

Optimization of the California Bearing Ratio of Lateritic Soil Stabilized with Rice Husk Ash using Scheffe's Method

Ike Ubadinanwa Elvis* & Ike Charles Chinwuba

Department of Civil Engineering, Enugu State University of Science and Technology, Enugu, Nigeria.

**Corresponding author: Elvisike5@gmail.com*

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ABSTRACT

In a bid to minimize the adverse effects of agricultural waste materials in the environment and also encourage for its optimal usage, this study aims at developing model constants for optimizing the California Bearing Ratio (CBR) of Lateritic Soils stabilized with Rice Husk Ash (RHA) using Scheffe's method. The study practically involved sampling of the materials, laboratory testing and model formulation. The natural soil was subjected to standardized laboratory testing to determine its index and Engineering properties. The scheffe's method provided mix ratios for the three mix components (lateritic soil, RHA and water) involved in the mix design. The method also employs the use of second-degree polynomials to generate the model constants. The CBR of the soil was the primary property of the soil considered in the study therefore, a CBR test was conducted on the natural soil and modified soil. The optimization technique carried out gave an optimal mix observed to be of mix ratio 1: 1.70: 0.25 for lateritic soil, RHA and water with a CBR value of 30%. The model formulated was further subjected to validation using the F-test and T-test statistical method and it was found adequate at a confidence level of 95%. Hence, the null hypothesis (H_0) was adopted. The formulated model being adequate can be used exclusively to predict the CBR of Lateritic soil Stabilized with Rice Husk Ash within the same locality.

Keywords: Optimization; California Bearing Ratio; Stabilization; Lateritic Soil; Scheffe's Method

INTRODUCTION

In a country faced with growing need for infrastructural development, Nigeria has seen its Engineering construction activities heightened across different states. These construction activities involving housing, roads, bridges are usually accompanied with various challenges of which Engineers are obligated to proffer professional solutions. A seemingly reoccurring challenge encountered in the civil engineering space is with weak soils.

Soil stabilization has been the conventional solution to this problem, with the cost of stabilization considerably high, engineers has sought to research on other methods or materials suitable for soil stabilization hence, extensive research has been conducted and are still ongoing on various industrial and agricultural waste materials suitable for stabilization of weak soils with intent to reducing cost of the construction process and by extension reducing the adverse effect of these waste materials on the environment. Soil stabilization is simply a method of improving the engineering properties of a soil. Stabilization can be achieved through physical mixing of the natural soil and stabilizing materials to achieve a homogeneous mix or by adding stabilizing material to an un-disturbed soil deposit and obtaining an interaction by letting it permeate through the soil voids, Abood et al. (2007).

This study considered the use of Rice Husk Ash (RHA) which is an agricultural waste material from Rice Milling, in the stabilization of lateritic soil. Research on RHA such Rathan et al (2016), Srinivas (2017), Ayegbokiki et al (2018) and Pornkasem et al (2018) has also enabled for its classification as a pozzolana. In ASTM (1976), it defined pozzolana as a siliceous or siliceous and aluminous material which in itself possesses little or no cementitious value but will, in finely divided form and in the presence of moisture, chemically react with calcium hydroxide at ordinary temperatures to form compounds possessing cementitious properties.

Furthermore, stabilization procedures requires for blending of the soil and stabilizing material, as such, these material are to be adequately proportioned in other to obtain an effective blend of both materials. The scheffe's optimization method essentially provides an effective mix design approach for blending various materials together and also provides models for prediction of a desired soil property based on a mix proportion.

STUDY OBJECTIVES

1. Sampling of Lateritic soil and Rice Husk Ash.
2. Classification of the soil based on its Engineering properties.

3. Determination of the California Bearing Ratio of the Lateritic soil.
4. Stabilization of the lateritic soil with proportions of Rice Husk Ash (RHA).
5. Formulation of model constants for optimizing the California Bearing Ratio of the lateritic soil.
6. Validation of the model (test for adequacy) using a Statistical Test method.

