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Power Division Analysis of Optical Single Mode Branching Waveguides

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ABSTRACT

The power division behavior of single mode branching waveguides is studied theoretically. The dependence of the power division on the branching angle and on the coupling length is shown. A new qualitative formula relating these important parameters is presented. The calculated results for various power divisions are plotted. Certain power divisions are found to have certain respective limiting branching angles above which the desired power divisions can no longer be obtained.

ABSTRAK

Sifat-sifat pembahagian kuasa di dalam pandu gelombang-pandu gelombang bercabang satu mod adalah diselidiki secara teorinya. Kebergantungan pembahagian kuasa ini terhadap sudut cabang dan juga terhadap panjang gandingan ditunjukkan. Satu formula kualitatif baru yang menghubungkan parameter-parameter penting ini telah didapati dan dipersembahkan di sini. Hasil-hasil pengiraan bagi pelbagai pembahagian kuasa adalah dilakarkan. Adalah didapati bahawa pembahagian-pembahagian kuasa tertentu mempunyai had-had sudut cabangnya yang tertentu di mana pada sudut yang melebihi had ini, pembahagian kuasa yang dikehendaki tidak akan diperolehi lagi.

INTRODUCTION

Branching optical waveguides are expected to play bigger roles in future optical technology applications. Their basic functions of power dividing and mode splitting make them very attractive for optical communications and optical computing, especially in optical network applications, where they can be used as switches and/or signal routers. In fact, today they can be found in practically all electro-optical channel waveguide switch designs [1]. Among others, Mach-Zehnder interferometers and Y-junctions are employing the branching waveguide principles. Single mode Y-junctions are widely used as 3 dB couplers for the guided light in optical push-pull modulators [2]. Presently, analyses on the branching waveguides are readily available [3-6]. However, the stresses are only on the mechanisms of the power flow in various types of branching waveguides and no clear relation between the power divisions (among the waveguides) and the branching angles are provided. Whereas, these two parameters are undeniably the two most obvious and immediate parameters in branching waveguide systems. In this paper, analysis is made on the 3-branch optical channel waveguide system consisted of three identical single mode waveguides as shown in

Figure 1. It is assumed that the system is lossless. The analysis is based on the coupled mode theory results on the parallel waveguide system [7-9]. The equation relating the power division to the branching angle and the coupling length is derived and the results are presented and plotted.

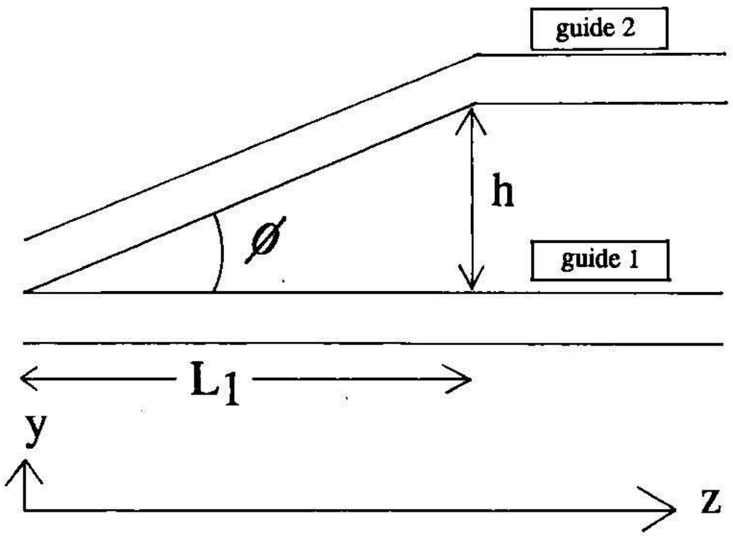

FIGURE 1. Schematic diagram of a branching waveguide system to show the relationship between branching angle, ϕ and coupling length, L_1, for a linear rate of departure, f(z) at $0 \leq z \leq L_1$.

THEORY

The analysis is based on the linearly branching waveguide system as modeled in Figure 1.

Since a parallel waveguide system can be considered as a special case of branching waveguide systems, the analysis is built up from the results of the former. It was found that the power flow expression in a parallel waveguide system takes the form as given in [8].

$$I_1(z) = |E_1(z)|^2 = |E_2(z=0)|^2 \sin^2 Kz + |E_1\,(z=0)|^2 \cos^2 Kz - 2E_1\,(z=0)\,E_2^*\,(z=0) \quad (1a)$$

$$I_2(z) = |E_2(z)|^2 = |E_1(z=0)|^2 \sin^2 Kz + |E_2\,(z=0)|^2 \cos^2 Kz - 2\;E_1(z=0)E_2^*(z=0) \quad (1b)$$

where $E_{1,2}$ are the respective electric field in waveguides 1 and 2, * is the notation for phase conjugate and κ is the coupling coefficient. With the initial conditions of $I_1(z{=}0){=}1$ and $I_2(z{=}0){=}0$ (only guide 1 is excited) equations 1a and 1b become

$$I_1(z) = 1\cos^2(Kz) \tag{2a}$$

$$I_2(z) = 1\cos^2(Kz) \tag{2b}$$

The coupling coefficient κ was derived by A. Yariv [2] to have the form of

$$K = \frac{2\delta^2\gamma e^{-\gamma h}}{\beta w(\delta^2 + \gamma^2)} \tag{3}$$

where δ and γ are the evanescent field decay constants in the substrate and in the guide respectively, β is the propagation constant, h is the distance between the guides and w is the width of the guide.

