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Use of Strain-life Models with Wavelet Bump Extraction (WBE) for Predicting Fatigue Damage

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ABSTRACT

This paper presents the use of strain-life fatigue damage models to observe the cycle sequence effects in the wavelet-based fatigue data editing algorithm. This algorithm is called Wavelet Bump Extraction (WBE), which was developed to produce a shortened signal by extracting fatigue damaging events from the original signal with the retention of the original cycle sequences. Current industrial practice uses the Palmgren-Miner linear damage rule to predict the fatigue life or fatigue damage under variable amplitude (VA) loadings. Using VA loadings, however, this rule does not have load interaction accountability in the analysis. Thus, a more suitable approach has been identified for predicting fatigue damage of VA loadings, i.e. the Effective Strain Damage (ESD) model. In this study, the cycle sequence effect observation was implemented in both analytical and experimental works using the WBE extracted VA loadings. The study includes the comparison between the experimental and the analytical (using four strain-life fatigue damage models: Coffin-Manson, Morrow, Smith-Watson-Topper and ESD) fatigue damage. The smallest average in the fatigue damage difference was found when using the ESD strain-life model, suggesting the suitability of the this model for analysing VA fatigue loadings.

Keywords: Fatigue, variable amplitude, WBE, cycle sequence effects, ESD Model.

ABSTRAK

Kertas kerja ini membincangkan penggunaan model terikan-hayat lesu untuk mengkaji kesan jujukan kitaran dalam algoritma penyuntingan data lesu berasaskan teori gelombang-isyarat. Algoritma ini yang dikenali sebagai Wavelet Bump Extraction (WBE) diguna untuk menghasilkan suatu isyarat terpangkas iaitu dengan *memilih hanya isyarat dari peristiwa lesu bermagnitud musnah daripada gelombang isyarat asal dan di samping itu, mengekalkan jujukan kitaran isyarat asal pada isyarat yang terhasil. Amalan industri masa kini menggunakan kaedah linear Palmgren-Miner bagi meramalkan jangka hayat lesu. Akan tetapi kesan jujukan kitaran yang wujud dalam kes lesu bebanan amplitud berbagai tidak diambil kira dalam kaedah tersebut. Justeru itu, model terikan hayat lesu Effective Strain Damage (ESD) yang mempertimbangkan kesan jujukan kitaran dirasakan lebih sesuai untuk digunapakai. Dalam kajian ini, pemerhatian terhadap kesan jujukan kitaran dilakukan secara analitik dan eksperimen dengan menggunakan isyarat beban yang terhasil daripada WBE. Perkadaran jangka hayat lesu dari keputusan eksperimen dibandingkan dengan keputusan analitik yang menggunakan empat model terikan-hayat yakni Coffin-Manson, Morrow, Smith-Watson-Topper dan ESD. Ramalan model ESD memberikan bandingan perbezaan purata hayat lesu yang terkecil, menunjukkan kesesuaian model ini dalam analisis lesu beban amplitud berbagai.*

Kata kunci: Model terikan-hayat lesu, amplitud pelbagai, algoritma Wavelet Bump Extraction, kesan jujukan kitaran beban amplitud berbagai, model Effective Strain Damage.

INTRODUCTION

Fatigue life prediction is important in the design process of vehicle structural components, with the essential input variable for fatigue is the load history. Practically, automobile manufacturers go to great lengths to instrument vehicles and subject them to a variety of driving conditions. By necessity, vehicle development requires accelerated fatigue testing and this is often accomplished by correlating test tracks with public road data. Both roads and test tracks generate variable amplitude (VA) load time histories. Loads that are predicted to do little or no damage can be eliminated and large amplitude cycles are retained. The process can be performed using a wavelet-based fatigue data editing, known as Wavelet Bump Extraction (WBE) algorithm which was developed by Abdullah et al. (2003; 2004) with the retention of the loading cycle sequences.

The situation where the order of loading affects the fatigue life is called a sequence effect. Sequence effects exist both in the crack initiation stage and crack propagation stage (Fuchs and Stephens 1980). Sequence effects are also related to overload and underload condition in VA loadings (DuQuesnay et al. 1993; Topper and Lam 1997). When overloads were inserted in the small cycle or below the material fatigue limit, the small cycles following the overloads contributed to the fatigue damage accumulation. Considering the importance of sequence effects in the fatigue damage prediction under VA loadings (Dowling 1999), therefore, a suitable approach was introduced (DuQuesnay et al. 1993) in order to have closer fatigue life predictions when compared to the respective experimental findings.

In this paper the cycle sequence effects are analysed using the VA loadings extracted by the WBE algorithm. The analytical fatigue lives using four strain-life models were compared to experimental findings in order to observe the load interaction effects in fatigue damage prediction. To date, there are no current studies that are related to the analysis of cycle sequence effects using the loadings extracted from fatigue data editing algorithms. This paper, finally, discusses one of the stages for validating the WBE effectiveness, i.e. fatigue cycle sequence effects observation.

BACKGROUND OF THE WAVELET BUMP EXTRACTION (WBE) ALGORITHM

The Wavelet Bump Extraction (WBE) algorithm was designed to identify and extract fatigue damaging events in order to produce a shortened loading. This shortened loading should have equivalent fatigue damage and global signal statistical values to the original input loading. Using WBE, large amplitude segments were extracted with the retention of the original load cycle sequences. WBE has three main stages (Figure 1): the wavelet decomposition process, the identification and extraction of the fatigue damaging events, and the production of a mission signal.