Uwaezuoke and Onyia, (2018) developed models using scheffe's method for the optimization of lateritic soils stabilised with quarry dust. The study expressed the behaviour of the various mix components in a simplex lattice. The optimal mix proportion obtained from the model was 1:2.75:0.135 (lateritic soil: Quarry dust: Water) with a CBR value of 18.2% which indicated an increase of 56.9% in the CBR value of the original lateritic soil. The model prediction was also compared to the experimental results and it was concluded to be adequate at 5% significance level.

Oguaghamba et al. (2019) investigated the Expansive soil sub-grade of Imo-Clay Shale formation traversing Amuro-Okigwe section of Owerri-Okigwe road identified for its intermittent flexible pavement failure. Samples of this sub-grade were collected and mixed with MIPA (Male Inflorescence of Oil Palm Ash) at varying proportions in line with the Scheffe's model approach to improve the natural engineering properties of the soil. Hence, models for the maximum dry density (MDD) and California bearing ratio (CBR) were developed. The models showed good correlations with the experimental results in the control tests, as they possess less than 3% differences. In the CBR results, 10.5% MIPA content in the soil gave the greatest strength (CBR value) of 8.13%

Nwaobakata and Ohwerhi (2020) investigated on the effect of meta-kaolin as a pozzolona on concrete materials. Apparently, this study tested the efficacy of this pozzolan on stabilization processes. This research study was thus aimed at evaluating and predicting the CBR of meta-kaolin stabilized lateritic soil. The Scheffe's simplex theory was used in the development of mix design and optimization model development. The CBR test of the modified soil sample was determined and compared with the unmodified counterpart. Results revealed that the meta-kaolin improved the properties of the lateritic soil. The CBR optimization model developed in this study also proved adequate at 5% level of significance from the F-statistics carried out and can be used reliably in the prediction of the CBR given any arbitrary mix ratio and vice versa.

This study basically applied the scheffe's method for developing optimization model constants for lateritic soils stabilised with RHA. Other literatures which employed Scheffe's method of optimization in determining the properties of soils stabilised with different waste materials are; Okonkwo (2015), Attah and Okafor (2020).

METHODOLOGY

Sampling of the lateritic soil was done at a depth of 1.2m within the premises of the Institute of Management and Technology (IMT), Enugu, Enugu State. The soil sample was carefully inserted into nylon bags to prevent loss of moisture and was subsequently taken to the Civil Engineering laboratory in the Enugu State University of Science and Technology (ESUT). The Rice Husk was sourced from a local rice milling station in Umuomaku, Orumba south, Anambra State. This husk was incinerated to obtain the by-product known as Rice Husk Ash (RHA). Standardized laboratory test as per BS 1377 were conducted on the natural soil to ascertain its index and engineering properties.

Similarly, soil classification was done as per American Association of State Highway and Transport Officials (AASHTO) and the Unified Soil Classification System (USCS). Natural moisture content test, Atterberg limits test, Specific gravity test, Compaction test and California bearing ratio (CBR) test were all conducted on the natural soil. A further CBR test was conducted on the modified soil which was used for the optimization technique.

LABORATORY PROCEDURES

Liquid Limit

The liquid limit of a soil is the moisture content expressed as a percentage by weight of the oven dried. In other words the moisture content above which the soil behaves like a viscous liquid. The Cassagrande apparatus was used for the determination of liquid limit as recommended in BS 1377: Part 2: 1990. The soil was sieved with 425 μ m sieve and water added in successive stages. The liquid limit is the water content at which 25 blows close a groove of about 13mm length.

Plastic Limit

This is the boundary between the plastic state and the semi-solid state. The sample was sieved through 425 μ m sieve and water was added to about 20g of the filtrate soil in order to mould it. The moulded soil was broken into smaller samples and each of them rolled on a glass plate using the fingers to a thread of 3mm diameter. The plastic limit is described as the water content when a thread of soil being rolled shear at 3mm diameter.

