.g. Findeisen and co-workers, 1980; Tatjewski,1985; Wu, Shao and Li, 1986; Shao and Roberts 1983), the new techniques are designed to give a precise optimal solution. Various structures have been developed depending upon the type of information feedback available from the real process and the manner in which the information is utilised.

In this paper, consideration is given to a strategy where the coordination task is split into two nested iterative loops. This is a development from previous work (Brdyś and Roberts1985; Chen, Brdyś and Roberts 1986; Brdyś and co-workers 1986) where the inner loop involves model based computations only while the outer loop requires measurements from the real process. It is only during the outer loop iterations that the controller set points are changed. This has an important practical advantage over the single loop coordination strategy arising from a reduction in the number of alterations to real subprocess controller set points in order to achieve optimality. However, this reduction is achieved at a cost of increasing the number of purely model based iterations. Since these iterations are performed within a hierarchical structure the total amount of information exchanged between the structure units which is required to find the solution turns out to be increased. An obvious way to alleviate this drawback is to improve the numerical properties of the inner loop optimisation problem. The paper presents a modification of the performance index which effectively achieves this improvement. The modification consists of in an augmentation of the outer loop performance index by appending a suitable convexifying term which, in contrast to previous methods e.g., Tatjewski (1985), is fully decomposable. It is shown that the resulting augmented model based double iterative loop strategies are applicable to non-convex problems.

PRELIMINARIES

Following the same methodology as Findeisen and co-workers (1980) it is assumed that the controlled system, inclusive of its follow-up controllers, is described in a decomposed way by the set of subsystem input-output mappings.

$$F_{*i}: C_i \times U_i \to Y_i \in \overline{1, N},$$

where N is the number of subsystems and C_i, U_i and Y_i are finite dimensional spaces, given by

$$y_i = F_{*i}(c_i, u_i), \; i \in \overline{1, N},$$

where the variables c_i, u_i, y_i are the i-th subsystem control, interaction input and interaction output, respectively, and

$$c_i \in C_i, \; u_i \in U_i, \; Y_i \in Y_i$$

The subsystems are interconnected with assumed structure equations

$$u_i = H_i y, \; i \in 1, N$$

where H_i are interconnection matrices, $y \triangleq (y_1, .., y_N) \in Y \triangleq Y_1 \times ... \times Y_N$.

The subsystem equations and the structure equations can be written jointly as:

$$y = F_*(c, u), \; u = Hy \tag{1}$$

where

$$c \triangleq (c_1, ..., c_N) \in C \triangleq C_1 \times ... \times C_N,$$

$$u \triangleq (u_1, ..., c_N) \in U \triangleq U_1 \times ... \times U_N.$$

We shall assume that for each $c \in C$, there exists only one solution of the equation

$$y = F_*(c, Hy)$$

Hence, the global system may be described by the mapping $K_* : C \to Y$ and

$$y = K_*(c) = (K_{*1}(c), ..., K_{*N}(c)).$$

The system relations are not known exactly and we have at our disposal only their approximate models.

$$F_i : C_i \times U_i \times A_i \to Y_i, \; i \in \overline{1, N},$$

i.e.,

$$y_i = F_i(c_i, u_i, \alpha_i),\ i \in \overline{1, N},$$

where A_i is a finite dimensional space and $\alpha_i \in A_i$ is the i-th subsystem model parameter variable. However, the interconnection relationships are assumed to be known exactly.

Each subsystem is assumed to be subjected to local constraints which are output independent as:

$$(c_i, u_i) \varepsilon CU_i,\ i \in \overline{1, N},$$

which can be written jointly in a form

$$(c, u) \in CU \triangleq CU_1 \times ... \times CU_N. \tag{2}$$

A fundamental assumption is that the system model is point parametric on a set CU (Brdyś, 1983). That is for every point $(\bar{c},\bar{u})$ CU there exists a A such that $F_*(\bar{c},\bar{u}) = F(\bar{c},\bar{u},\bar{\alpha})$. With each subsystem a known local performance function is associated:

$$Q_i : C_i \times U_i \times Y_i \to R,\ i \in \overline{1, N}.$$

The overall performance index of the system Q: C x U x Y - > R is assumed to have the additive form:

$$Q(c,u,y) = \sum_{i=1}^{N} Q_i(c_i, u_i, y_i) \tag{3}$$

The optimising control problem is to minimize Eq. (3) by finding set points (controls) subject to Eqs. (1) and (2). That is:

$$\min_{c,\ u,\ y} Q\ (c,u,y)$$

Subject to

$$y = F_*(c, u),\ u = Hy \tag{4}$$
$$(c, u) \in CU$$

It has been shown that the problem is equivalent to the following one (Brdyś, 1983):

$$\min_{c,\ u,\ \alpha} q\ (c, u, \alpha)$$

Subject to

$$F(c, u, \alpha) = K_*(c)$$

$$u = HF(c, u, \alpha)$$

$$(c, u) \in CU, \alpha \in A,$$

where

$$q(c, u, \alpha) \triangleq Q(c, u, F(c, u, \alpha)).$$

In order to use system output measurements and to decouple the parameter estimation problem from the model based optimisation problem we further transform the problem to the equivalent form (Brdyś and Roberts, 1986):

$$\min_{c, u, v, \alpha} q(c, u, \alpha) \tag{5}$$

Subject to

$$u - HK_*(v), F(v, HK_*(v), \alpha) = K_*(v).$$
$$(c, u) \in CU, v = c$$

PPRESENTATION OF AUGMENTED DOUBLE ITERATIVE LOOP STRATEGIES

Clearly, the optimisation problem described by Eqs. (5) is equivalent to:

$$\min_{c, u, v, \alpha} [q(c, u, \alpha) + 1/2\rho \| c-v \|^2]$$

Subject to

$$u = HK_*(v), F(v, HK_*(v), \alpha) = K_*(v), \tag{6}$$
$$(c, u) \in CU, v = c,$$

where $\rho > 0$.

For given values of v, α and price vector $p \in U$, Lagrangian analysis as performed by Bradyś and co-workers (1986), provides the following optimization problem:

$$\min_{(c, u) \in CU} [q(c,u,\alpha) + 1/2\rho \| c-v \|^2 + p^T([u-HF(c,u,\alpha)] - \lambda^T c] \tag{7}$$

where

$$\lambda^T(v, \alpha, p) = q_c'(v, HK_*(v), \alpha) - q_*'_c(v, HK_*(c)) + [q_u'(v, HK_*(v), \alpha) - q_u'(v,HK_*(v))]\, HK_*'(v) + p^T(g'c(v,HK_*(v),\alpha) - g_*'_c(v,HK_*v))] + [g'_u(v,HK_*(v), \alpha) - g_*'_u v,HK_*(v))]\, HK'_*(v)). \quad (8)$$

and where

$$q_*(c,u) \triangleq Q\,(c,u,K_*(c)) \quad (9)$$
$$g_*(c, u) \triangleq u - HK_*(c) \quad (10)$$
$$g(c,u,\alpha) \triangleq u - HF\,(c, u, \alpha) \quad (11)$$

The term $1/2\rho\|\, c\text{-}v\, \|^2$, with a suitably chosen value of ρ, improves the properties of the optimisation problem. In particular, it convexifies the problem.