WBE uses the orthogonal wavelet transform by means of the 12th order Daubechies wavelets which were chosen as the basis functions. Each wavelet level describes the time behaviour of the signal in a frequency band. Using the WBE algorithm, fatigue damaging events are identified in wavelet groups. A wavelet grouping stage permits the user to cluster wavelet levels into a single region of significant signal vibrational energy.

A bump, which is an oscillatory transient with a monotonic decay envelope either side of a peak value (Figure 2a), is identified in each wavelet group by means of an automatic trigger level (Figure 2b). Bump identification is performed by means of a search which identifies the points at which the signal envelope inverts from decay behaviour. After all the bumps are identified in all the wavelet groups, bump segments are extracted (Figure 2c) by removing the original time history of the complete section between the start and the end of the bump. The extracted bump segments are combined to produce a mission time history which has equivalent signal statistics and fatigue damage potential of the original loading.

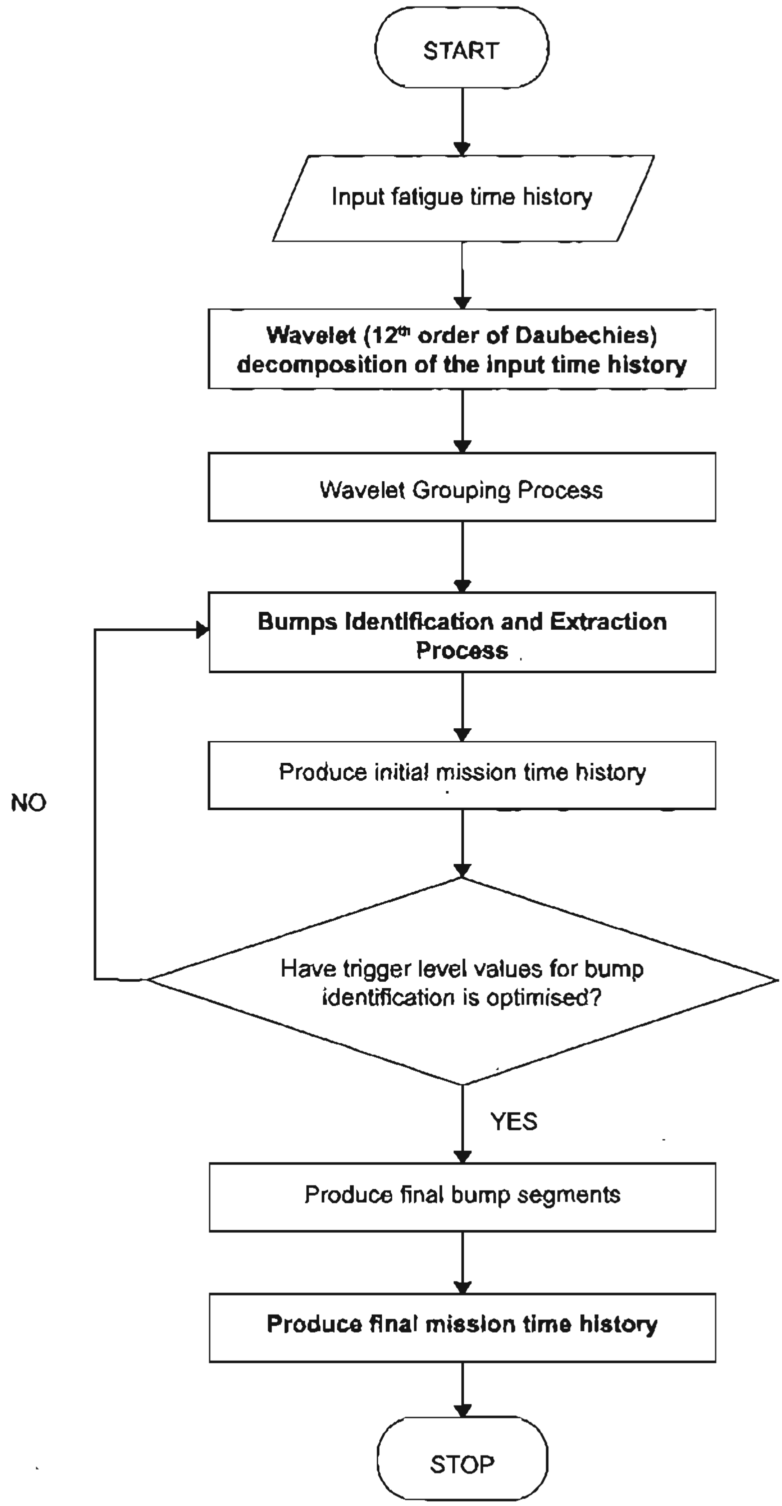


FIGURE 1. Flowchart of the wavelet bump extraction (WBE) algorithm

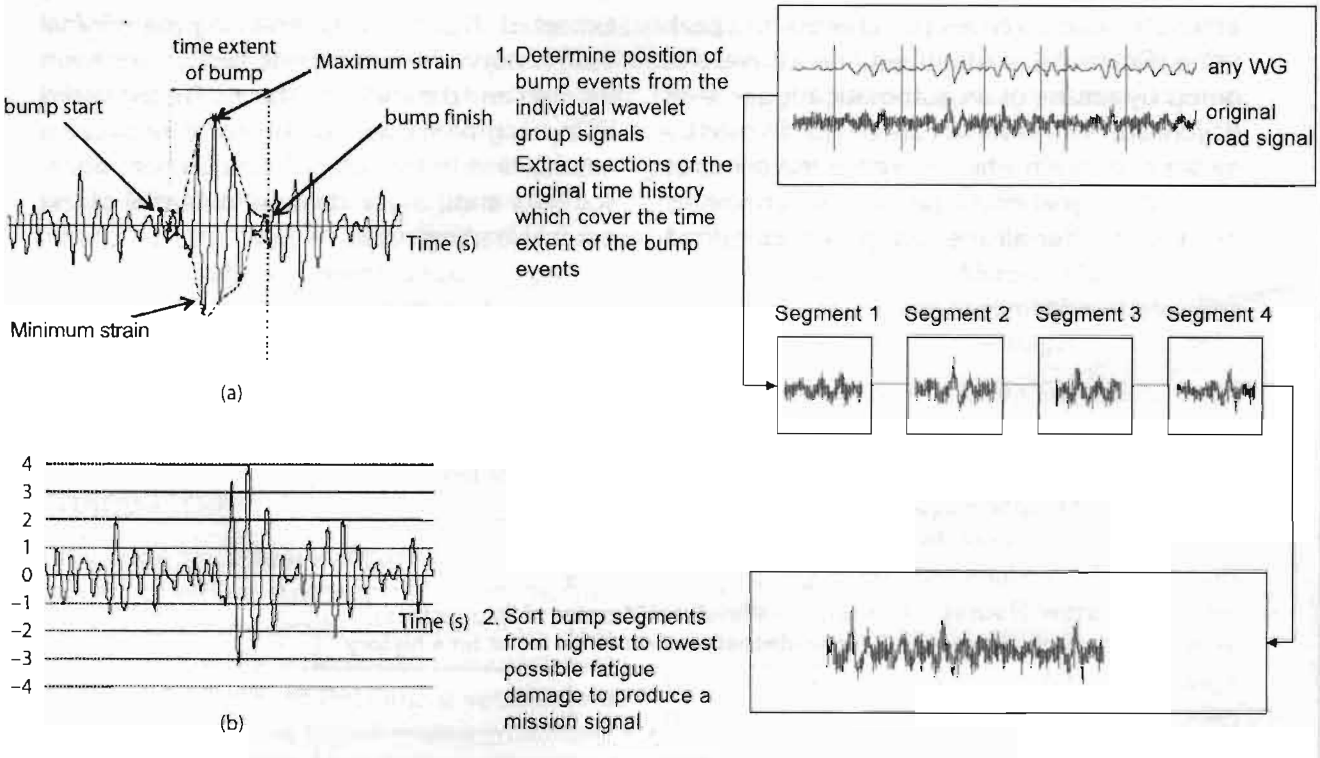


FIGURE 2. (a) Decay enveloping of a bump, (b) Possible trigger level values, (c) Production of a mission signal.

FATIGUE LIFE PREDICTION

Current industrial practice for fatigue life prediction is to use the Palmgren-Miner (PM) linear damage rule (Palmgren 1924; Miner 1945). This rule is normally applied with strain-life fatigue damage models for analyzing shorter fatigue life problems. The strain-life fatigue life behaviour considers plastic deformation that occurs at the localised region where fatigue cracks begin with the influence of a mean stress. The first strain-life model is the Coffin-Manson relationship (Coffin 1954; Manson 1965) and is defined as

