Jurnal Kejuruteraan SI 4(1) 2021: 117-128
https://doi.org/10.17576/jkukm-2021-si4(1)-15

Design of Aircraft Lateral Control Laws Simulation for Teaching and Learning

(Reka bentuk Simulasi Kawalan Lateral Pesawat untuk Pengajaran dan Pembelajaran)

Zuhairi Abdul Rashid[a,*], Valérie Budinger[b], Philippe Pastor[b], Syed Mohd Fairuz Syed Mohd Dardina & Norzaima Nordin[a]

[a]*Faculty of Engineering, National Defense University of Malaysia, Sg. Besi Camp, Kuala Lumpur, Malaysia.*
[b]*Institut Supérieur de L'aéronautique et de L'espace (ISAE-SUPAERO)*

[*]*Corresponding author: maidiana@upnm.edu.my*

Received 15 March 2021, Received in revised form 18 April 2021
Accepted 30 August 2021, Available online 30 September 2021

ABSTRACT

This paper studies the design of a simulator to simulate the general behaviour of an aircraft in its lateral motion. The study will be in two parts: the aircraft lateral natural system and the design of the control laws. In the aircraft lateral natural system, all modes will be discussed including the Pure Roll mode, the Dutch Roll mode, and the Spiral mode. In the design of the control laws, two types of control laws were discussed which are the yaw damper and the poles placement method. The result is a complete simulator for aircraft lateral motion which allows the user to simulate and study the lateral characteristic of an aircraft given the desired input. The simulator resulted from this study is suitable to be used for teaching and learning as the changes in parameters yield instantaneous results that can be displayed.

Keywords: Aircraft simulator; lateral mode; control laws

ABSTRAK

Kertas kajian ini mengkaji pembangunan simulator yang mampu untuk melaksanakan simulasi keatas tingkah laku pesawat pada paksi lateral. Kajian yang dilaksanakan terbahagi kepada dua bahagian iaitu kajian terhadap system tingkah laku natural pesawat pada paksi lateral dan pembangunan kawalan pesawat. Dalam kajian system tingkah laku natural pesawat, kesemua mod penerbangan akan dibincangkan termasuk mod "pure roll", mod "Dutch Roll", dan mod "Spiral". Dalam pembangunan kawalan pesawat pula, dua jenis teknik pembangunan kawalan akan dibincangkan iaitu kaedah "yaw damper" dan kaedah "poles placement". Hasil daripada pembangunan ini adalah satu simulator yang lengkap untuk paksi lateral yang membolehkan pengguna untuk melaksanakan simulasi dan mengkaji tingkah laku pesawat pada paksi lateral apabila sesuatu input diberikan. Simulator yang terhasil dari kajian ini sesuai untuk digunakan dalam pengajaran dan pembelajaran kerana apabila perubahan parameter dilakukan, perubahan akan berlaku secara serta merta dan akan dipaparkan kepada pengguna.

Kata kunci: Simulasi pesawat; mod lateral; kejuruteraan kawalan

INTRODUCTION

This project paper explains how the project is done using the mathematical approach with the given parameter and estimation. The outcomes of this project are to have a reference model on how an aircraft generally fly in lateral mode and to have a sample model of a control law that can be implemented on an aircraft for lateral control. The control laws in this project are not implemented on an actual aircraft but solely designed for study purposes. The final result of this project will be a user-friendly simulator that will be used as a teaching aid for a better understanding of how an aircraft function. The objective of this project is to design an aircraft lateral simulation tool using MATLAB Graphical User Interface Development Environment (GUIDE) Tool. The simulation is designed to simulate the following:

1.Aircraft natural lateral mode.

2. Aircraft controlled mode using yaw damper.

3. Aircraft controlled mode using control feedback gain.

The aircraft chosen for this project is the Lockheed C-5A aircraft and the reference data for this aircraft is obtained in the book entitled "Introduction to Aircraft Flight Mechanics" by Thomas, R.Y. et al. (2014). The aircraft can be modelled as a linearized state-space representation as in Equation (1).

$$\begin{pmatrix} \dot{\beta} \\ \dot{r} \\ \dot{p} \\ \dot{\emptyset} \end{pmatrix} = \begin{pmatrix} \frac{Y_\beta}{V} & -\cos\alpha & \sin\alpha & \frac{g\cdot\cos\theta}{V} \\ N_\beta & N_r & N_p & 0 \\ L_\beta & L_r & L_p & 0 \\ 0 & \tan\theta & 1 & 0 \end{pmatrix} \cdot \begin{pmatrix} \beta \\ r \\ p \\ \emptyset \end{pmatrix} + \begin{pmatrix} \frac{Y_{\delta a}}{V} & \frac{Y_{\delta r}}{V} \\ N_{\delta a} & N_{\delta r} \\ L_{\delta a} & L_{\delta r} \\ 0 & 0 \end{pmatrix} \cdot \begin{pmatrix} \delta a \\ \delta r \end{pmatrix} \tag{1}$$

$$Y = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \beta \\ r \\ p \\ \emptyset \end{pmatrix}$$

METHODOLOGY

Where in Equation (1), the state matrix composes of sideslip angle, yaw rate, roll rate, and bank angle which represented by β, r, p, and Ø respectively. The state-space also contain variables for airspeed and angle-of-attack which represented by V and α respectively. The inputs for the state-space are the ailerons deflection angle and rudder deflection angle which represented by δa and δr respectively. The Y_β, $Y_{\delta a}$, $Y_{\delta r}$, N_β ,N_r ,N_p ,$N_{\delta a}$,$N_{\delta r}$,L_β, L_r,L_p,$L_{\delta a}$, and $L_{\delta r}$ represent the lateral-directional derivatives which have the value as in Table 1.

The simulation is done as if the aircraft is flying at sea level with the lateral-directional derivative for the C-5A at sea level given as in Table 1.

The final part will discuss other features embedded in the simulator.

INPUT PARAMETER FOR SIMULATION

The main inputs parameter for the simulator will be the ailerons and rudder deflection which control the lateral dynamics of the aircraft. However, other parameters also need to be considered or calculated as the inputs parameters for the simulator. Refer back to Equation (1), as the model of the aircraft depends on the airspeed (V), angle of attack (α), and pitch angle (θ), which means that the simulation results should also be base from these parameters as well.

TABLE 1. Lateral-directional derivative for C-5A aircraft at sea level

Lat-Dir Derivatives	Value	Lat-Dir Derivatives	Value
Y_β (m/s²)	-6.466	$Y_{\delta r}$ (m/s²)	1.4143
L_β (1/s²)	-0.585	$L_{\delta r}$ (1/s²)	0.00974
L_p (1/s)	-0.329	$N_{\delta r}$ (1/s²)	-0.141
L_r (1/s)	0.256	$Y_{\delta a}$ (m/s²)	0
N_β (1/s)	0.167	$L_{\delta a}$ (1/s²)	0.264
N_p (1/s)	-0.0184	$L_{\delta r}$ (1/s²)	0.0138
N_r (1/s)	-0.12		

Based on the linearized state-space model and the lateral directional derivative for the C-5A aircraft, the response of the aircraft will be simulated according to the variation of its control stick/column, paddles, and airspeed. The output of the simulator will be in form of the graph representing the response of β, r, p, and Ø.