The parameter estimation problem is concerned with the determination of parameters $\hat{\alpha}(v)$ which satisfy

$$F(v, HK_*(v), \hat{\alpha}(v)) = K_*(v) \quad (12)$$

At a given $(v, \hat{\alpha}(v), p)$ a solution of the optimisation problem described by Eq. (6) with $\alpha = \alpha(v)$ is denoted by $\hat{c}(v,p)$ and $\hat{u}(v,p)$. Clearly, if the regularity conditions are satisfied at every point of a set CU then v_* at any solution (v_*, p_*) of the set of equations:

$$\hat{c}(v, p) = v, \quad (13)$$
$$\hat{u}(v, p) = HF(\hat{c}(v,p), \hat{u}(v,p), \hat{\alpha}(v)), \quad (14)$$

satisfies the Kuhn-Tucker first order necessary conditions for optimality corresponding to the expanded problem (5). The optimality properties of the solutions of Eqs. (13) and (14) are exactly the same as in the case of the non-augmented double iterative loop techniques investigated by Brdyś and co-workers (1986). Following that contribution, the principle of the model based double loop technique consists in separating Eq. (13) from Eq. (14). This separation is realized in such a manner that each time the new values v^k of v is produced within a *outer loop* to satisfy Eq. (13) it is sent to an *inner loop* where Eq. (14) is solved for price p. The solution is denoted by p^k. Next, $\hat{c}^k \triangleq \hat{c}\,(v^k, p^k)$ is sent to the outer loop and a new value of v^{k+1} of v is determined according to.

$$v^{k+1} = v^k + \varepsilon\, [\hat{c}^k - v^k] \quad (15)$$

where ε is a positive number suitably chosen in order to guarantee converange.

It is important to note that the inner loop problem is purely model based (see Eqs.(7),..., (11)). The following two level iterative scheme is proposed to solve this problem:

$$p^{n+1} = p^n + \delta[\hat{u}(v^k,p^n) - HF(\hat{c}(v^k, p^n), \hat{u}(v^k,p^n), \hat{\alpha}(v^k))] \tag{16}$$

where $\delta > 0$, n denotes the number of the inner loop iteration under a prescribed v^k by the outer loop.

Since the optimisation problem described by Eq. (7) is fully decomposable, the vectors $u(\hat{v}^k, p^n)$ and $\hat{c}(v^k, p^n)$ are computed by solving N independent optimisation problems. However, because price p enters the performance index in (6) also through the modifier $\lambda(v, \hat{\alpha}(v, p))$ (see (8)) then the strategy (16) is not of a gradient type.

Consequently, there are no immediate sufficient conditions for the convergence of this strategy. We provide such conditions in the next subsection. Furthermore, existence of the inner loop solutions may be restricted. In order to overcome these potential difficulties we shall modify Eqs. (13) and (14) in the following manner. The optimisation problem Eq. (17), is replaced by

$$\min_{(c,\,u)\,\epsilon\,CU} \quad [q(c, u, \hat{\alpha}(v)) + p_1^T g(c, u, \hat{\alpha}(v)) - \lambda^T (v,\hat{\alpha}(v), p_2)c], \tag{17}$$

where in comparison with Eq. (7), the price affecting term $p_1^T g(c, u, \hat{\alpha}(v))$ is distinguished from the price affecting term $\lambda^T(v, \alpha(v), p_2)c$.

The solution of Eq. (17) with respect to c and u under given (p_1, p_2) and v is denoted by $\hat{c}(v, p_1, p_2)$ and $\hat{u}(v, p_1, p_2)$. Clearly, the set of Eqs. (13) and (14) is equivalent to:

$$\hat{c}(v, p_1, p_2) = v, \tag{18}$$

$$p_1 - p_2 \tag{19}$$

$$\hat{u}(v, p_1, p_2) = HF(\hat{c}(v, p_1, p_2), \hat{u}(v, p_1, p_2), \hat{\alpha}(v)) \tag{20}$$

Now the inner loop task is to solve Eq. (20) with respect to p_1 for given values of v and p_2. The solution is denoted as $\hat{p}_1(v, p_2)$. The outer loop task is to solve Eqs. (18) and (19) with respect to v and p_2. Notice, that the previous inner loop strategy described by Eq. (16) applied to p_1 is of the gradient type and the resulting iterative scheme for solving the inner loop problem is nothing else than the interaction balance iterative scheme (e.g., Findeisen and co-workers, 1980). However, the resulting second double iterative loop technique requires additional iterations of the price vector:

$$p_2^{k+1} = p_2^k + \varepsilon\,[\hat{p}_1^k - p_2^k] \tag{21}$$

where ϵ is exactly the same as in Eq. (15).

The modifier λ described by Eq. (8) may either be determined centrally as part of the outer loop task or found within the information exchange.

CONVERANGE ANALYSIS

Convergence analysis for the general nonlinear situation has been found to be intractable at the present time and it has been necessary to restrict the analysis

to the linear quadratic and unconstrained case. It has also been necessary to assume that the performance index is not output dependent. Hence, when the iterative schemes are applied to more general nonlinear problems, the convergence results are valid in a local sense only. However, it is expected that they will provide insight into these general problems. The performance index is considered to have the form:

$$Q(x) = 1/2[x-d]^T M[x-d], \tag{22}$$

where $x \triangleq (c, u)$,

$$F(c,u,\alpha) = D_1 c + D_2 u + P(\alpha), \tag{23}$$
$$F_*(c,u) = D_{*1} c + D_{*2} U + d_*, \tag{24}$$

where M is a symmetric matrix, and matrices D_1, D_2 and $P(\alpha)$ are chosen so that the model is point parametric.