$$\varepsilon_a = \frac{\sigma'_f}{E}(2N_f)^b + \varepsilon'_f(2N_f)^c \tag{1}$$

where ε_a is the strain amplitude, σ_a is the stress amplitude, σ'_f is the fatigue strength coefficient, E is the modulus of elasticity, N_f is the number of cycles to failure, b is the fatigue strength exponent, ε'_f is the fatigue ductility coefficient and c is the fatigue ductility component.

Some of the realistic service situations involve non-zero mean stresses. Two mean stress effect models are used in the strain-life fatigue damage analysis, i.e. Morrow and Smith-Watson-Topper (SWT) strain-life models. Mathematically, the Morrow's model is defined by (Morrow 1968)

$$\varepsilon_a = \frac{\sigma'_f}{E}\left(1-\frac{\sigma_m}{\sigma'_f}\right)(2N_f)^b + \varepsilon'_f(2N_f)^c \tag{2}$$

where σ_m is the mean stress. The SWT strain-life model is mathematically defined as (Smith et al. 1970)

$$\sigma_{max}\varepsilon_a = \frac{(\sigma'_f)^2}{E}(2N_f)^{2b} + \sigma'_f\varepsilon'_f(2N_f)^{b+c} \tag{3}$$

where σ_{max} is the maximum stress.

Several limitations were found in the implementation of the PM linear damage rule. The fatigue damage is accurately calculated for constant amplitude (CA) loadings, but it may lead to the erroneous prediction for VA loadings (Dowling 1999). It is because the PM linear damage rule assumes no load sequence effect and lacks load interaction accountability. Considering the importance of sequence effects in VA loadings, therefore, a suitable approach was identified. Several studies related to the fatigue life prediction on metal components under VA loadings have been carried out by Fatemi and Yang (1998), such as a model derived from random vibration theory, the fracture mechanics approach, etc. In spite of better improvement in fatigue life prediction contributed by these models, however, they were difficult to associate for general use.