For simplification, the assumption was made as the aircraft is in a level flight which indicates that flight path angle, "γ" , is equal to zero. This will result in "θ" equal to "α" (Angelo, M. 2016).

$$\theta = \alpha \qquad \text{Assuming that } \gamma = 0°$$

There is also a significant relationship between airspeed and angle-of-attack which is written in form of lift equation as in Equation (2) (Anderson, J. 2015).

$$L = W = 0.5 \cdot \rho S V^2 C_L. \tag{2}$$

$$C_L = C_{L_\alpha}(\alpha - \alpha_0) \tag{3}$$

In Equation (2) the lift force acting on aircraft is equivalent to the weight of the aircraft where in this case is the C-5A aircraft weighing at 3 240 000 N. The lift coefficient, CL, can be obtained through Equation (3) defined as:

It is observed in Equation (3) that the lift coefficient is a function of angle-of-attack. To obtain the $C_{L\alpha}$ and α_0 , the lift curve for C-5A aircraft in Figure 1 is analyzed.

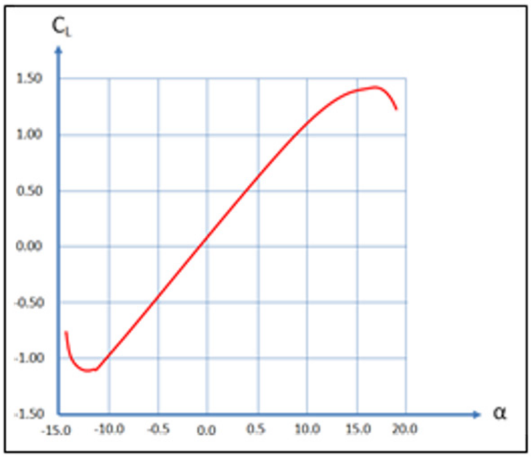


FIGURE 1. Lift curve for C-5A aircraft [1]

The value for $C_{L\alpha}$ is obtained by measuring the slope of the lift curve, taken only at the linear part between "$-10°\leq\alpha\leq10°$, which the resulted value of $C_{L\alpha}$ is 0.16. The value for α_0 is obtained by looking at the value for "α" at C_L=0. This will be the value of α_0=-2°. The combination and rearrangement of the lift equation in Equation (2), and lift coefficient equation in Equation (3), will give the angle-of-attack equation which as a function of the airspeed as in Equation (4) defined as:

$$\alpha(V)= (2W/0.16\rho_0 SV^2)\text{-}2 \qquad (4)$$

For α in the range of $-10°\leq\alpha\leq10°$.

Since the largest angle-of-attack will occur at the lowest speed, this will subsequently define the lowest limitation boundary for the speed. At $\alpha\leq10°$, the lowest limit for the speed will be V_0=75m/s or V_0=146 kt. The maximum speed allowable for C-5A aircraft at sea level is at 0.25 Mach number or at 85.75 m/s or 166.68 kt. This will give the airspeed range for the aircraft of 75 m/s ≤ V ≤ 85.75m/s or 146kt ≤ V ≤ 167kt. For simulation purposes, the range for airspeed allocated is in the range between 140kt ≤ V ≤ 160kt. The overall nonlinear parameter for this simulator is presented in Table 2.

TABLE 2. Nonlinear inputs parameter for simulation

Parameter	Range/Equation	Note
Airspeed, V	140 kt≤V≤160 kt	User-defined
Angle of Attack, α α_0=-2°	$\alpha(V)=\frac{\rho SV^2}{2W}+\alpha_0$	Calculated
Pitch Angle, θ	θ=α	Calculated
Ailerons Deflection, δa	-25°≤V≤25°	User-defined
Rudder Deflection, δr	-25°≤V≤25°	User-defined

From Table 2, there will be three parameters needed to be defined by the user for the simulator and two parameters will be calculated based on the assumption at level flight.

SIMULATION LAYOUT

The graphical user interface (GUI) layout of the simulator is simple with six panels for input and display. The input for the simulator is in the form of sliding bar and text box, while the output is in the form of a graph, figure, and text display. The GUI is designed in MATLAB software (The MathWorks, Inc. 2017).

The proposed layout of the simulator is shown in Figure 2 where the layout can be divided into six panels that have functions as follows:

1. The graph panel; where all the graphs and figures are shown.

2. The control panel; where the input of airspeed, paddles and control stick are given.

3. The stability panel; where the user can select which type of simulation to perform as well as showing the main parameter for the selected simulation.

4. The simulation time panel; where the end time for each simulation is set.

5. The control feedback panel; where the user can select between neutral aircraft, yaw damper, or control feedback.

6. The push to start panel; where the user can execute the simulation. This panel will also display the value for the main parameter.

The simulator will be done based on two selections of condition for the C-5A aircraft. The two conditions are natural condition and controlled condition.

A servo command block is added to the Simulink block diagram in Figure 3 to simulate the actuator response of the aircraft. This was not the actual servo response of the aircraft but rather just for realization. The transfer function for the servo command block as per Equation (5) and Equation (6)

$$\text{Ailerons/stick}=\delta_a/\delta_{a_c}=1/(0.1s+1) \quad (5)$$

$$\text{Rudder/paddles}=\delta_r/\delta_{r_c}=1/(0.15s+1) \quad (6)$$

For natural condition, the simulation will be based on the linearized state-space model of the aircraft. For the controlled condition, the simulation can be done either using a yaw damper controller or a feedback gain controller.