It is assumed that an inverse

$$[HD_{*2} - I]^{-1}$$

exists and consequently the following matrices are well defined:

$$B \triangleq [-HD_1, I-HD_2], \tag{25}$$
$$B_* \triangleq \left[\left[HD_{*2} - I\right]^{-1} HD_{*1}, I\right] \tag{26}$$
$$\tilde{B} \triangleq [I - HD_2]\, B_* \tag{27}$$

We shall further assume that the matrix B_* has full rank, and that the second order sufficient conditions for optimality hold for the optimising control problem (4), that is

$$x^T M x > 0 \text{ for every } x \in CxU,\ x \neq 0 \text{ such that } B_* x = 0 \tag{28}$$

Notice that this implies that there is a unique solution $\bar{c}$ to the control problem and a unique corresponding price vector $\bar{p}$ associated with the constraint $g_*(c, u) = 0$. Consequently, there is a unique solution $(\bar{c}, \tilde{p})$ of the set of Eqs. (13) and (14), where

$$\tilde{p} = [(I - HD_2)^{-1}]^T \bar{p} \tag{29}$$

Notice also that the non-convex case is not excluded from the considerations. It is assumed that the value of ρ in (7) is chosen such that (clearly, such choice always exists)

$$M_\rho \triangleq M + \rho I > 0. \tag{30}$$

CONVERGENCE THEOREM A: ASSUME

$$BM_\rho^{-1} \tilde{B}^T > 0. \tag{31}$$

Then:

1. The inner loop problem described by Eq. (14) is well-defined. There exists such number $\bar{\delta} > 0$ that for any $(0, \bar{\delta})$ the iterative scheme (16) is convergent for every v.
2. The outer loop iterative scheme described by Eq. (15) is convergent to the point $\bar{v}$ while the corresponding sequence of price vectors is convergent to $\tilde{p}$, for every value of ϵ such that

$$M_\rho - \frac{\epsilon}{2} M > 0 \tag{32}$$

A proof of this theorem is presented by Brdyś, Abdullah and Roberts (1986). Clearly, condition (32) can always be satisfied by choosing a sufficiently small ϵ. It can be seen that a convexfying term exists in Eq. (7) and thus $\rho > 0$ may allow larger values of the step coefficient ϵ to be employed which makes the outer loop iterative scheme more efficient. This has been fully confirmed by numerical simulations. Since $\tilde{B} = B$ if the mathematical model is perfect condition (31) is satisfied in that particular situation.

We shall formulate now sufficient conditions for the convergence of the second double iterative loop technique, that is with the relaxed inner loop problem.

Let us define matrix M_γ in the following way:

$$M_\gamma \triangleq M + \gamma \tilde{B}^T \tilde{B}, \tag{33}$$

where $\gamma > 0$ is chosen such that $M_\gamma > 0$.

Notice, that due to the assumption (27) such a choice always exits (e.g., Luenberger (1973)). It is assumed additionally, that the matrix B has full rank.

Convergence Theorem B: Assume:

$$\text{(i)} \quad M_\rho - \frac{\epsilon}{2} M_\gamma > 0 \tag{34}$$

$$\text{(ii)} \quad 2BM_\gamma^{-1} \tilde{B}^T - \tilde{B} M_\gamma^{-1} M_\rho M_\gamma^{-1} \tilde{B}^T - \tilde{B} M_\gamma^{-1} \tilde{B}_\gamma^T > 0 \tag{35}$$

Then the algorithm described by Eqs. (15) and (21) is well defined and generates a sequence (v^k, p^k) which is convergent to $\{ \tilde{v}, \tilde{p} \}$, where $\tilde{p}$ and $\bar{p}$ are related by Eq. (29).

A proof of this theorem is presented by Brdyś, Abdullah and Roberts (1986) and utilizes an technical idea Cohen (1980).

As previously, condition (34), can always been satisfied by choosing ϵ sufficiently small. Condition (35) can always been satisfied easily since the

second and third matrices in (35) are positive definite, $M_\rho > 0$ and $M_\gamma > 0$ and because B has full rank, then if condition (35) is satisfied condition (31) must also be satisfied. It follows from the proof of Theorem A that condition (31) is only required to preserve convergence of the inner loop iterative scheme. The proof of Theorem B shows that condition (35) is only required to preserve convergence of the outer loop iterative scheme of the second technique while there are no restrictions implied by the inner loop. Therefore, although the convergence properties of the relaxed inner loop are better, the outer loop imposes such restrictive conditions that the overall convergence conditions of the second technique are more restrictive, in the linear quadratic case. It is expected that in some more general non linear situations the second technique may be more efficient due to the unquestionably better efficiency of the inner loop. This is, however, a problem which is under current research. Finishing this section let us examine condition (35) when $M > 0$. The values $\rho = O$ and $\gamma = 0$ are then allowed and the condition takes the form

$$2B\, M^{-1} \tilde{B}^T - \tilde{B} M^{-1} \tilde{B}^T - 1/2\, \epsilon\, \tilde{B} M^{-1} \tilde{B}^T > 0 \qquad (36)$$

which is precisely the same as obtained by Brdyś and co-workers (1986), for the non augmented technique. Notice also that (36) can always be satisfied by choosing ϵ sufficiently small if the mathematical model is perfect.

EXAMPLE RESULTS

Two typical examples of a large scale interconnected processes have been employed to investigate the convergence properties of the augmented model based double iterative loop algorithm.

Example 1:

The model and reality equations are:

$$\begin{aligned} y_{11} &= 1.7\, c_{11} - 0.5\, c_{12} + u_{11} + \alpha_{11} \\ y_{21} &= 2.2\, c_{21} - 0.2 c_{22} + 0.5\, u_{21} + \alpha_{21} \\ y_{*11} &= 2.0\, c_{11} - c_{12} + 0.5\, u_{11} + 0.15\, u_{11}\, c_{11} \\ y_{*21} &= 2.2\, c_{21} - 0.2 c_{22} + 0.5\, u_{21} \end{aligned}$$

with interconnections

$$\begin{bmatrix} u_{11} \\ u_{21} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} y_{11} \\ y_{21} \end{bmatrix}$$

and constraints $c_{11}^2 + c_{12}^2 \leq 1,\ 0 \leq u_{11} \leq 0.5$
$0.5 c_{12} + c_{22} \leq 1$

The output independent performance index is employed:

$$Q = 1/2(u_{11} - 1)^2 + 1/2\, c_{11}^2 + 1/2 c_{12}^2 + 1/2(u_{21} - 2)^2 + 1/2 c_{21}^2 + 1/2\, c_{22}^2$$

The input-output relations, the local constraints and structure equations are the same as in the Example 1.

Computer simulation studies have been performed on the above examples to investigate the performance of the augmented model based double iterative loop technique. Initially, simulations were performed to determine suitable values of ρ in equation (6) and ϵ in equations (15) and (21). The results presented are optimum in terms of speed of convergence for the particular case examined.