The use of overload events with a simple linear damage model (DuQuesnay et al. 1993) has been found to be suitable for this purpose. This strain-life fatigue damage model, or known as Effective Strain Damage (ESD), is based on crack growth and crack closure that works well for a wide variety of materials, load spectra, component geometries, strain magnitudes and mean-strain effects. This model was developed for safe-life or life to crack detection, so that fatigue damage can be analysed based on the short crack growth.

The ESD model is mathematically defined as follows:

$$E\Delta\varepsilon^* = A(N_f)^B \tag{4}$$

where E is the elastic modulus of the material, $\Delta\varepsilon^*$ is a net effective strain range for a closed hysteresis loop that is related to fatigue crack growth. A and B are the material constants, and Nf is the number of cycles to failure. The magnitude of $E\Delta\varepsilon^*$ for a given cycle is a function of crack-opening stress, Sop, level and it is dependant on the prior stress and strain magnitudes in the loading history. The expression of $E\Delta\varepsilon^*$ can be expanded to

$$E\Delta\varepsilon^* = E\left(\varepsilon_{max} - \varepsilon_{op}\right) - E\varepsilon_i \tag{5}$$

where ε_{max} and ε_{op} are the maximum strain and the crack-opening strain of the particular cycle, respectively. ε_i is the intrinsic fatigue limit strain range under the VA loading condition.

In order to consider the cycle sequence effects in the fatigue life calculation, a decay parameter, m, is used to define the change in a crack-opening stress between two adjacent cycles. ΔS_{op} was first defined as

$$\Delta S_{op} = m(S_{ss} - S_{cu}) \tag{6}$$

where S_{cu} is the current opening stress and S_{ss} is the steady-state opening stress. S_{cu} is defined as the S_{op} value of the previous cycle. S_{ss} is defined as

$$S_{ss} = \alpha S_{max}\left(1 - \left(\frac{S_{max}}{S_y}\right)^2\right) + \beta S_{min} \tag{7}$$

where α and β are the material constants, S_{max} is the maximum stress of the previous largest cycle in the time history, S_{min} is the minimum stress of the previous largest cycle and S_y is the cyclic yield stress.

The damage caused by each cycle of the repeated loading is calculated by reference to the material strain-life curve. The N_f value can be obtained from Equation (1) – (4) for all listed strain-life models. Finally, the fatigue damage (*D*) potential for one cycle is defined as

$$D = \frac{1}{N_f} \tag{8}$$

APPLICATION OF THE WBE ALGORITHM USING VARIABLE AMPLITUDE LOADINGS

The effectiveness of the WBE algorithm was evaluated using a VA strain loading which was measured on a van suspension arm while driving over a pavé test track. The type of road surface is shown in Figure 3. The suspension arm pavé test data set was chosen because it contained many small amplitude, high frequency and transient events in the signal background. The signal was sampled for 23,000 discrete points at a sampling rate of 500 Hz. Figure 4a shows a plot of the acquired 46-second time history. Using the WBE algorithm the bump segments that produced the majority of fatigue damage were identified and extracted, so as to achieve the mission signal which matched the original data within ±10% in terms of root-mean-square and kurtosis value (Figure 4b). The segments were combined to produce the 18.8-second mission signal (Figure 4c).

LABORATORY FATIGUE TESTING AND RESULTS

The material chosen for the test sample was BS 080A42 steel, which is often used in the suspension components of passenger cars. Smooth specimens were used for the fatigue tests and the specimen geometry is shown in Figure 5. The specimens were hand-polished with 60-1000 grades of silicone carbide abrasive papers and finished with 6-μm diamond compound in order to produce an hourglass profile. A servo-hydraulic test machine was used in displacement control mode for all tests.

A strain-life curve, as shown in Figure 6a, was obtained from CA fatigue tests at seven different strain amplitudes. Using the CA fatigue test data and a tensile test data, the mechanical properties is presented in Table 1. The cyclic mechanical properties were obtained using CA fatigue test data with Equation (1) and (4). Uniaxial VA fatigue