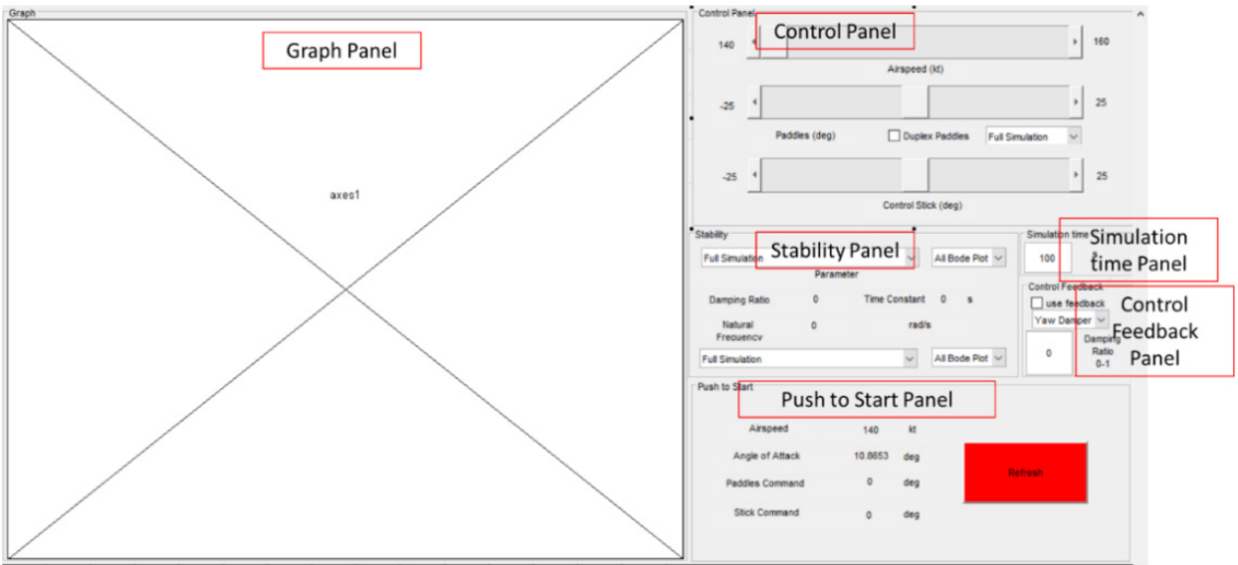


FIGURE 2. C-5A Simulator layout

NATURAL AIRCRAFT SIMULATION

The natural aircraft simulation was modeled in Simulink block diagram as shown in Figure 2 where the input for the simulation is the control stick and paddles command of the aircraft. The output is β, r, p, and ∅.

The example of a full simulation for the natural aircraft is as shown in Figure 4. This result was simulated with an airspeed of 140 kt and control stick deflected at 1°.

The simulator also allows user to observe the estimated response for each of the lateral flight mode of the aircraft. The estimation equation for each lateral flight mode is tabulated in Table 3 (Nandan, K. et al. 2017).

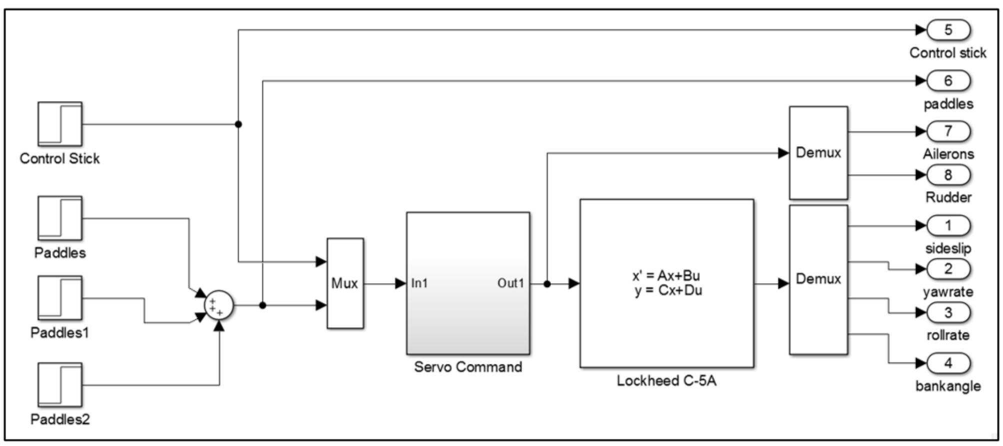


FIGURE 3. Simulink block diagram for the natural dynamic of C-5A aircraft

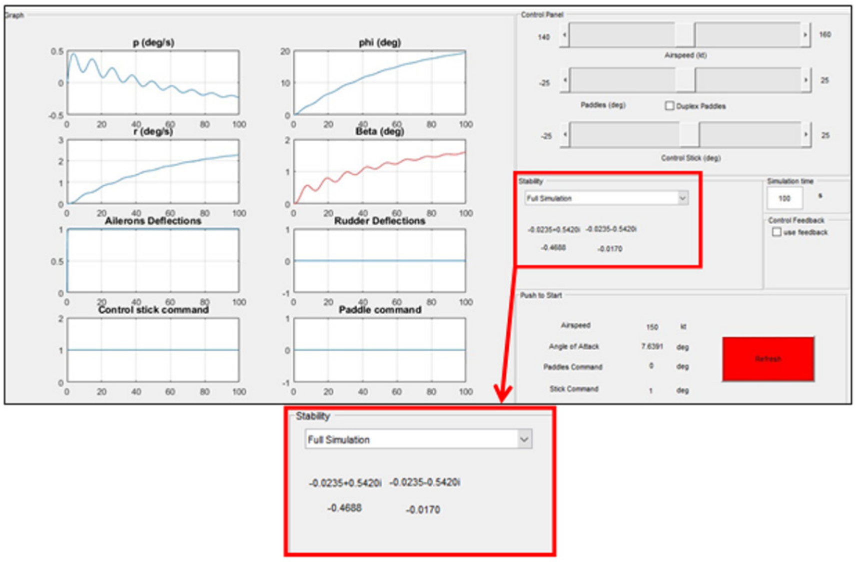


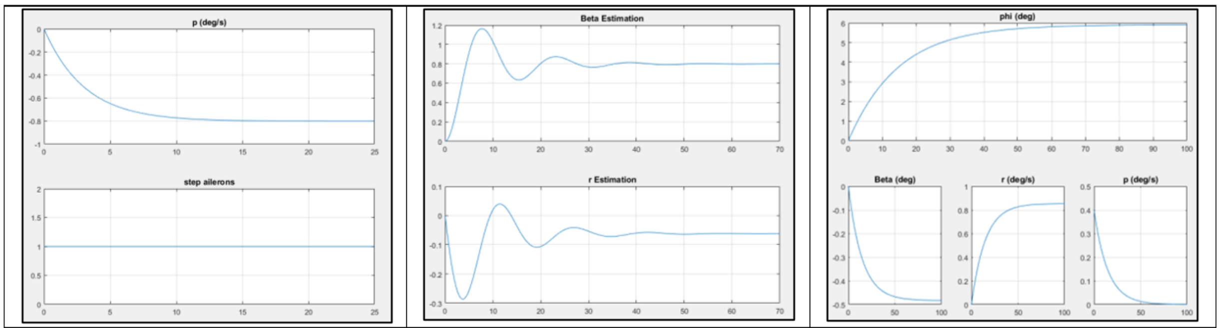
FIGURE 4. Example of natural aircraft simulation