Fig. 1(a), compares the convergence of performance Q of Example 1 obtained during the iterations of the augmented model based double iterative

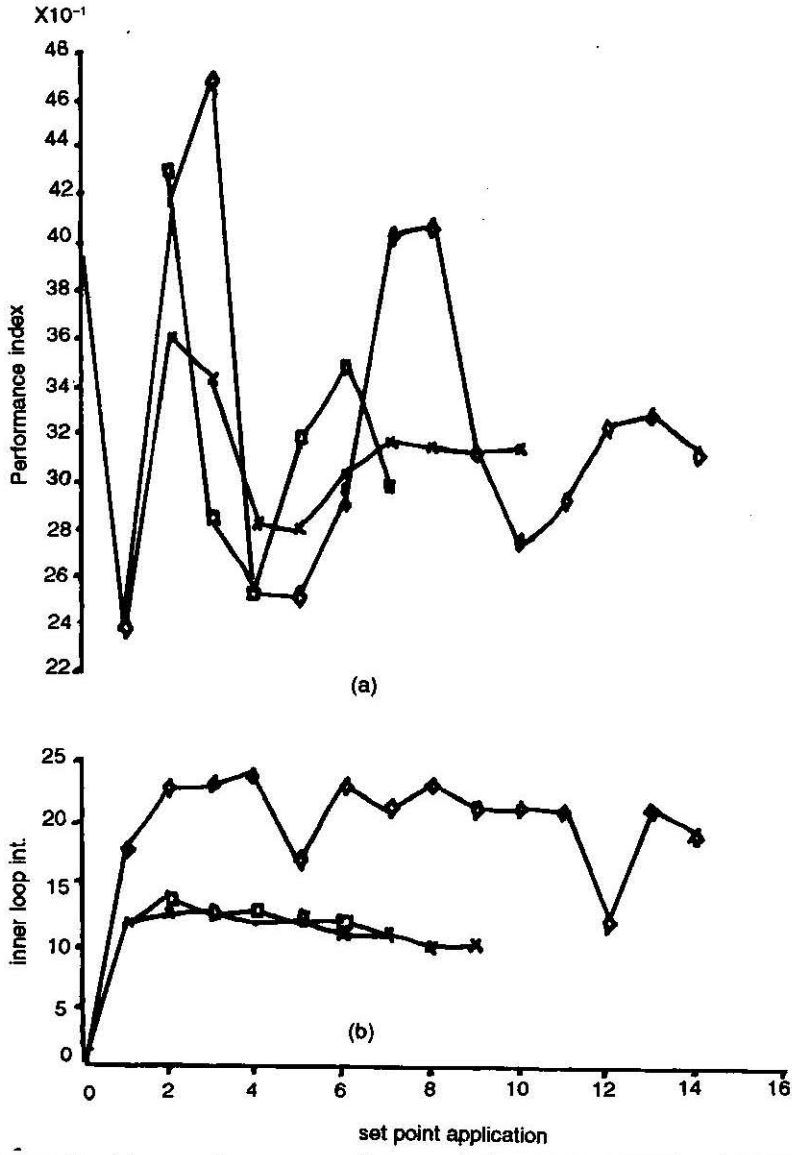

FIGURE 1. (a) rate of convergence for example 1 on MDIS, AMDIS1 and AMDIS2
(b) number of inner iteration required for each set point application (convex)

strategy, AMDIS, with the previous model based strategy, MDIS, where a normal Lagrangian is applied (Brdys and Roberts, 1986). Without augmentation (MDIS), 14 set-point applications and 285 information exchanges are needed to achieve convergence. When augmentation is used where price is not distinguished, AMDIS1, the set-point application and information exchanges are reduced to 10 and 113 respectively. When price is distinguished, AMDIS2, the set-point applications reduce to 7 which is about half the amount needed when MDIS is applied. Therefore, AMDIS is more efficient than MDIS

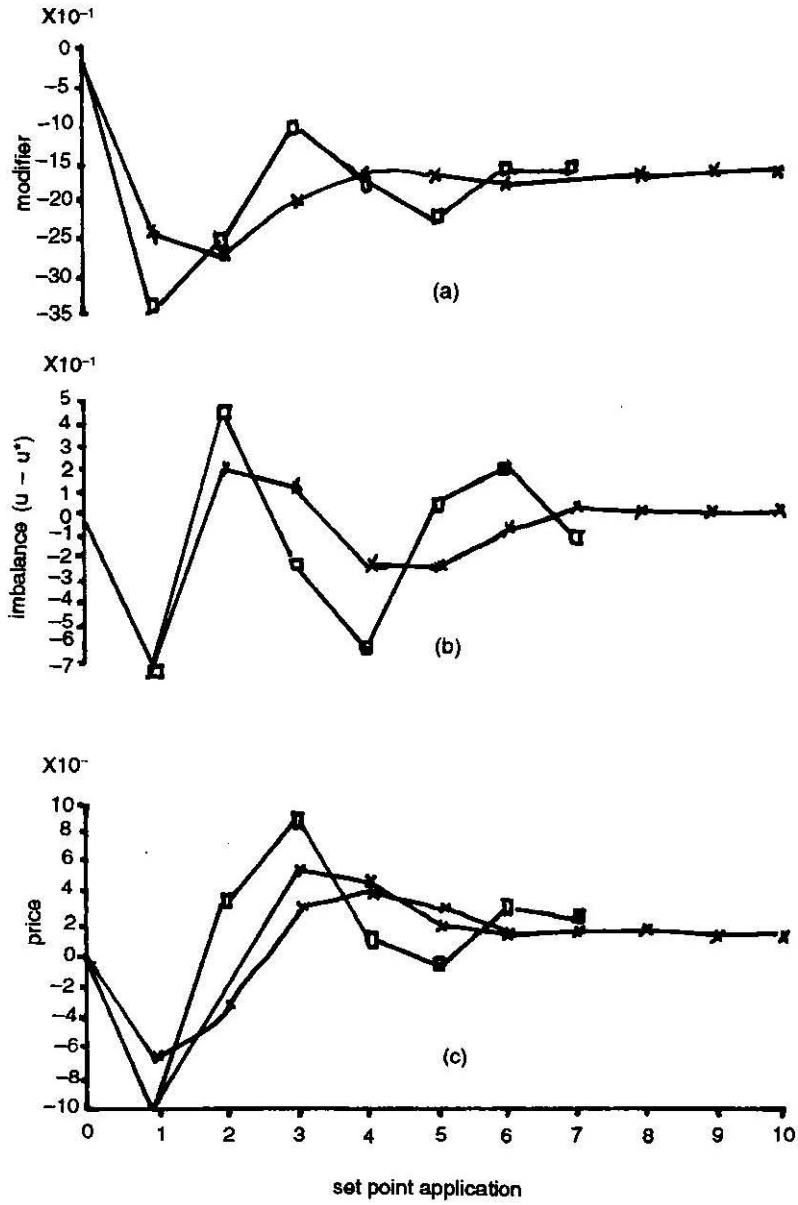

FIGURE 2. a typical (a) modifiers, (b) imbalance, (c) price for each set point application on example 1 (convex)

since significant reduction in both set-point applications and information exchanges are obtained. Since Example 1 is an example of a convex problem, it appears that by applying augmentation to convex problems reduced the set-point application and information exchanges significantly.