Standard Proctor Compaction

This test is to determine the maximum dry density and the optimum moisture content relative to a compactive effort. This test established the optimum moisture content to be used for some other performance test like California bearing ratio which requires compaction. As specified by BS 1377:1990 (Standard Proctor) was adopted. A cylindrical

metal mould of about 1000cm³ volume and a rammer of 2.5kg weight with a height drop of 300mm was used as the given compactive effort. Twenty-five (25) blows were given on each layer of three (3) and moisture content samples were taken from the top and bottom of the mould. The optimum moisture content was taken as the moisture content at which the maximum dry density was attained.

California Bearing Ratio

In the test as given in BS 1377: Part 2: 1990, a specimen is compacted into the CBR mould. The specimens were prepared in 3 (three) layers and rammed with each layer receiving 61 (sixty-one) blows. The load required to cause a circular, 49.65mm in diameter, to penetrate the specimen at a specified rate of 1.25mm per minute is then measured. From the test results, the CBR value is calculated. This is done by expressing the corrected values of forces on the plunger for a given penetration as a percentage of a standard force of value 0.590kN. The 2.5mm and 5.0mm penetration caused by 13.24kN and 19.96kN loads respectively were used in comparing the loads that caused the same penetration on the specimens.

Scheffe's Method

The Scheffe's optimization method was employed in this study to formulate model constants for predicting the CBR of the lateritic soil stabilized with RHA. The method simply involves representing the various mix components present in the mix design on a simplex. Onyelowe et al 2018, described a simplex as a structural representational shape of a line or planes joining assumed positions of constituent materials (atoms) of a mixture. Scheffe 1963 described the factor space as having a simplex with (q-1) where q is the number of components in the mixture which subsequently determines the geometry of the simplex. Hence, a mix design having two mix components will have its geometry as a straight line, a three mix component will be an equilateral triangle while a four component mix design will have its geometry as a tetrahedron. For emphasis, one of the limitations observed in the scheffe's method is the fact that it does not consider more than one property of the soil at a time.

q = 2 → Straight Line

q = 3 → Equilateral Triangle

q = 4 → Tetrahedron

This work essentially involved a three mix component (lateritic soil, Rice husk ash and Water). Therefore, its simplex is an equilateral triangle of which each individual component is represented at the vertex of the triangle and a blend of two components is represented at the line joining two vertex as shown in figure 1.

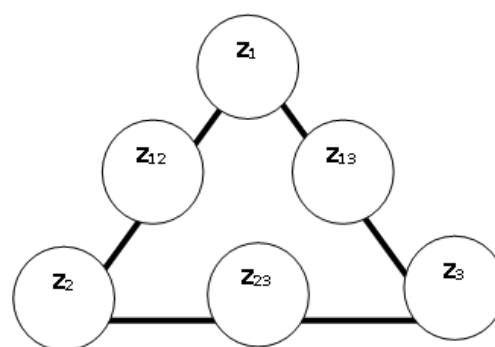


FIGURE 1. A Simplex Triangle

The components are denoted as follows;

- Z_1 = Proportion of the Lateritic Soil
- Z_2 = Proportion of the Rice Husk Ash
- Z_3 = Proportion of Water
- Z_{12} = Proportion of (Soil & Rice Husk Ash)
- Z_{13} = Proportion of (Soil & Water)
- Z_{23} = Proportion of (Rice Husk Ash & Water)

Also, the number of responses or observation is deduced from the equation given by Scheffe 1958 as;

$$N = \frac{(q+n-1)!}{n!(q-1)!} \quad (1)$$

Where,

- N = Number of Observations Required
- q = Number of Mixture Components
- n = Degree of the Polynomials

Hence, given a three component mix in this study and subsequently employing Eqn. 1. We have that;

$$N = \frac{(3+2-1)!}{2!(3-1)!} = \frac{4!}{2!2!} = 6$$

Therefore, the number of responses or observation required is six.