TABLE 3. Estimation equations for lateral flight mode

Pure Roll Mode	Settling Time	$\tau=-\frac{1}{L_p}$
	Roll Response	$p=(p_0-p_e)e^{-t/\tau}+P_e$
	Steady State Value	$p_e=-\frac{L_{\delta_a}}{L_p}\cdot\delta_a$
Dutch Roll Mode	Settling Time	$\tau=\frac{1}{\zeta\cdot\omega_n}$
	Natural Frequency and Damping Ratio	$\lambda=-\frac{N_r+\frac{Y_\beta}{V}}{2}$ $\omega_0=\sqrt{-N_\beta(-\cos\alpha)+\frac{Y_\beta N_r}{V}}$ $\zeta=\frac{\lambda}{\omega_0}$ $\omega_n=\sqrt{{\omega_0}^2-\lambda^2}$
	Dutch Roll Response	$\begin{pmatrix}\dot\beta\\ \dot r\end{pmatrix}=\begin{pmatrix}\frac{Y_\beta}{V} & -\cos\alpha\\ N_\beta & N_r\end{pmatrix}\cdot\begin{pmatrix}\beta\\ r\end{pmatrix}+\begin{pmatrix}\frac{Y_{\delta_r}}{V}\\ N_{\delta_r}\end{pmatrix}\cdot\delta_r$
	Settling Time	$\tau=\frac{V}{g}\cdot\frac{N_\beta\cdot L_p}{N_\beta\cdot L_r-N_r\cdot L_\beta}$
	Steady State Value	$\beta(\infty)=\frac{N_rL_{\delta_a}-N_{\delta_a}L_r}{N_\beta L_r-N_rL_\beta}\cdot\delta_a+\frac{N_rL_{\delta_n}-N_{\delta_n}L_r}{N_\beta L_r-N_rL_\beta}\cdot\delta_r$ $r(\infty)=\frac{N_{\delta_a}L_\beta-N_\beta L_{\delta_a}}{N_\beta L_r-N_rL_\beta}\cdot\delta_a+\frac{N_{\delta_r}L_\beta-N_\beta L_{\delta_r}}{N_\beta L_r-N_rL_\beta}\cdot\delta_r$ $\phi(\infty)=-\frac{V}{g}\cdot\left(\frac{Y_\beta}{V}\cdot\beta(\infty)-r(\infty)+\frac{Y_{\delta_a}}{V}\cdot\delta_a+\frac{Y_{\delta_r}}{V}\cdot\delta_r\right)$
	Spiral Response	$\beta=\beta(\infty)\cdot(1-e^{-t/\tau})$ $r=r(\infty)\cdot(1-e^{-t/\tau})$ $p=\frac{\varnothing(\infty)}{\tau}\cdot e^{-t/\tau}$ $\varnothing=\varnothing(\infty)\cdot(1-e^{-t/\tau})$

Example of estimated lateral mode simulation is as shown in Figure 5. This simulation allows user to analyze graphically each mode that exists in lateral flight.

Based on the natural simulation in Figure 4 and estimated lateral mode simulation in Figure 5, there is oscillation at the output for C-5A aircraft natural mode. This is due to the small damping of the Dutch Roll mode (Micheal, V.C. 2013). This phenomenon remains even with changes of the airspeed. The Dutch roll can be further damped using a yaw damper configuration (Rob, M. 2017) or feedback gain configuration.



(a) Estimated Roll Mode	(b) Estimated Dutch Roll Mode	(c) Estimated Spiral Mode

FIGURE 5. Example of estimated lateral mode simulation

YAW DAMPER

The yaw damper is designed to damp the Dutch Roll mode oscillation by introducing a feedback gain (k), a washout filter between the yaw rate output (r), and the paddles input. This is illustrated by the diagram in Figure 6.

The feedback gain (k) will change the yaw rate coefficient (CNr) and can be calculated by using Equation (7) (Guillaume, C. 2016).

However, for modern aircraft with fly-by-wire system, the automatic has taken place where the control system is much more complex with a faster computer and a lot more sensors.

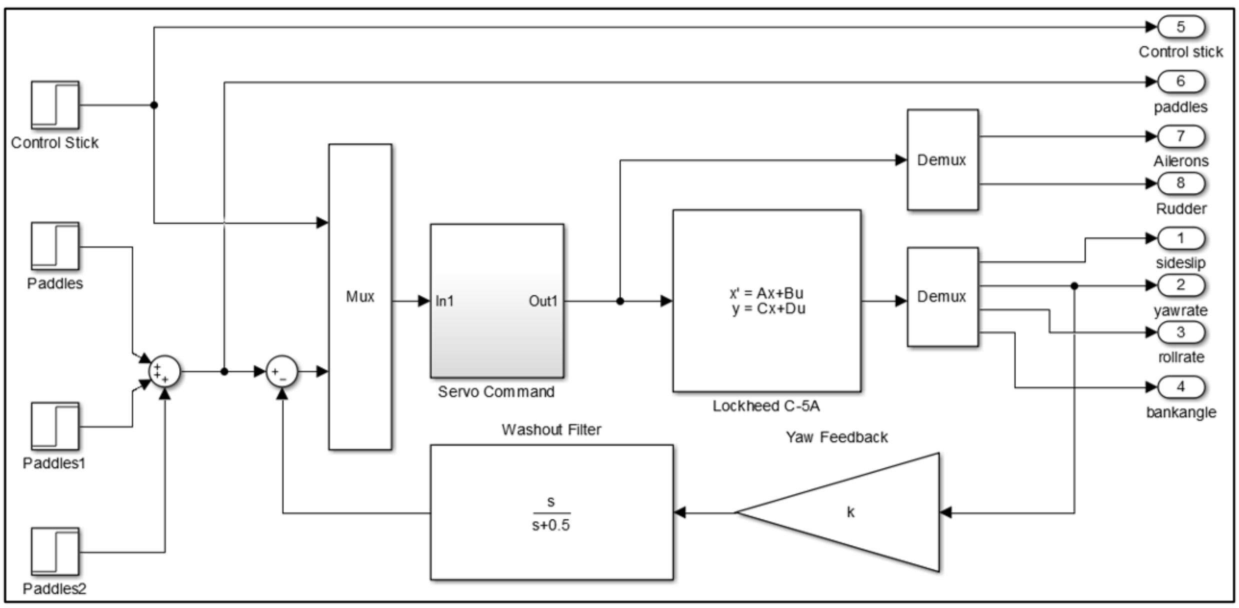
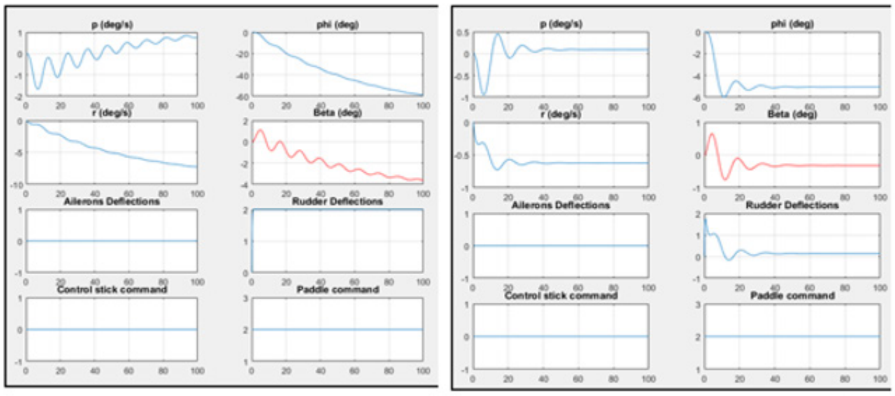