The two proposed strategies are also applied to Example 2 which is non-convex and the simulated results are shown in Figs. 3 and 4. Fig. 3(a) compares the convergence performance of AMDIS1 to AMDIS2. A better performance is obtained with AMDIS1 where 14 set-point applications are required compared to 22 set-point applications needed by AMDIS2 and there is also reduction in information exchanges. Therefore in these particular

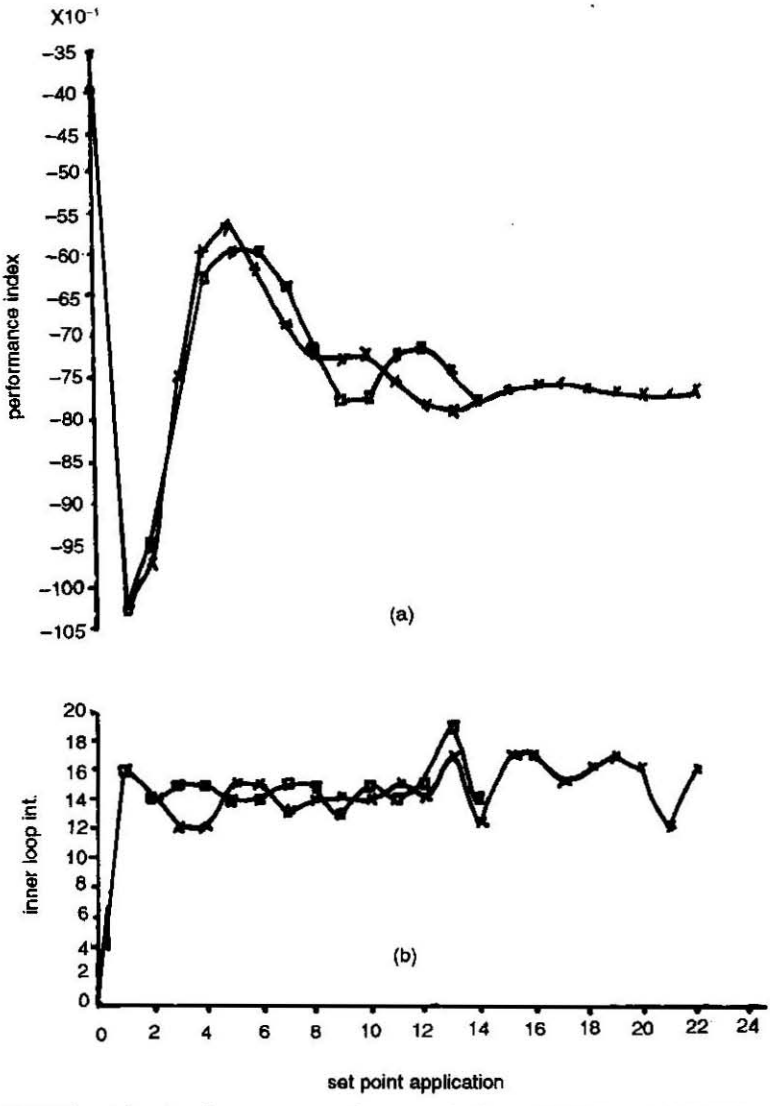

FIGURE 3. (a) rate of convergence for example 2 on AMDIS1 and AMDIS2
(b) number of inner iteration required for each set point application (non convex)

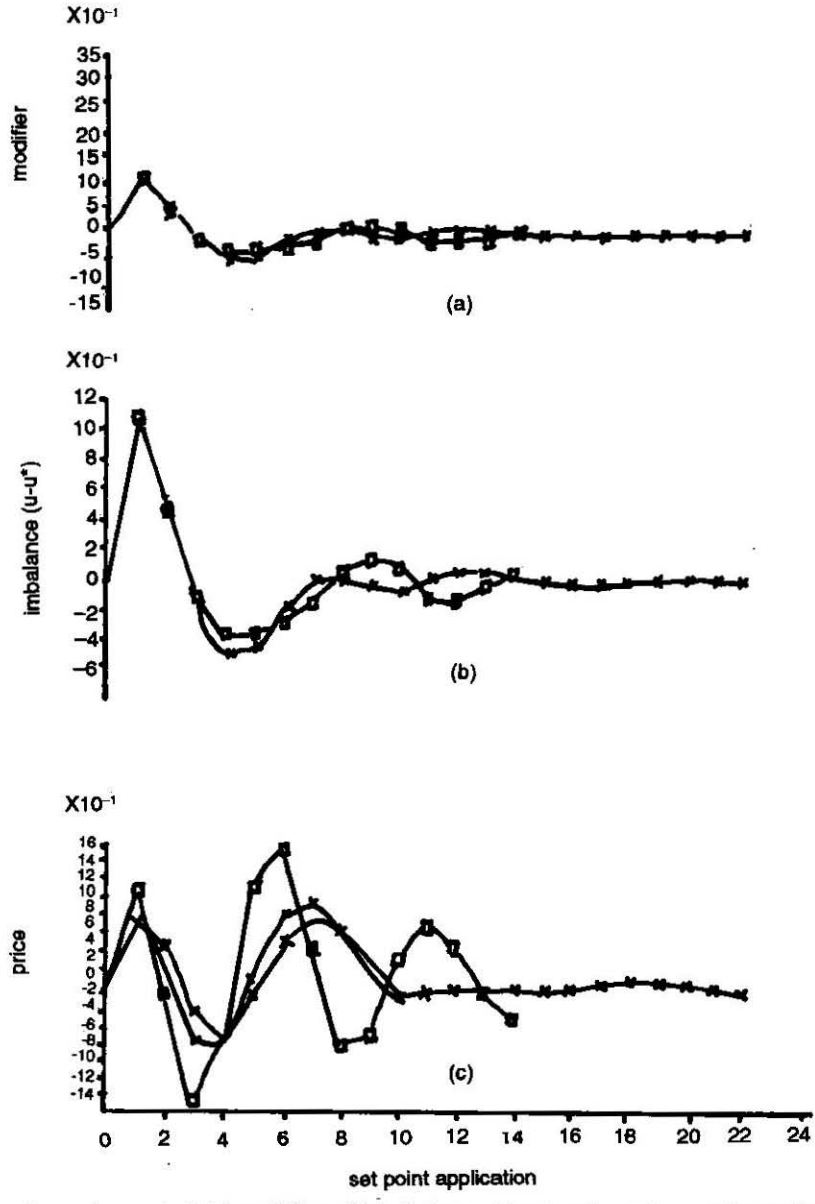

FIGURE 4. typical (a) modifiers, (b) imbalance, (c) price for each set point application on example 2 (non convex)

examples AMDIS1 gives a better result in both cases of convex and non-convex problems.