FIGURE 3. The pavé test track used to measure fatigue road loading

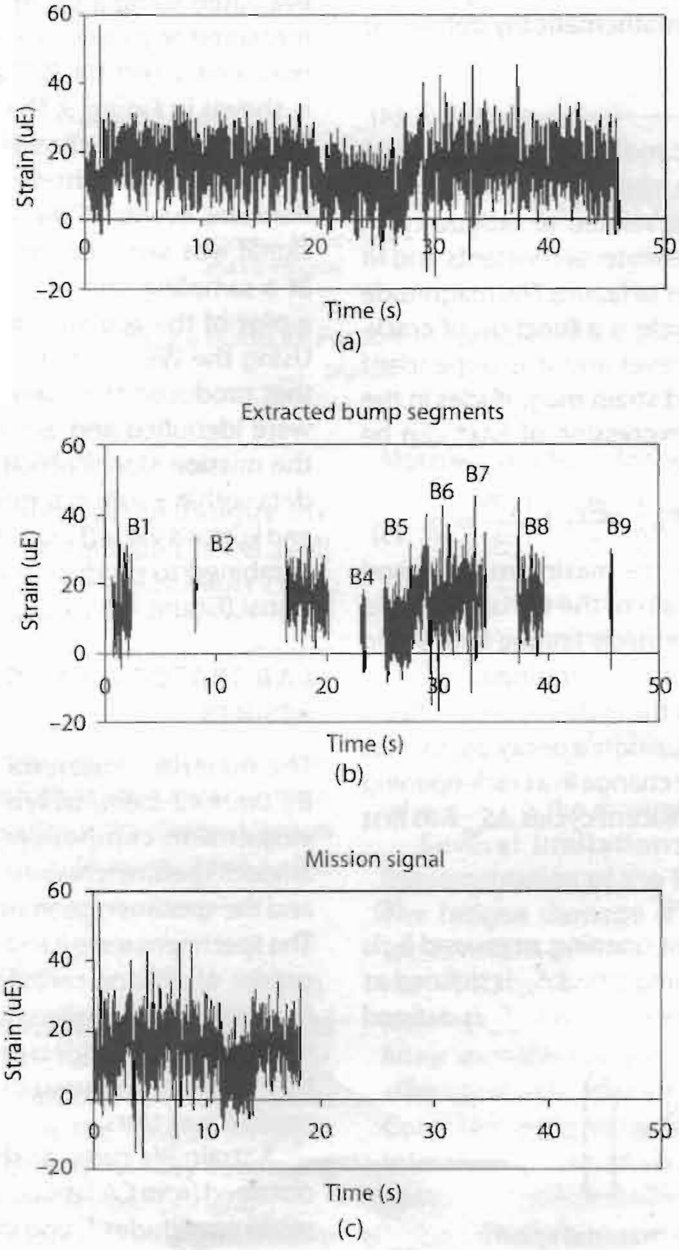
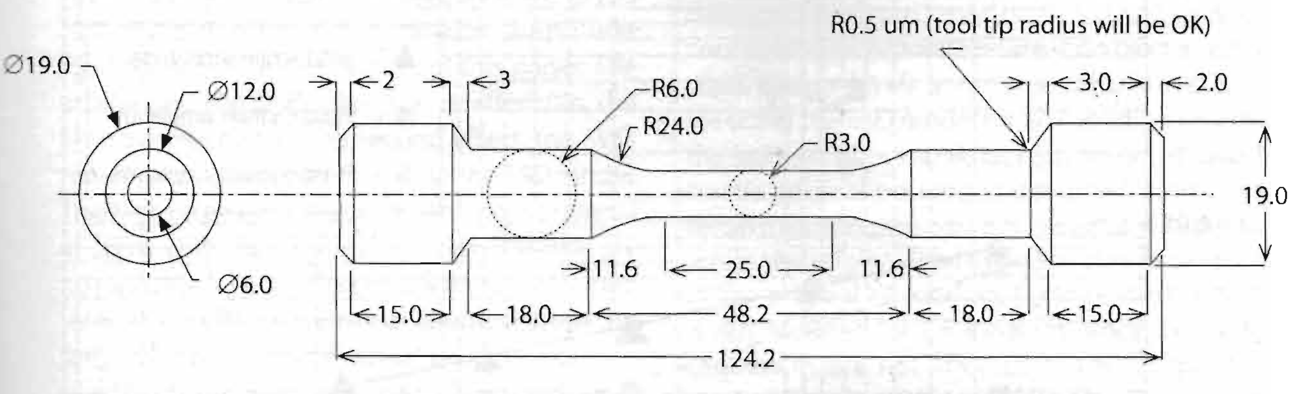


FIGURE 4. Plot of time histories: (a) Experimetally measured VA loading, (b) Extracted bump segments with the bump segment label numbers, (c) The mission road loading



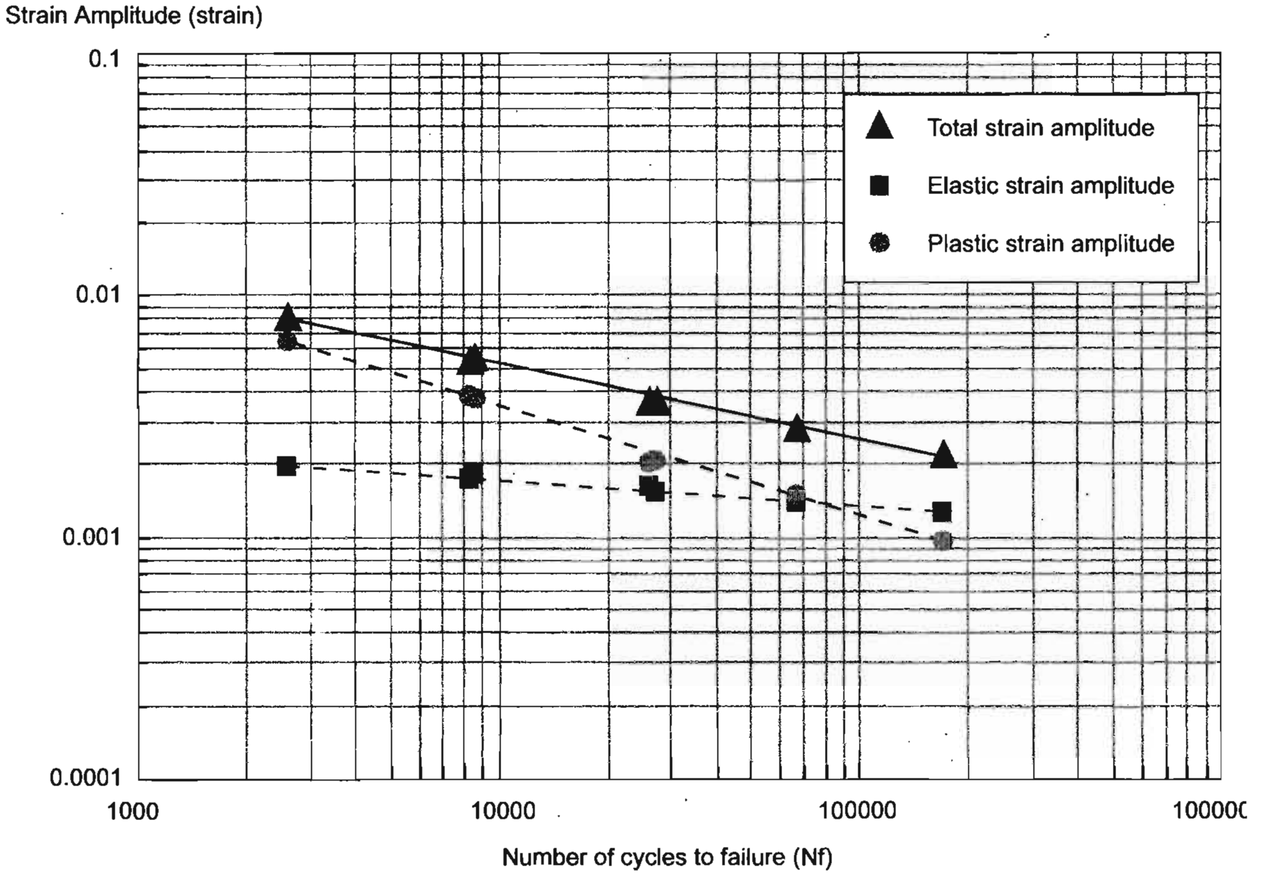
(a)