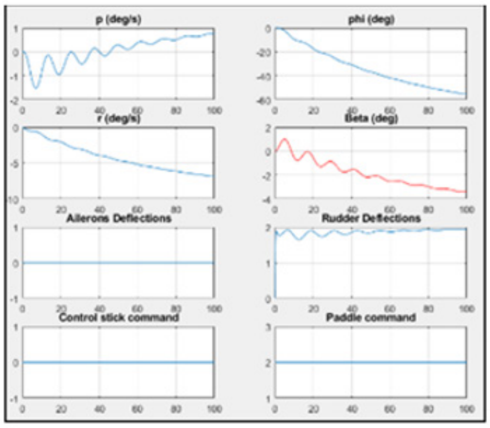
FIGURE 6. Yaw damper block diagram.

The inertia value, I_{zz} is 8.33×10^7 kg·m^2, and wing chord for C-5A aircraft, l is 9.4 m. The response with only yaw feedback gain will be different from the natural aircraft response. In order to have a similar response but damped in Dutch Roll, the washout filter is added. For this simulator, a simple first-order filter is added. The response of the simulation is shown in Figure 7.

Figure 7 shows the responses when simulated with a damping ratio of ζ=0.7. The response of yaw damper with washout filter shows that the response is quite similar to the natural response but fairly damped the Dutch roll oscillation. This simulation is an ideal way to show how the yaw damper was designed to damp oscillation while still maintaining the same response as a natural aircraft.



(a) Natural Aircraft (b) Yaw rate feedback



(c) Yaw rate feedback with washout filter

FIGURE 7. Different response between natural aircraft response (a), yaw rate feedback response (b), and yaw rate feedback with washout filter response (c)

FEEDBACK CONTROLLER

The feedback controller for this simulator was designed by utilizing the pole placement method with state coupling and input-output coupling (Guillaume, C. 2016). The Simulink diagram for the feedback controller is as shown in Figure 8.

The feedback gain vector (K) in Figure 8 is calculated using pole placement method where the pole is set as in Table 4. The pre-command gain vector (Kprec) is calculated so that the input will control the Side slip angle (β) and bank angle (ϕ) respectively.

TABLE 4. Desire poles location and their characteristic for poles placement method

Poles	Damping Ratio	Natural Frequency	Time Constant
0.8+j0.8	ξ=0.7	ω_n=1.13 rad/s	T=1.25s
-0.8-j0.8			
-0.75			"T=1.33 s"
-1.2			"T=0.83 s"

The feedback gain (K) in Figure 8 is calculated first through state-space representation using Equation (8) and Equation (9) (Norman, S.N. 2015).

$$[A\text{-}\lambda_i I \quad B]\begin{bmatrix} v_i \\ \text{-}w_i \end{bmatrix}=0 \tag{8}$$

$$K=\text{-}w_i \cdot v_i^{-1} \tag{9}$$

Variable A and B in Equation (8) are the state matrix and input matrix defined previously in Equation (1). The variable v_i is the Eigenvectors while wi is a combination of feedback gain (K) and v_i. The value for $[v_i \;\; w_i]^T$ can be obtained using the null function in MATLAB. This must be done for all desired poles in Table 4. For example, the value of $[v_i \;\; w_i)]^T$ for poles at p_1=-0.8 + 0.8i is obtained in MATLAB as follows:

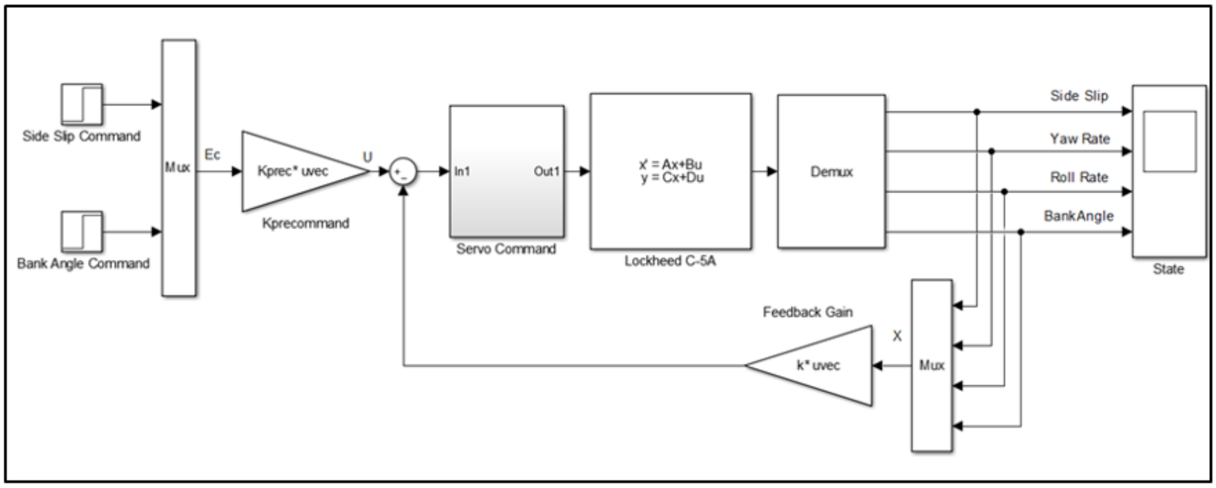


FIGURE 8. Simulink diagram for C-5A feedback controller

```
K1 = null([A-(p1)*(eye(4)) B])

K1 =
   0.0204 + 0.0049i     0.0014 + 0.1202i
  -0.0097 - 0.0171i     0.0992 + 0.0870i
  -0.1339 - 0.2134i    -0.0574 - 0.0084i
  -0.0505 + 0.2197i     0.0294 + 0.0226i
   0.9406 + 0.0023i    -0.0013 + 0.0230i
  -0.0102 + 0.0063i     0.9813 + 0.0026i
```

The value of K1 is the value of $[v_1 \ \ w_1]^T$ for p_1=-0.8 + j0.8. To obtain the value of the feedback gain (K), all value of $[v_i \ \ w_i]^T$ for all poles need to be calculated separately and solve using Equation (9). This will result in two values for K as in Equation (10) and Equation (11).