CONCLUSIONS

A model based double iterative loop strategy based on an augmented Langrangian which is applicable to non-convex problems has been presented.

Unlike previous methods the augmentation is readily decomposable without the requirement for local linearization. Furthermore, the real optimal final performance is achieved, not a sub-optimal one. Optimality and convergence conditions have been derived and investigated on two simple examples. The convergence conditions assume linear models, linear reality and a quadratic performance index and hence, are valid only in a local sense when the technique is applied to nonlinear systems. However, the simulation studies have demonstrated the utility of the method when the real process is nonlinear. Although the strategy described assumes that the performance and constraints are output independent it is expected that the method can be readily extended to include more general cases and this is a subject for further research. The technique is also recommended for convex problems due to the significant reduction in information exchanges and set-point changes possible compared to those needed by the non-augmented method MDIS. The first augmented strategy AMDIS1 has an important practical advantage over the second technique AMDIS2 in that it enables the optimum steady state operation to be achieved and maintained with a significant reduction in the number of on-line controller set point changes possible compared to those needed by the non-augmented method MDIS. The first augmented strategy AMDIS1 has an important practical advantage over the second technique AMDIS2 in that it enables the optimum steady state operation to be achieved and maintained with a significant reduction in the number of on-line controller set point changes.

APPENDIX A

Proof of the Convergence Theorem A

Using (22) and (23) we obtain

$$g_*(c,u) = B_* \begin{bmatrix} c \\ u \end{bmatrix} - b_*,$$

$$g(c, u, \alpha) = B \begin{bmatrix} c \\ u \end{bmatrix} - HP(\alpha)$$

where $b_* = [I - HD_{*2}]^{-1} Hd_*$

Let v^k is given. Denote $u_*^k = HK_*(v^k)$ since

$$g(v^k, u_*^k), \alpha^k) - g_*(v^k, u_*^k)$$

then

$$HP(\hat{\alpha}k) = B \begin{bmatrix} v^k \\ u_*^k \end{bmatrix}$$

and condition

$$g(\hat{c}^k, \hat{u}^k, \hat{\alpha}^k) = 0$$

can be written as

$$B\left(\begin{bmatrix} c^k \\ u^k \end{bmatrix} \begin{bmatrix} v^k \\ u_*^k \end{bmatrix}\right) = 0 \qquad (A.1)$$

Let us denote

$$\hat{x}^k \triangleq \begin{bmatrix} \hat{c}^k \\ \hat{u}^k \end{bmatrix} \text{ and } z^k = \begin{bmatrix} v^k \\ u_*^k \end{bmatrix}$$

Since $M_\rho > 0$ (see Eq. (28)) then the sufficient and necessary condition for $\hat{x}^n$ to be a solution of the inner loop infirmal problem described by Eqs. (7) and (8) can written as follows:

$$M_\rho(\hat{x}^n - z^k) + \tilde{B}Mz^k + {}^T p^n - Md = 0 \qquad (A.2)$$

where n denotes number of the inner loop iteration corresponding to the k-th iteration of the outer loop.

Eq. (A.2) yields that

$$\hat{x}n = M_\rho^{-1}\tilde{B}^T p^n + (I - M_\rho^{-1}M)z^k + M_\rho^{-1}Md \qquad (A.3)$$

Therefore, and due to (A.1) the iterative scheme (16) can be written as:

$$p^{n+1} = p^n + \delta B(\hat{x}^n - z^k) =$$

$$= [I - \delta BM_\rho^{-1}\tilde{B}^T]p^n + (I - M_\rho^{-1}M - \delta B)z^k + M\rho^{-1}Md \qquad (A.4)$$

Since the last two terms on the right side of the Eq. (A.4) are constant the sequence (p^{n+1}) is convergent to p^k (if it is convergent then, due to the structure of (16) it must converge to (p^k) from arbitrary chosen starting point if and only if the following discrete dynamical system

$$p^{n+1} = [I - \delta BM_\rho^{-1}B^T]p^n \qquad (A.5)$$

is asymptotically stable in the large at a point $p = 0$.

Since the matrix $BM^{-1}\tilde{B}^T$ is not symmetric then a positiveness of this matrix does not immediately guarantee an existence of a suitable value of sufficient for the stability. Let us define

$$Z(p) \triangleq ||p||^2$$

as a candidate to be Liapunov function.

Let us denote

$$A(\delta) \triangleq [I - \delta B M_p^{-1} \tilde{B}^T]$$

and

$$W \triangleq B M_p^{-1} \tilde{B}^T$$

$$\begin{aligned} Z(A(\delta)p) &= p^T p - \delta p^T W p - \delta p^T W^T p + \delta^2 p^T W p = \\ &= p^T p - 2\delta p^T W p + \delta^2 p^T W^T W p \leqslant ||p||^2 - 2\delta\lambda_{min}(W)\,||p||^2 + \\ &+ \delta^2 \lambda_{max}(W^T W)\,||p||^2 = \varphi(\delta) \end{aligned}$$

where $\varphi(\delta) \triangleq 1 - 2\delta\lambda_{min}(W) + \delta^2\lambda_{max}(W^T W)$

Since $\varphi(0) = 1$ and because $\lambda_{min}(W) > 0$ and $\lambda_{max}(W^T W) > 0$ then one can easily verify that there exists such value δ that for every $\delta \varepsilon (C, \delta)$ the following holds:

$$0 < \varphi(\delta) < 1$$

and consequently

$$Z(A(\delta)) < ||p||^2 = Z(p)$$

for such value of δ.

It has proved now that for $\delta \varepsilon (0, \overline{\delta})$ the function Z(.) described by Eq. (A.6) is the Liapunov function corresponding to the system described by Eq. (A.5).

Therefore, the assertion 1 has now been proved.