For the case of branching waveguides, the power flow expression will be slightly different from equations 1a and 1b which were for the parallel waveguide systems. This is due to the fact that, as shown by equation 3, κ is dependent on the interguide length, h which is a constant in parallel waveguide systems, but changes with z in the branching waveguide systems. Thus, the power flow equation in the branching waveguide systems will be a function of z as expressed below.

$$I_1(z)\cos^2\alpha(z) \tag{4a}$$

$$I_2(z)\cos^2\alpha(z) \tag{4b}$$

$$\text{where } \alpha(z) = \int_0^z K(z)dz \tag{5}$$

If the separation between the guides varies in such a way that α(z) approaches a finite value with increasing z, the oscillatory behavior of equation 5 disappears [10] and the power carried by each guide will approach a constant value determined by the value of α(∞).

$$I_1(\infty) = \cos^2\alpha(\infty) = A \tag{6a}$$

$$I_2(\infty) = \cos^2\alpha(\infty) = B \tag{6b}$$

where A and B are constants.

Now, the task is to carry out the integration in equation 5 to determine α(z) and essentially its relation to the power flow in the branches. To do this we have to express κ(z) as a function of various parameters of the waveguides. By assuming that all the material properties and the dimensions except the separation between the guides are independent of z, we can, express equation 3 as

$$K(z) = \zeta e^{\gamma c(z)} \tag{7}$$

where $\zeta = \dfrac{2\delta^2}{\beta w(\delta^2 + \gamma^2)}$ is the constant term as in equation 3.

In equation 7, $c(z) = c_o + f(z)$ where c_o is the initial separation at $z = 0$ and $f(z)$ is the function for the rate of departure of the upper guide from being parallel. In this case c_o is taken to be 0 and $f(z)$ a linear function of z such that

$$f(z) = uz \tag{8}$$

where u is a constant. Thus equation 7 becomes

$$K(z) = \zeta e^{-\gamma uz} \tag{9}$$

The integration of equation 5 from 0 to z yields

$$\alpha(z) = \frac{\zeta}{\gamma u}\left[1 - e^{-\gamma uz}\right] \tag{10}$$

From equation 10 we can see that $\alpha(z)$ is dependent on the rate of separation between the guides which in turn is linearly dependent on the direction of propagation z. Realizing that the branching angle is a more obvious parameter of a branching waveguide system, we are now going to rewrite equation 10 in terms of branching angle, ϕ. As shown in Figure 1, the branching angle ϕ is given by

$$\tan\phi = \frac{uz}{z} = u \tag{11}$$

Substitution of equation 11 into equation 10 yields

$$\alpha(z) = \frac{\zeta}{\gamma \tan\phi}\left[1 - e^{-\gamma z \tan\phi}\right] \tag{12}$$

Thus, the expression of power carried by guides 1 and 2 respectively are

$$I_1(\phi, z) = \cos^2 \alpha(\phi, z) = \left\{\frac{\zeta}{\gamma \tan\phi}\left[1 - e^{-\gamma z \tan\phi}\right]\right\} \tag{13a}$$

$$I_2(\phi, z) = \sin^2 \alpha(\phi, z) = \left\{\frac{\zeta}{\gamma \tan\phi}\left[1 - e^{-\gamma z \tan\phi}\right]\right\} \tag{13b}$$

Equations 13a and 13b show the dependence of power on the branching angle, ϕ and on the distance of propagation along z (thus, the coupling length, $z = L_t$).

Our next and final task is to find the relation between ϕ and L_t at certain power divisions between the waveguides. For instance, if we need the power carried by the upper guide (guide 2) to be $I_2 = 0.25\ I_{in}$, $I_2(z)$ in equation 13b must be set to be equivalent to 0.25, considering unity input power I_{in}=1

$$\sin^2\left\{\frac{\zeta}{\gamma \tan\phi}\left[1-e^{-\gamma z \tan\phi}\right]\right\} = 0.25 \tag{14a}$$

or

$$\frac{1}{\tan\phi}\left[1-e^{-\gamma L_1 \tan\phi}\right] = \frac{0.524\gamma}{\zeta} \tag{14b}$$

at z = L_1, the coupling length. Now, let us define the constant term on the right side of equation 14b as