Both values of the feedback gain K (K_a and K_b) is simulated using Simulink diagram in Figure 9 and the output is observed. The simulation was done separately for stick command and paddles command. This is to observe the output response for each input when excited separately. The result of the simulation is as shown in Figure 10 and Figure 11.

The simulation results in Figure 10 and Figure 11 shows that both feedback gains produce a unique response even though the poles resulted in the two gains is the same. The output responses do not align with pilot desire condition where when pilot triggers the stick input (for banking) it is desirable for the aircraft to roll without side-slip, and if the pilot trigger the paddles input, it is desirable for the aircraft to side-slip without a roll (Ajay, K.K. et al. 2019).

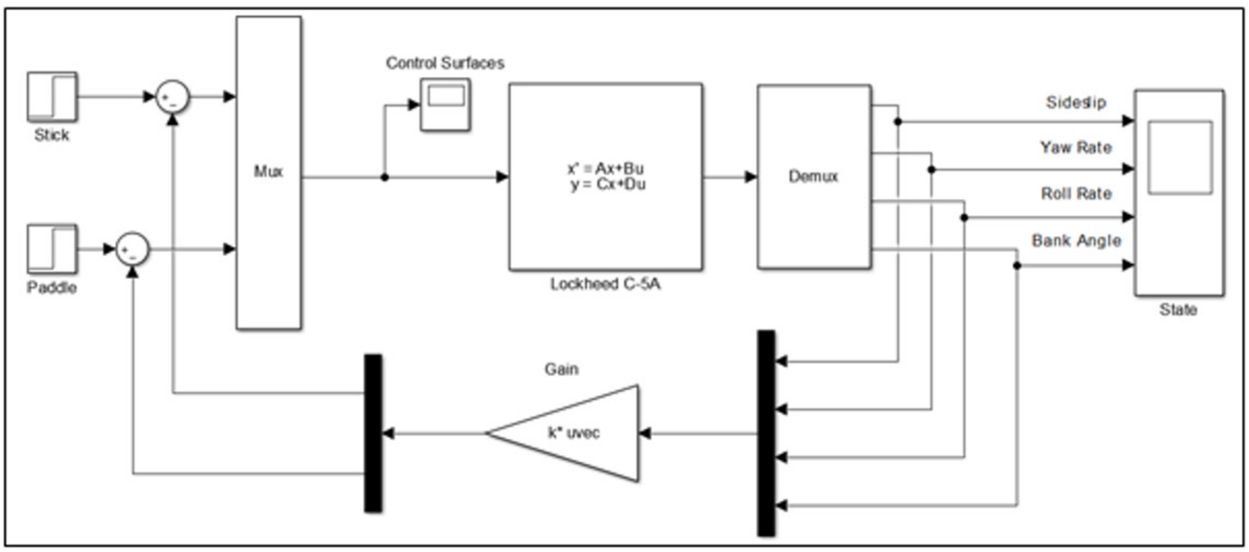


FIGURE 9. Simulink diagram for testing the obtained feedback gain, K

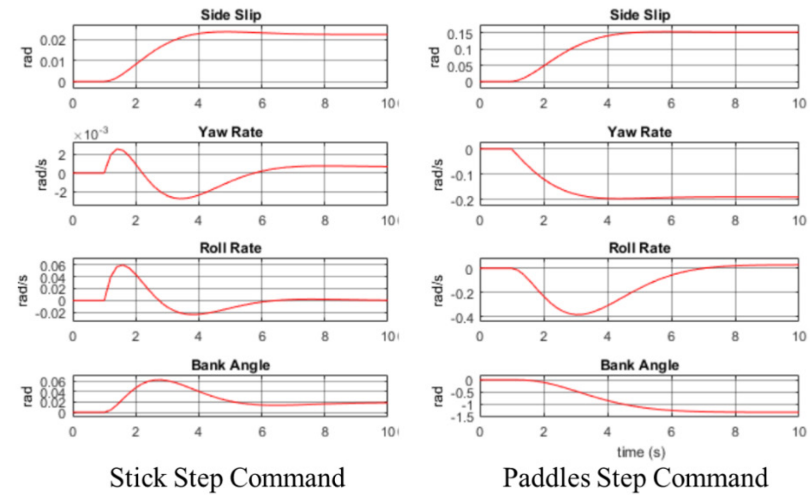


FIGURE 10. Simulation results with feedback gain K_a

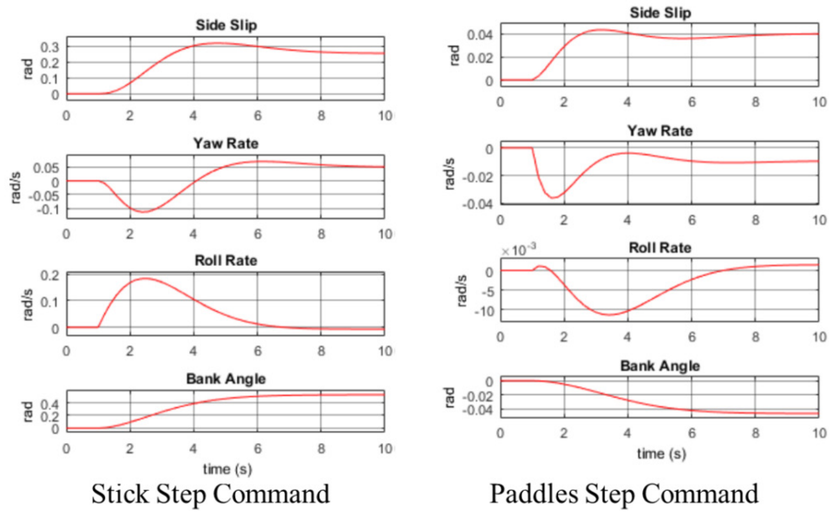


FIGURE 11. Simulation results with feedback gain K_b

The simulation results in Figure 10 and Figure 11 show that there is a coupling between outputs where every output will have a high influence on each other. In simpler words, when pilots desired a side-slip, there will also exist a large portion of bank angle. To reduce the effect of output coupling, the coupling/decoupling method needs to be used before finishing the poles placement calculation. This method is done by forcing some elements in the eigenvector (vi) to have value 1 or 0. For an aircraft, it is desirable to decouple eigenvalue associated with pure roll mode and spiral mode from the side slip and to decouple eigenvalue associated with Dutch roll mode from the bank angle. It is also desirable to couple eigenvalue associated with pure roll mode and spiral mode from the bank angle and to couple the eigenvalue associated with Dutch roll mode to the sideslip. This sets of the association are illustrated by the following:

$v_i = \begin{bmatrix} v_{i1} \\ v_{i2} \\ v_{i3} \\ v_{i4} \end{bmatrix}$	$v_{i1} \rightarrow \beta$
	$v_{i2} \rightarrow r$
	$v_{i3} \rightarrow p$
	$v_{i4} \rightarrow \emptyset$

With $X = [\beta\ r\ p\ \emptyset]^T$ and vi is the eigenvector associated with eigenvalue or poles, p_i. It is desirable to have eigenvector with the elements in Table 5. Where each element for eigenvector is associated with vector X.