Under prescribed by outer loop value z^k of z the inner loop generates the solution (x^k, p^k) satisfying the following equations:

$$M_p(\hat{x}^k - z^k) + Mz^k + \tilde{B}^T p^k - Md = 0 \qquad (A.7)$$

and

$$B(\hat{x}^k - z^k) = 0 \qquad (A.8)$$

The optimizing control problem solution $z = (\bar{c}, HK_*(\bar{c}))$ satisfies the following equations:

$$M\bar{z} + B_*^T \bar{p} - Md = 0 \tag{A.9}$$

$$B_*\bar{z} - b_* = 0 \tag{A.10}$$

Eq. (A.9) can be also written as (see Eqs. (29) and (26)):

$$M\bar{z} + \tilde{B}^T \tilde{p} - Md = 0 \tag{A.11}$$

Conditions (A.7), (A.8) (A, 10) and (A.11) constitute a basis for further considerations. Using (15) we can express (A.7) and (A.8) in terms of points generated by the outer loop as follows:

$$\frac{1}{\epsilon} M_\rho(z^{k+1} - z^k) + Mz^k + \tilde{B}^T p^k - Md = 0 \tag{A.12}$$

and

$$B(z^{k+1} - z^k) = 0 \tag{A.13}$$

Multiplying the equality (A.12) by $(z^k - z^{k+1})^T$ and utilizing the fact that $B(z^k - z^{k+1}) = 0$ we obtain

$$\frac{1}{\epsilon}(z^{k+1} - z^k)^T M_\rho (z^{k+1} - z^k) + (z^k - z^{k+1})^T Mz^k - \\ -(z^k - z^{k+1})^T Md = 0 \tag{A.14}$$

Notice that due to (A.11) the following holds:

$$(z^{k+1} - zk)^T Md = (z^{k+1} - z^k)^T (Mz + \tilde{B}^T \tilde{p}) = (z^{k+1} - z^k)^T Mz$$

which together with (A.14) yields

$$\frac{1}{\epsilon}(z^{k+1} - z^k)^T M_\rho (z^{k+1} - z^k) + (\bar{z} - z^k)^T M(z^{k+1} - z^k) =$$

$$= -\frac{1}{\epsilon}(z^{k+1} - z^k)^T M_\rho (z^{k+1} - z^k) + 1/2(z^k - \bar{z})^T M(z^k - \bar{z}) -$$

$$- 1/2(z^{k+1} - \bar{z})^T M(z^{k+1} - \bar{z}) + 1/2(z^{k+1} - z^k)^T M(z^{k+1} - z^k) \tag{A.15}$$

Since $B_*(z^k - \bar{z}) = 0$ and $B_*(z^{k+1} - \bar{z}) = 0$ then (A.15) can be written in the following equivalent form:

$$\frac{1}{2\epsilon}(z^{k+1} - z^k)^T[\epsilon M - 2M\rho](z^{k+1} - z^k) + 1/2(z^k - \bar{z})^T M_\gamma (z^k - \bar{z})$$

$$-1/2(z^{k+1} - \bar{z})^T M_\gamma (z^{k+1} - \bar{z}) = 0 \tag{A.16}$$

where

$$M_\gamma \triangleq M + \gamma B_*^T B, \ \gamma > 0 \tag{A.17}$$

Let us notice that to the assumption (27) there existis such value of that (e.g., Luenberger (1973))

$$M_\gamma > 0 \tag{A.18}$$

Finally, let us define the following functional:

$$T(z) \triangleq 1/2(z - \bar{z})^T M_\gamma \ (z - \bar{z}) \tag{A.19}$$

where γ is chosen to satisfy the Eq. (A.19).

According to (A.16) and due to assumption (32) the sequence $\{T(z^k)\}$ is decreasing. Therefore, this sequence is bounded above by $(T(z^o)$ and consequently, due to (A.19) the sequence $\{z^k\}$ is bounded. A matrix B_* has full rank. Hence a matrix B has also full rank. Therefore, an inverse $[\tilde{B} M_\rho^{-1} \tilde{B}^T]^{-1}$ exists and (A.7) together with (15) yield

$$p^k = [\tilde{B} M_\rho \ \tilde{B}^T]^{-1}[Md - M_\rho \left(\frac{z^{k+1} - z^k}{\epsilon} + z^k \right) - Mz^k]$$

which implies that the sequence $\{p^k\}$ is also bounded. Therefore, the sequence $\{z^k, p^k\}$ has at least one convergent subsequence. The Eqs. (A.7) and (A.8) show that a limit of any subsequence of the sequence $\{z^k, p^k\}$ satisfies the Eqs. (A.10) and (A.11). It has been proved, however, that $(\bar{z}, \ \tilde{p})$ is the only one point to satisfy those inequalities. Therefore, a proof of the assertion 2 has now been completed.

APPENDIX B

PROOF OF THE CONVERGENCE THEOREM B

The necessary and sufficient conditions for x^k and p^k to be a solution of the inner loop problem corresponding to prescribed outer loop value (z^k, p_2^k) of (z, p_2) can be written as follows:

$$M_\rho (\hat{x}^k - z^k) + Mz^k + B^T(\hat{p}_1^k - p_2^k) + \tilde{B}^T p_2^k - Md = 0 \tag{B.1}$$

and

$$B(\hat{x}^k - z^k) = 0 \tag{B.2}$$

According to the assumptions $M_\rho > 0$ and matrix B has full rank, therefore, since

$$\begin{bmatrix} M_\rho & B^T \\ B & 0 \end{bmatrix} > 0$$

then there is a unique solution of (B.1) and (B.2) with respect to $(\hat{x}^k, \hat{p}_1^k)$. Hence, the itertive scheme (15), (21) is well defined. Conditions (B.1) and (B.2) constitute a basis for further considerations. Using (15) and (21) we an express (B.1) and (B.2) in terms of points generated by the outer loop as follows:

$$\frac{1}{\epsilon} M_\rho(z^{k+1} - z^k) + Mz^k + \frac{1}{\epsilon} B^T(p_2^{k+1} - p_2^k) + \tilde{B}^T p_2^k - Md = 0 \tag{B.3}$$

and

$$B(z^{k+1} - z^k) = 0 \tag{B.4}$$

Utilizing (A.11) and equality $\tilde{B}(z^k - \bar{z})$ we transform (B.3) to the following form:

$$\frac{1}{\epsilon} M_\rho(z^{k+1} - z^k) + M_\gamma (z^k - \bar{z}) + \frac{1}{\epsilon} B^T (p_2^{k+1} - p_2^k) + \tilde{B}^T(p_2^k - \tilde{p}) = 0 \tag{B.5}$$

Multiplying Eq. (B.5) by $(z^k - z^{k+1})^T$ and Eq. (B.4) by $(p_2^{k+1} - p_2^k)^T$, adding the resulting equalities we obtain:

$$-\frac{1}{\epsilon}(z^{k+1} - z^k)^T M_\rho (z^{k+1} - z^k) + (\bar{z} - z^k)^T M_\gamma (z^{k+1} - z^k) + (\tilde{p} - p_2^k)^T \tilde{B}(z^{k+1} - z^k) + p_2^{k+1} - p_2^k)^T \tilde{B}(z^k - \bar{z}) = 0 \tag{B.6}$$

The second term in (B.6) can be expressed as:

$$1/2(z^k - \bar{z})^T M_\gamma (z^k - \bar{z}) - 1/2 (z^{k+1} - \bar{z})^T M_\gamma (z^{k+1} - \bar{z}) + 1/2 (z^{k+1} - z^k)^T M_\gamma (z^{k+1} - z^k) \tag{B.7}$$