(b)

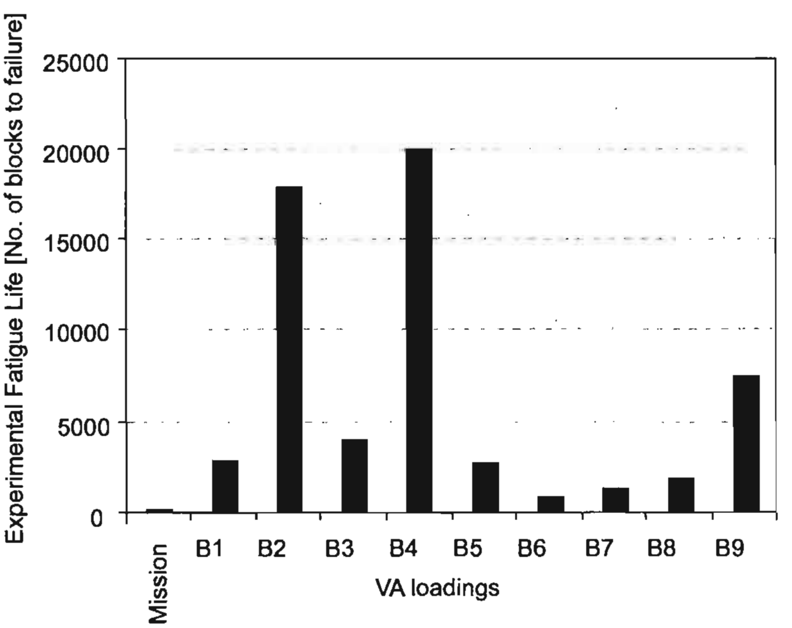
FIGURE 5. (a) The geometry of a smooth specimen - all in the unit of mm unless specified, (b) A specimen at the servo-hydraulic machine with a 25-mm extensometer

TABLE 1. Mechanical properties of BS 080A42 steel

Monotonic Mechanical Properties		**Cyclic Mechanical Properties**	
Ultimate tensile strength, S_u [MPa]	624	Fatigue strength coefficient, σ'_f [MPa]	1505
Modulus of elasticity, *E* [GPa]	210	Fatigue strength exponent, *b*	-0.144
Static yield stress [MPa]	342	Fatigue ductility coefficient, ε'_f	0.176
Area reduction, [%]	51.9	Fatigue ductility exponent, *c*	-0.400
Elongation [%]	28.4	Material constant *A* [GPa] in Equation (4)	119
		Material constant *B* in Equation (4)	-0.5
		$E\varepsilon_i$ in Equation (5) [MPa]	270
		Decay parameter *m* in Equation (6)	0.002
		Material constant α in Equation (7)	0.55
		Material constant β in Equation (7)	0.23



(a)



(b)

FIGURE 6. (a) Experimental strain-life curve, (b) Experimental fatigue lives

tests were performed using ten WBE extracted VA loadings (refer to Figure 4): nine WBE bump segments and the WBE mission signal. The purpose of these fatigue tests is to observe the fatigue lives produced by the actual original sequence of VA loadings, so that the original fatigue cycle sequence effects can be observed. Figure 6b presents the experimental fatigue life results.

DISCUSSIONS

Four strain-life models were used for the analysis of fatigue damage using variable amplitude loadings, which is the subject of this paper. The models are the Coffin-Manson relationship, the Morrow mean stress correction effect, the SWT mean stress correction effect and the ESD model. The first three are suitable for the conventional analysis with the CA loading. These models are not suitable for analysing VA loadings, as the load interaction effects are not accounted for in the fatigue life prediction. Despite this, they are often used for VA problems in practice. To properly treat VA loading histories, the ESD strain-life model was developed in order to perform for life-to-crack detection.