Table 5. Desire eigenvector elements for each eigenvalue

Eigenvector	$v_1=\begin{bmatrix}1\\ *\\ *\\ 0\end{bmatrix}$	$v_2=\begin{bmatrix}1\\ *\\ *\\ 0\end{bmatrix}$	$v_3=\begin{bmatrix}0\\ *\\ *\\ 1\end{bmatrix}$	$v_4=\begin{bmatrix}0\\ *\\ *\\ 1\end{bmatrix}$
Eigenvalue/ Poles	"-0.8+0.8i"	"-0.8-0.8i"	"-0.75"	"-1.2"
Lateral Mode	Dutch Roll Mode		Pure Roll Mode	Spiral Mode

To achieve the coupling/decoupling in Table 5, the existing $[v_i \ \ w_i)]^T$ need to be modified using Equation (11) to obtain v_1 and v_2, and Equation (12) to obtain v_3 and v_4.

$$\begin{bmatrix}v_{inew}\\ -w_i\end{bmatrix}=\begin{bmatrix}v_i\\ -w_i\end{bmatrix}\begin{bmatrix}v_{irow1}\\ v_{irow4}\end{bmatrix}^{-1}\begin{bmatrix}1\\ 0\end{bmatrix} \rightarrow v_{inew}=\begin{bmatrix}1\\ *\\ *\\ 0\end{bmatrix} \tag{12}$$

$$\begin{bmatrix}v_{inew}\\ -w_i\end{bmatrix}=\begin{bmatrix}v_i\\ -w_i\end{bmatrix}\begin{bmatrix}v_{irow1}\\ v_{irow4}\end{bmatrix}^{-1}\begin{bmatrix}0\\ 1\end{bmatrix} \rightarrow v_{inew}=\begin{bmatrix}0\\ *\\ *\\ 1\end{bmatrix} \tag{13}$$

For example, $[v_1 \ \ w_1]^T$ for p_1 is originally obtained as in Equation (14).

$$\begin{bmatrix}v_1\\ -w_1\end{bmatrix}=\begin{bmatrix}0.0204+0.0049i & 0.0014+0.1202i\\ -0.0097-0.0171i & 0.0992+0.0870i\\ -0.1339-0.2134i & -0.0574-0.0084i\\ -0.0505+0.2197i & 0.0294+0.0226i\\ 0.9406+0.0023i & -0.0013+0.0230i\\ -0.0102+0.0063i & 0.9813+0.0026i\end{bmatrix} \tag{14}$$

The $[v_1 \ \ w_1]^T$ is then modified using Equation (12) to have coupling between Dutch roll mode with side-slip and to have decoupling between Dutch roll mode with bank angle. This will result in the new $[v_{1new} \ \ w_{1new}]^T$ as in Equation (15).

$$\begin{bmatrix}v_{1new}\\ -w_{1new}\end{bmatrix}=\begin{bmatrix}1\\ 0.7011-0.8372i\\ -0.119+0.1336i\\ 0\\ 1.3323+0.5290i\\ -0.0393-7.9811i\end{bmatrix} \tag{15}$$

The coupling/decoupling process is then repeated for all eigenvector and then solved for feedback gain (K) using Equation (9). Due to the coupling/decoupling process, there will be only one feedback gain (K) matrix as shown in Equation (16).

$$K=\begin{bmatrix}-1.7753 & 1.6105 & 6.1339 & 3.4167\\ 6.7233 & -9.3483 & 1.1593 & 0.7470\end{bmatrix} \tag{16}$$

The value of K in Equation (16), is simulated using the Simulink block diagram in Figure 9 with the result in Figure 12.

Simulation results in Figure 12 shows that when the pilot triggers the stick, the aircraft will have a significant bank angle with a very small excitation of side-slip. And if the pilot triggers the paddles, the aircraft will have a significant side-slip with a very small excitation in bank angle. This will allow the aircraft to be flown in side-slip especially during a high crosswind landing. However, there is still steady-state error between stick command and bank angle, and between paddles with a side-slip angle. The steady-state error will be eliminated using pre-command gain (K_{prec}).

The pre-command gain (K_{prec}) is calculated to control the bank angle through control stick and to control side-slip through paddles where it is calculated by using Equation (17).

$$K_{prec}=\left(-\tilde{C}(A\text{-}BK)^{-1}\right)\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$$

Variables A and B in Equation (17) is the same state matrix and input matrix in Equation (1). The value of $\tilde{C}$ is a matrix that chooses the bank angle and side-slip as input for the system. The value for $\tilde{C}$ matrix is defined in Equation (18).

$$\tilde{C}=\begin{bmatrix}0 & 0 & 0 & 1\\ 1 & 0 & 0 & 0\end{bmatrix} \tag{18}$$

The result from the calculation of Equation (17) will result in the value of K_{prec} in Equation (19).

$$K_{prec}=\begin{bmatrix}3.3504 & 0.4292\\ -0.6366 & 8.9570\end{bmatrix} \tag{19}$$

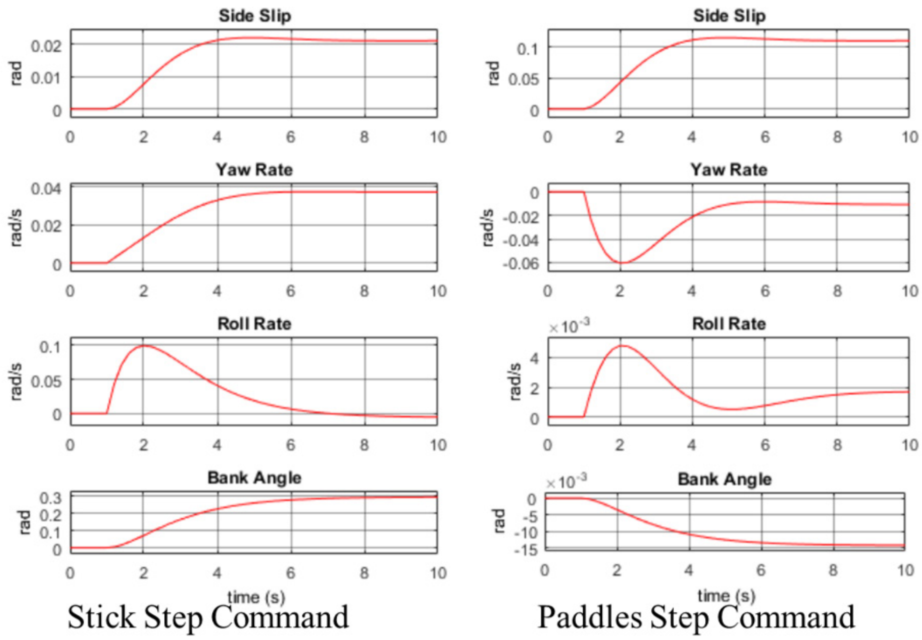


FIGURE 12. Simulation result with gain K from coupling/decoupling method

The value of feedback gain matrix (K), and pre-command gain matrix (Kprec) obtained in Equation (16) and Equation (19) is with a constant airspeed of 140 kt. However, as the state-space model of C-5A depends on the airspeed, the feedback gain matrix (K), and the pre-command gain matrix (Kprec) will also depend on the airspeed. This is illustrated in Equation (20) and Equation (21).