The third term in (B.6) is equal to

$$(\tilde{p} - p_2^k)^T \tilde{B}(\bar{z} - z^k) - (\tilde{p} - p_2^{k+1})^T \tilde{B}(\bar{z} - z^{k+1}) + (p_2^{k+1} - p_2^k)T \tilde{B}(z^{k+1} - z^k) + (p_2^k - p_2^{k+1})^T \tilde{B}(\bar{z}-z^k) \tag{B.8}$$

Let us now compute the term $\tilde{B}(\bar{z}-zk)$. The equalities (B.5) and (A.11) imply that

$$\bar{z}-z^k = \frac{1}{\epsilon} M_\gamma^{-1} M_p (z^{k+1} - z^k) + \frac{1}{\epsilon} M_\gamma^{-1} B^T (p_2^{k+1} - p_2^k) + M_\gamma^{-1} \tilde{B}^T (p_2^k - \tilde{p})$$

and

$$\tilde{B}(\bar{z}-z^k) = \frac{1}{\epsilon} \tilde{B} M_\gamma^{-1} M_p (Z^{k+1} - z^k) + \frac{1}{\epsilon} \tilde{B} M_\gamma^{-1} B^T (p_2^{k+1} - p_2^k) + \tilde{B} M_\gamma^{-1} B^T (p_2^k - \tilde{p})$$

Hence

$$2(p_2^k - p_2^{k+1})^T \tilde{B}(\bar{z}-z^k) = \frac{2}{\epsilon} (p_2^k - p_2^{k+1})^T \tilde{B} M_\gamma^{-1} M_p (z^{k+1}-z^k)$$

$$+ \frac{2}{\epsilon} (p_2^k - p_2^{k+1})^T \tilde{B} M_\gamma^{-1} B^T (p_2^{k+1} - p_2^k) +$$

$$2 (p_2^k - p_2^{k+1})^T \tilde{B} M_\gamma^{-1} \tilde{B}^T (p_2^{k+1} - p)$$

$$= -\frac{2}{\epsilon} (p_2^{k+1} - p_2^k)^T \tilde{B} M_p^{-1} M_p (z^{k+1} - z^k) -$$

$$\frac{2}{\epsilon} (p^{k+1} - p^k)^T \tilde{B} M_\gamma^{-1} B^T (p_2^{k+1} - p_2^k)$$

$$+ (p_2^k - \tilde{p}^k)^T \tilde{B} M_\gamma^{-1} \tilde{B}^T (p_2^k - \tilde{p}) - (p_2^{k+1} - \tilde{p})^T \tilde{B} M_\gamma^{-1} B^T (p_2^{k+1} - \tilde{p})$$

$$+ (p_2^{k+1} - p_2^k)^T \tilde{B} M_\gamma^{-1} \tilde{B}^T (p_2^{k+1} - p_2^k) \tag{B.9}$$

Finally, let us define the following function:

$$T(z,p_2) \triangleq 1/2(p_2 - \tilde{p})^T \tilde{B} M_\gamma^{-1} \tilde{B}^T (p_2 - \tilde{p}) + 1/2[M_\gamma (z - \bar{z}) + \tilde{B}^T (p_2 - \bar{p})]^T M_\gamma^{-1} [M_\gamma (z-\bar{z}) + \tilde{B}^T (p_2 - \tilde{p})] \tag{B.10}$$

We shall utilize now a technical idea from Cohen's proof of his Theorem 5.1 (Cohen, 1980).

Namely, applying (B.9), (B.8) and (B.7) to (B.6) we conclude that

$$T(z^k, p_2^k) - T(z^{k+1}, p_2^{k+1}) = (z^k - z^{k+1})^T[-\frac{1}{\epsilon}M_\rho - 1/2M_\gamma](zk - z^{k+1})$$

$$+2(p_2^k - p_2^{k+1})T[-\frac{1}{\epsilon}\tilde{B}M_\gamma^{-1}M_\rho - 1/2\,\tilde{B}](z^k - z^{k+1}) +$$

$$+(p_2^k - p_2^{k+1})^T[-\frac{2}{\epsilon}\tilde{B}M_\gamma^{-1}\tilde{B}^T - \tilde{B}M_\gamma^{-1}\tilde{B}^T](p^{k+1} - p^k) =$$

$$-\frac{1}{2\epsilon}[(z^k - z^{k+1}) + M_\gamma^{-1}\tilde{B}^T(p_2^k - p_2^{k+1})]^T[2BM_\rho - \epsilon M_\gamma][(z^k - z^{k+1})$$

$$+M_\gamma^{-1}\tilde{B}^T(p_2^k - p_2^{k+1})] + \frac{1}{\epsilon}(p_2^k - p_2^{k+1})^T[2\,\tilde{B}M_\gamma^{-1}B^T - BM_\gamma^{-1}M_\rho M_\gamma^{-1}\tilde{B}^T$$

$$-\frac{\epsilon}{2}\tilde{B}M_\gamma^{-1}\tilde{B}^T](p_2^k - p_2^{k+1}) \qquad \text{(B.11)}$$

According to (B.11) and due to the assumptions (i) and (ii) the sequence $\{T(z^k, p_2^k)\}$ is decreasing. Since $\tilde{B}M_\gamma^{-1}\tilde{B}^T > 0$ and $M_\gamma^{-1} > 0$. Hence, $\{T(z^k, p_2^k)\}$ converages and consequently

$$T(z^k, p_2^k) - T(z^{k+1}, p_2^{k+1}) \underset{k \to \infty}{\longrightarrow} 0$$

Therefore, (B.11) implies that

$$\{(z^k - z^{k+1}, p_2^k - p_2^{k+1})\} \underset{k \to \infty}{\longrightarrow} 0$$

Since $B_* z^k - b_* = 0$ then

$$\tilde{B}z^k - (I - HD_2)b_* = 0 \qquad \text{(B.12)}$$

Eqs. (B.3) and (B.12) imply that the sequence $\{z^k, p_2^k\}$ converges to a point $(\bar{z}, \bar{p}_2)$ satisfying the following equations:

$$M\bar{z} + B^T\bar{p}_2 - Md = 0 \qquad \text{(B.13)}$$

and

$$\tilde{B}\bar{z} - (I - HD_2)b_* = 0 \text{ or } B_*\bar{z} - b_* = 0 \qquad \text{(B.14)}$$

Since there is only one solution of the Eqs. (B.13) dn (B.14) then

$$\bar{\bar{z}} = \bar{z} \text{ and } \bar{\bar{p}}_2 = \tilde{p}$$

and a proof of theorem B has now been completed.

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