$$\psi = \frac{0.524\gamma}{\zeta} \tag{15}$$

Thus, equation 14b becomes

$$\psi \tan\phi = 1 - e^{-\gamma L_1 \tan\phi} \tag{16a}$$

or

$$L_1 = -\frac{\ln(1-\psi\tan\phi)}{\gamma\tan\phi} \tag{16b}$$

Equations 16a and 16b present the relation between the coupling length, L_1 and the branching angle, ϕ for the power carried by guide 2, $I_2 = 0.25$. To generalize these equations for any desired division of power, such that $I_2 = MI_{in}$, where $0 \le M \le 1$, equation 16b can be written as

$$L_1 = -\frac{\ln\left(1-\frac{\gamma}{\zeta}\sin^{-1}\left(\sqrt{M}\right)\tan\phi\right)}{\gamma\tan\phi} \tag{17}$$

by rewriting equation 15 as $\frac{\gamma\sin^{-1}\left(\sqrt{M}\right)}{\zeta} = \psi$ and substituting it into equation 16b. The calculated results for several specific M are plotted in Figure 2 for $\frac{\gamma}{\zeta} = 23.864$ (at λ = 0.8 μm, guide depth, d = 1.8 μm, guide width, w = 4 μm) [9].

Note that, since L_1 is a scalar quantity, equation 17 is valid only if

$$0 \le 1 - \frac{\gamma}{\zeta}\sin^{-1}(M)\tan\phi \le 1 \tag{18a}$$

or

$$0 \le \phi \le \tan^{-1}\left(\frac{\zeta}{\gamma\sin^{-1}(M)}\right) \tag{18b}$$

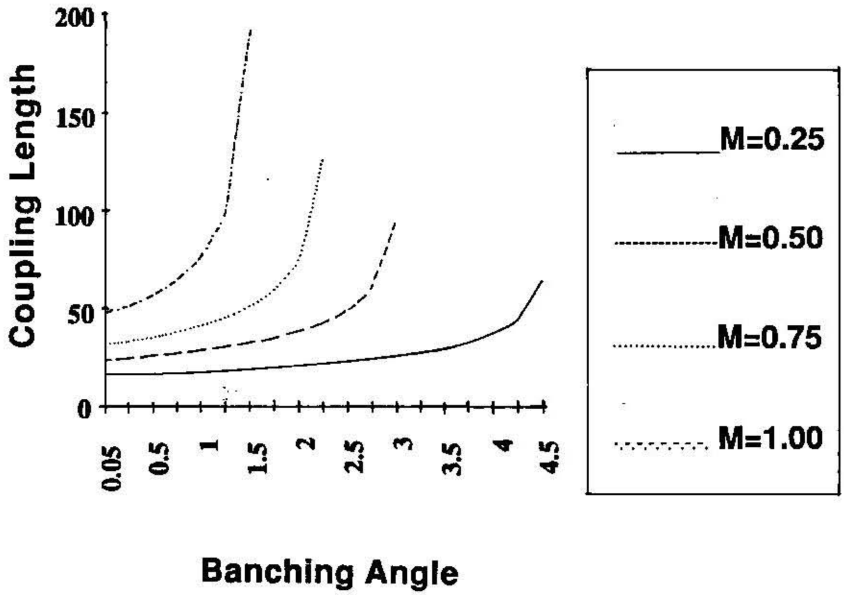

FIGURE 2. Coupling length, L_l (9μm) versus branching angle (degree) for various power fractions M where $M = I_2/I_{in}$

Equations 18a and 18b set the limit of ϕ for certain division of power, M between the guides in Figure 1.

Figure 2 shows the branching angles and the corresponding coupling lengths for several power divisions (M = 0.25, 0.50, 0.75, 1.00) between the two guides. The values for any other power divisions ($0 \leq M \leq 1$) can be obtained by using equation 17. It is clear that the greater value of coupling lengths correspond to the smaller branching angles for greater M. The figure also shows that at a fixed branching angle, longer coupling length gives greater power division M and this is expected to continue until the coupling length reaches a value that corresponds to an interguide gap, h at which the mode coupling is no longer significant and the power division is maintained. However, there are branching angle limits set by the asymptotic increment of the curves. For example, as the coupling length increases the power division approaches M = 1 for the branching angles less than about 1.5° and so on.

SUMMARY AND CONCLUSION

The characteristics of branching waveguides make them very important to optical technology especially in the fast pacing fields of optical networks and optical computing. The power division between the two single-mode branches (refer to Figure 1) is found to vary with the coupling length and the branching angle as clearly shown by equation 17. Equation 17 is new and believed to be the most straight-forward in relating the power division to the coupling length and the branching angle. The results for several power divisions are plotted in Figure 2 and found to be consistent with the existing analyses [10-11]. The greater the branching angle, the longer the coupling length is needed to maintain the particular power division. However, as the analysis in the paper is made for an ideal system, the losses especially the radiation loss which is associated with the branching angle are disregarded here. Thus, experiments are highly suggested for more practical results.

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