In the ESD model the damage parameter that is treated is the effective strain range. The fatigue damage is analysed based on short crack growth concepts to incorporate retardation by changing crack closure levels. In order to calculate fatigue life using the ESD model for a VA loading history, the fatigue cycles should be reconstructed based on the original position in the time history. This reconstruction procedure is required in order to retain the original load cycle sequences present in the original VA loading. Cycle reconstruction is not required for CA loadings, since the cycle sequences are not affected in the fatigue life prediction using this type of loading. The fatigue cycle reconstruction consists of converting a time history into a series of peak-valley reversals. These reversals were then rainflow counted (Matsuishi

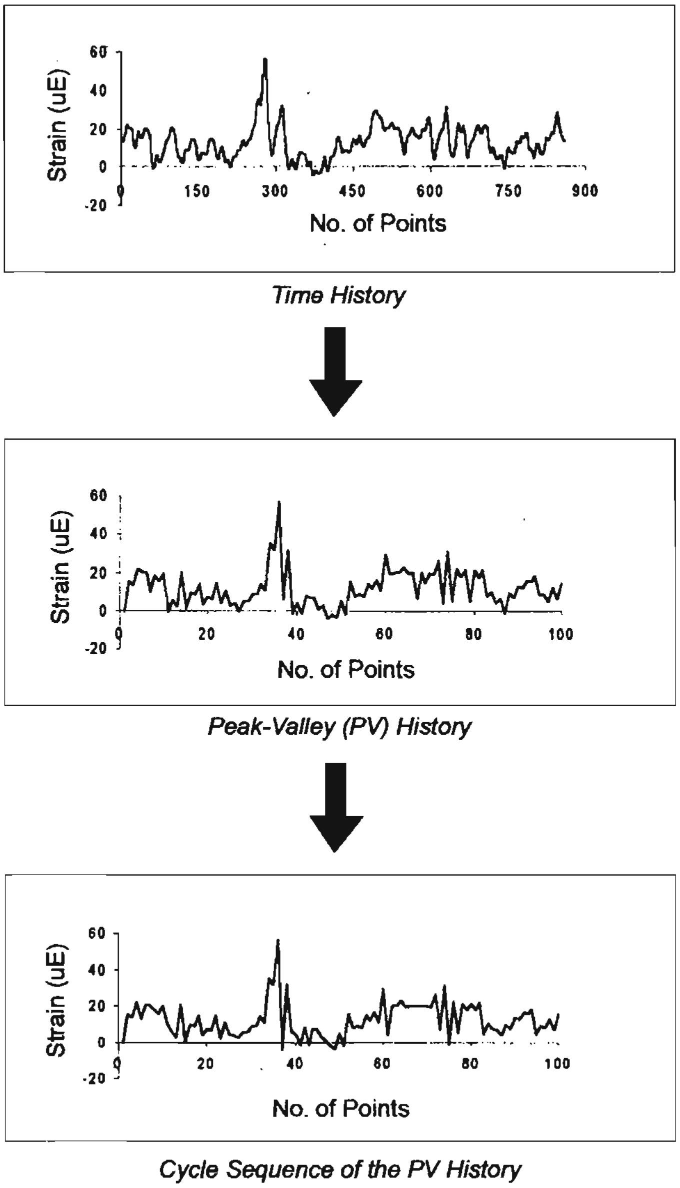


FIGURE 7. Time history reconstruction for the first bump segment (B1) of the signal

and Endo 1968) in order to extract fatigue cycles. The cycles were then sorted based on the peak-valley history in order to produce a similar pattern to the original load sequences. For example, a reconstructed cycle history is shown in Figure 7 for the first bump segment (B1) of the signal.

Table 2 shows the fatigue damage values of ten VA loadings calculated using four strain-life models. In addition, the findings obtained from the laboratory fatigue testing were also presented in this table. From the information in Table 2, the calculated fatigue damage for each bump segment and mission signal was compared to the respective experimental value. The total average fatigue damage difference (AD) between the Morrow strain-life model and experiment gave the highest value of 566%. Using the ESD model, the smallest difference was found to be at 21%. These results show a better accuracy of the ESD model to predict the fatigue damage of VA loadings compared to the other models.

Figure 8 shows the correlation of fatigue lives between experiment and all four strain-life models, for which each data point represents a

TABLE 2. Fatigue damage obtained using strain-life models and experiment

SIGNAL	Fatigue Damage [damage/block]				
	Coffin-Manson	Morrow	SWT	ESD	Experiment
B1	1.6×10^{-4}	9.6×10^{-5}	1.7×10^{-4}	5.1×10^{-4}	3.5×10^{-4}
B2	9.3×10^{-6}	8.8×10^{-6}	1.0×10^{-5}	5.1×10^{-5}	5.6×10^{-5}
B3	6.6×10^{-5}	5.6×10^{-5}	7.1×10^{-5}	4.2×10^{-4}	2.5×10^{-4}
B4	9.9×10^{-6}	6.1×10^{-6}	1.0×10^{-5}	3.9×10^{-5}	5.0×10^{-5}
B5	7.0×10^{-5}	4.1×10^{-5}	7.3×10^{-5}	3.7×10^{-4}	3.6×10^{-4}
B6	2.8×10^{-4}	1.6×10^{-4}	2.9×10^{-4}	1.1×10^{-3}	1.2×10^{-3}
B7	1.5×10^{-4}	9.6×10^{-5}	1.6×10^{-4}	6.5×10^{-4}	7.3×10^{-4}
B8	1.0×10^{-4}	6.8×10^{-5}	1.1×10^{-4}	4.5×10^{-4}	5.1×10^{-4}
B9	2.6×10^{-5}	1.8×10^{-5}	2.8×10^{-5}	1.2×10^{-4}	1.3×10^{-4}
Mission	9.5×10^{-4}	9.2×10^{-4}	5.0×10^{-4}	3.1×10^{-3}	4.9×10^{-3}
*AD [%]	364	566	388	21	- *na* -