$$K(V)=\begin{bmatrix} K_{1,1}(V) & K_{1,2}(V) & K_{1,3}(V) & K_{1,2}(V) \\ K_{2,1}(V) & K_{2,2}(V) & K_{2,3}(V) & K_{2,4}(V) \end{bmatrix} \quad (20)$$

$$K_{prec}(V)=\begin{bmatrix} K_{prec_{1,1}}(V) & K_{prec_{1,2}}(V) \\ K_{prec_{2,1}}(V) & K_{prec_{2,2}}(V) \end{bmatrix} \quad (21)$$

Both values of K and Kprec were calculated with the variation of airspeed, (V). The results is a polynomial function for every element in K and Kprec as shown in Table 6 and Table 7, respectively.

The polynomial function for each element for K and Kprec as shown in Table 6 and Table 7 will be inserted into the simulator so that the simulator can operate at different airspeed variation. The control laws are tested with a variation of airspeed and the result shows that it will always have the same response and poles location regardless of the change of the airspeed.

OTHER FEATURES

In completing the simulator, two other features aside from the controller design are added. This include as follows:

1. Bode Plot. The Bode Plot features are added so that the user can see the Bode diagram of the system with respect to input and output chosen by them.

TABLE 6. The polynomial equation for feedback gain, K variation with airspeed, V

$K_{i,j}$	n order	Polynomial function	Error
$K_{1,1}$	3	$-7.63\times10^{-7} V^3+4.34\times10^{-4}V^2-0.09V+4.19$	1.91×10^{-5}
$K_{1,2}$	3	$-5.31\times10^{-7}V3+2.91\times10^{-4}V^2-0.06V+5.17$	1.59×10^{-5}
$K_{1,3}$	1	$-4.71\times10^{-4}V+6.2$	9.55×10^{-5}
$K_{1,4}$	3	$1.93\times10^{-7}V^3-9.23\times10^{-5}V^2+0.01V+2.7$	1.01×10^{-5}
$K_{2,1}$	3	$7.28\times10^{-7}V^3-3.94\times10^{-4}V^2+0.08V+1.86$	2.53×10^{-5}
$K_{2,2}$	2	$3.67\times10^{-5}V^2-0.02V-7.6$	4.14×10^{-5}
$K_{2,3}$	3	$7.31\times10^{-7}V^3-3.86\times10^{-4}V^2+0.07V-2.92$	2.42×10^{-5}
$K_{2,4}$	3	$1.09\times10^{-6}V^3-5.94\times10^{-4}V^2+0.11V-6.12$	3.21×10^{-5}

TABLE 7. The polynomial equation for pre-command, Kprec variation with airspeed, V

$K_{preci,j}$	n order	Polynomial function	Error
$K_{prec1,1}$	3	$4.78\times10^{-7}V^3-2.47\times10^{-4}V^2-0.04V+0.82$	1.91×10^{-5}
$K_{prec1,2}$	3	$-1.43\times10^{-6}V^3+7.81\times10^{-4}V^2-0.15V+6.72$	4.42×10^{-5}
$K_{prec2,1}$	3	$1.10\times10^{-6}V^3-6.28\times10^{-4}V^2+0.13V-9.27$	2.47×10^{-5}
$K_{prec2,2}$	3	$-4.73\times10^{-7}V^3+3.05\times10^{-4}V^2-0.07V+14.91$	9.93×10^{-7}

2. Pole Location Plot. This feature will plot the pole location on the S-plane so that the user can analyze the effect of the pole location toward the system as a whole.

3. Duplex paddles mode. These feature allows the user to analyze what will happen to the aircraft when duplex paddles are used to excite the Dutch roll mode.

With the addition of these features, the developed simulator is complete and can be easily integrated as part of teaching and learning tools for those interested in improving basic understanding and concept of aircraft control laws.

CONCLUSION

The C-5A aircraft lateral mode simulator is designed to simulate the natural lateral mode of the aircraft and control mode of the aircraft. The important aspects in the natural lateral mode of the aircraft have been emphasized including the Pure Roll mode estimation, the Dutch Roll mode estimation, and the spiral mode estimation. The duplex paddles mode will simulate how the pilot is exciting the Dutch roll mode and the Bode plot will allow the user to analyze the dynamic stability of the aircraft.

The control laws designed for the aircraft is only for study purposes and not the actual control laws implemented in the C-5A aircraft. However, the yaw damper design and the poles placement method design of control laws will allow the user to see how the control laws affect the original system and what are the important parameters that need to be taken care of in designing control laws for an aircraft in general.

The computation of the controller was performed without taking into account the actuator bandwidth for sake of simplicity. This work could be extended to include the actuator transfer function in the controller design. However, the resulted simulator can be used for teaching and learning purposes to improve basic understanding and concept of aircraft control laws.

DECLARATION OF COMPETING INTEREST

None

REFERENCE

Ajay, K.K., Mark, A.P. & David, R. 2019. *Conceptual Aircraft Design: An Industrial Approach*. West Sussex: John Wiley & Sons, Inc.

Anderson, J.D. 2015. *Introduction to Flight*. 8th edition. New York: Mc Graw Hill Education.

Angelo, M. 2016. Flight Mechanics: *Theory of Flight Paths*. 1st Edition. Mineola: Dover Publications Inc.

Guillaume, C. 2016. Flight Characteristics – Automatic Basic. Toulouse: AIRBUS France.

Micheal, V.C. 2013. *Flight Dynamics Principles*. Oxford: Elsevier Ltd.

Nandan, K.S. & Ananthkrishnan, N. 2017. *Advanced Flight Dynamics with Elements of Flight Control.* Boca Raton: CRC Press.

Norman, S.N. 2015. *Control Systems Engineering*. Singapore: John Wiley & Sons, Inc.

Rob, M. 2017. How It Works: Yaw Damper. https://www.flyingmag.com/how-it-works-yaw-damper/. (accessed 10 October 2018).

The MathWorks, Inc. 2017. *MATLAB® Apps Building*. Natick: The MathWorks, Inc.

Thomas, R., Steven, L., David, E., Wayne, F. & James, K. 2014. *Introduction to Aircraft Flight Mechanics. 2nd edition*. Reston: American Institute of Aeronautics and Astronautics.