Model Formulation

Scheffe uses polynomials of the second degree to adequately describe the responses of the property or characteristics of a mixture to the difference in proportions of its mix components. A polynomial of n degree in q variables has C_{q+n}^n coefficient (Scheffe 1963). If a mixture has a total of q components and Z_i is the proportion of the i_{th} component in the mixture such that $Z_i \geq 0$ ($i = 1, 2, \dots, q$), then the summation of the component proportion is a whole unity, i.e.

$$Z_1 + Z_2 + Z_3 = 1 \text{ or } \sum Z_i - 1 = 0 \quad (2)$$

There are two basic components involved in Scheffe's design mix that aided for the formulation of the model. The components which are; Pseudo and Real Components are denoted by "Z and X" respectively throughout this study.

The relationship between “Z and X” as expressed by Scheffe is given as;

$$[X] = [A] [Z] \quad (3)$$

Where;

[X] Represents Real Mix Ratio Matrix
[A] Represents Coefficient of the Matrix
[Z] Represents Pseudo Mix Ratio Matrix

The response of the system denoted or represented by “Y” is obtained using the equation of polynomials as shown below;

$$Y = b_0 + \sum b_i Z_i + \sum b_{ij} Z_i Z_j + \sum b_{ijk} Z_i Z_j Z_k + \dots e \quad (4)$$

Considering a three pseudo component mix with two degrees, the response ‘Y’ is of the form;

$$Y = b_0 + \sum b_i Z_i + \sum b_{ij} Z_i Z_j \quad (5)$$

The general polynomial equation for a ternary system is given as;

$$Y = b_0 + b_1 Z_1 + b_2 Z_2 + b_3 Z_3 + b_{11} Z_1^2 + b_{12} Z_1 Z_2 + b_{13} Z_1 Z_3 + b_{22} Z_2^2 + b_{23} Z_2 Z_3 + b_{33} Z_3^2 \quad (6)$$

Multiplying Eqn. 2 by b_0 ;

$$\begin{aligned} b_0 &= b_0 Z_1 + b_0 Z_2 + b_0 Z_3 \\ b_0 &= b_0 (Z_1 + Z_2 + Z_3) \end{aligned} \quad (7)$$

Furthermore, multiplying Eqn. 2 by Z_1, Z_2 and Z_3 successively and re-arranging gives;

$$\begin{aligned} Z_1^2 &= Z_1 - Z_1 Z_2 - Z_1 Z_3 & Z_1 &= Z_1^2 + Z_1 Z_2 + Z_1 Z_3 \\ Z_2^2 &= Z_2 - Z_1 Z_2 - Z_2 Z_3 & Z_2 &= Z_2^2 + Z_1 Z_2 + Z_2 Z_3 \\ Z_3^2 &= Z_3 - Z_1 Z_3 - Z_2 Z_3 & Z_3 &= Z_3^2 + Z_1 Z_3 + Z_2 Z_3 \end{aligned} \quad (8)$$

Substituting Eqn. 7 and Eqn. 8 into Eqn. 6 gives;

$$Y = (b_0 + b_1 + b_{11})Z_1 + (b_0 + b_2 + b_{22})Z_2 + (b_0 + b_3 + b_{33})Z_3 + (b_{12} - b_{11} - b_{22})Z_1 Z_2 + (b_{13} - b_{11} - b_{33})Z_1 Z_3 + (b_{23} - b_{22} - b_{33})Z_2 Z_3 \quad (9)$$

Let,

$$\beta_1 = b_0 + b_1 + b_{11} \quad (10)$$

$$\beta_{ij} = b_{ij} - b_{ii} - b_{jj} \quad (11)$$

Re-writing Eqn. 9 to arrive at a reduced second degree polynomial with three variables;

$$Y = \beta_1 Z_1 + \beta_2 Z_2 + \beta_3 Z_3 + \beta_{12} Z_1 Z_2 + \beta_{13} Z_1 Z_3 + \beta_{23} Z_2 Z_3 \quad (12)$$