*AD is the average fatigue damage difference between a specified strain-life model and experiment.
na – not available

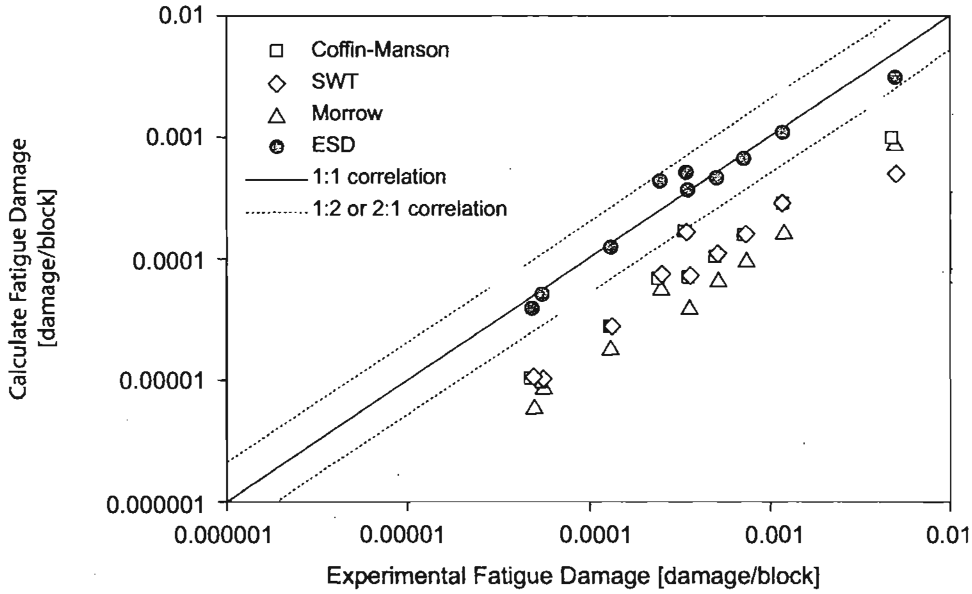


FIGURE 8. Fatigue damage correlation between the calculation and the experiment using the WBE extracted loadings

loading condition in Table 2. It is found that the correlated fatigue lives produced by the ESD model and experiments were distributed around the 1:1 line and within the range of ± a factor of 2. However, the correlation points produced from the data of the three other strain-life models (Coffin-Manson, Morrow and SWT) were located outside the range of ± a factor of 2. The findings in Figure 8 suggested that the ESD model provided the closest correspondence between the predicted fatigue life and the experimental fatigue life.

Finally, it is concluded from these findings that ESD is again the most suitable strain-life model to predict fatigue damage of VA loadings. Hence, it is proven that the WBE algorithm can be efficiently applied with the ESD strain-life model for extracting the fatigue damaging events, whilst retaining the original cycle sequence effects in the bump segments and the mission signal.

CONCLUSION

1. The Effective Strain Damage (ESD) strain-life model was identified as a suitable strain-life model accounting for the cycle sequence effects. The average fatigue life difference between the ESD model and experiment was 21%, which is the smallest compared to the other models.
2. A closer correspondence between the analytical and the experimental findings was obtained when the fatigue damage was predicted using the ESD model. It is found that the respective fatigue life correlation points were distributed around the 1:1 correlation line (Figure 8).
3. The combination of WBE and ESD provide a novel wavelet-based fatigue data editing technique which accurately extracts the most fatigue damaging events (bump segments) and preserves of the original load cycle sequence within the fatigue loadings.

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NOMENCLATURE

A	Material constant for the ESD strain-life model [MPa]
b	Fatigue strength exponent
B	Material constant for the ESD strain-life model
c	Fatigue ductility component
D	Fatigue damage parameter
E	Modulus of elasticity [MPa]
N_f	Fatigue life [Number of cycles or blocks to failure]
S_{max}	Maximum stress amplitude [MPa]
S_{min}	Minimum stress amplitude [MPa]
S_{op}	Crack opening stress [MPa]
S_u	Ultimate strength of material [MPa]
S_y	Yield strength of material [MPa]
α	Material constant for steady state opening stress equation
β	Material constant for steady state opening stress equation
$\Delta\varepsilon^*$	Net effective strain range for a closed hysteresis loop
$\Delta\varepsilon_{eff}$	Effective strain range
ε'_f	Fatigue ductility coefficient
ε_a	Total strain amplitude
ε_i	Intrinsic strain fatigue limit under VA loading

ε_{op} Crack opening strain
σ'_f Fatigue strength coefficient
σ_{max} Maximum stress [MPa]

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