Therefore, Eqn. 12 is the model by Scheffe’s method of simplex lattice design used in this study for predicting the CBR values of the lateritic soil at various mix proportions. The coefficient of the second degree polynomial is also obtained when the lattice coordinates of the pseudo components in the simplex triangle is known. Considering Figure 1, it is deduced that at any vertex of the triangle, only one component is present with the boundary lines having binary components. Hence, Z_1, Z_2 and Z_3 have coordinates as follows;

$$Z_1 = (1, 0, 0)$$

$$Z_2 = (0, 1, 0)$$

$$Z_3 = (0, 0, 1)$$

Substituting the above lattice coordinate into Eqn. 12 gives the coefficient of the second degree polynomial as;

$$\begin{aligned} \beta_1 &= Y_1 \\ \beta_2 &= Y_2 \\ \beta_3 &= Y_3 \end{aligned} \quad (13)$$

Also,

$$\begin{aligned} \beta_{12} &= 4Y_{12} - 2Y_1 - 2Y_2 \\ \beta_{13} &= 4Y_{13} - 2Y_1 - 2Y_3 \\ \beta_{23} &= 4Y_{23} - 2Y_2 - 2Y_3 \end{aligned} \quad (14)$$

Therefore, Eqn. 13 and Eqn. 14 represents the coefficients of the second degree polynomial.

Transformation Matrix

The model usually being expressed in pseudo components as seen in Eqn. 12 and having its relationship between the real components and pseudo components as stated in Eqn. 3, the value for the matrix [A] is deduced from the first three mix ratios representing values for the matrix [X], of which are;

$$\begin{aligned} X_1 &= [1 : 1.50 : 0.10] \\ X_2 &= [1 : 1.65 : 0.15] \\ X_3 &= [1 : 1.70 : 0.25] \end{aligned}$$

The corresponding pseudo mix ratios are of an identity matrix form thus;

$$\begin{aligned} Z_1 &= [1 : 0 : 0] \\ Z_2 &= [0 : 1 : 0] \\ Z_3 &= [0 : 0 : 1] \end{aligned}$$

An expanded form of Eqn. 3 can now be written as;

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} \quad (15)$$

Therefore, the values of the real and pseudo mix ratios are substituted into Eqn. 15 at each run and the resulting equation is solved.

For the first run, the matrix is of the form;

$$\begin{bmatrix} 1 \\ 1.50 \\ 0.10 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (16)$$

Solving Eqn. 16 gives the following coefficients of matrix A

$$\begin{aligned} a_{11} &= 1 \\ a_{21} &= 1.50 \\ a_{31} &= 0.10 \end{aligned}$$

For the second run, the matrix is of the form;

$$\begin{bmatrix} 1 \\ 1.65 \\ 0.15 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (17)$$

Solving Eqn. 17 gives the following coefficients of matrix A

$$\begin{aligned} a_{12} &= 1 \\ a_{22} &= 1.65 \\ a_{32} &= 0.15 \end{aligned}$$

For the third run, the matrix is of the form;

$$\begin{bmatrix} 1 \\ 1.70 \\ 0.25 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (18)$$

Solving Eqn. 18 gives the following coefficients of matrix A

$$\begin{aligned} a_{13} &= 1 \\ a_{23} &= 1.70 \\ a_{33} &= 0.25 \end{aligned}$$

Therefore, assembling the coefficients obtained thus from Eqn. 16 to Eqn. 18 yields the following coefficient matrix, A;

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1.50 & 1.65 & 1.70 \\ 0.10 & 0.15 & 0.25 \end{bmatrix} \quad (19)$$

Value Computation for Actual Components

The values of the actual components (X_i) of the mixture are computed through the product of the values from matrix A and values from matrix Z.

For A_{12} , substituting the values of Z_i yields;

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1.50 & 1.65 & 1.70 \\ 0.10 & 0.15 & 0.25 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \\ 0 \end{bmatrix} \quad (20)$$

Solving Eqn. 20 gives the values of the real components;

$$\begin{aligned} X_1 &= 1 \\ X_2 &= 1.575 \\ X_3 &= 0.125 \end{aligned}$$

For A_{13} , substituting the values of Z_i yields;

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1.50 & 1.65 & 1.70 \\ 0.10 & 0.15 & 0.25 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0 \\ 0.5 \end{bmatrix} \quad (21)$$

Solving Eqn. 21 gives the values of the real components;

$$\begin{aligned} X_1 &= 1 \\ X_2 &= 1.60 \\ X_3 &= 0.175 \end{aligned}$$

For A_{23} , substituting the values of Z_i yields;

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1.50 & 1.65 & 1.70 \\ 0.10 & 0.15 & 0.25 \end{bmatrix} \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix} \quad (22)$$

Solving Eqn. 22 gives the values of the real components;

$$\begin{aligned} X_1 &= 1 \\ X_2 &= 1.675 \\ X_3 &= 0.20 \end{aligned}$$

The values of the actual and pseudo components of the mix design at different experimental points or runs as determined above are presented in Table 1.

TABLE 1. Mix Design for Trial Points

Runs	Actual Components			Responses	Pseudo Components		
	X1 Soil	X2 RHA	X3 Water		Z1 Soil	Z2 RHA	Z3 Water
1	1	1.50	0.10	Y_1	1	0	0
2	1	1.65	0.15	Y_2	0	1	0
3	1	1.70	0.25	Y_3	0	0	1
4	1	1.575	0.125	Y_{12}	0.5	0.5	0
5	1	1.60	0.175	Y_{13}	0.5	0	0.5
6	1	1.675	0.20	Y_{23}	0	0.5	0.5

Value Computation for Control Points

The mixture proportions for the six control points presenting real components and pseudo components are generated as follows; the control points are denoted as C_1 , C_2 , C_3 , C_{12} , C_{13} , and C_{23} .

For control point C_1 ;

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1.50 & 1.65 & 1.70 \\ 0.10 & 0.15 & 0.25 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.25 \\ 0.25 \end{bmatrix} \quad (23)$$

Solving Eqn. 23 gives the values for C_1 as;

$$\begin{aligned} X_1 &= 1 \\ X_2 &= 1.5875 \\ X_3 &= 0.15 \end{aligned}$$

For control point C_2 ;

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1.50 & 1.65 & 1.70 \\ 0.10 & 0.15 & 0.25 \end{bmatrix} \begin{bmatrix} 0.25 \\ 0.5 \\ 0.25 \end{bmatrix} \quad (24)$$

Solving Eqn. 24 gives the values for C_2 as;

$$\begin{aligned} X_1 &= 1 \\ X_2 &= 1.625 \\ X_3 &= 0.1625 \end{aligned}$$

For control point C_3 ;

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1.50 & 1.65 & 1.70 \\ 0.10 & 0.15 & 0.25 \end{bmatrix} \begin{bmatrix} 0.25 \\ 0.25 \\ 0.5 \end{bmatrix} \quad (25)$$

Solving Eqn. 25 gives the values for C_3 as;

$$\begin{aligned} X_1 &= 1 \\ X_2 &= 1.6375 \\ X_3 &= 0.1875 \end{aligned}$$

For control point C_{12} ;

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1.50 & 1.65 & 1.70 \\ 0.10 & 0.15 & 0.25 \end{bmatrix} \begin{bmatrix} 0.6 \\ 0.2 \\ 0.2 \end{bmatrix} \quad (26)$$

Solving Eqn. 26 gives the values for C_{12} as;

$$\begin{aligned} X_1 &= 1 \\ X_2 &= 1.57 \\ X_3 &= 0.14 \end{aligned}$$

For control point C_{13} ;

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1.50 & 1.65 & 1.70 \\ 0.10 & 0.15 & 0.25 \end{bmatrix} \begin{bmatrix} 0.2 \\ 0.6 \\ 0.2 \end{bmatrix} \quad (27)$$

Solving Eqn. 27 gives the values for C_{13} as;

$$\begin{aligned} X_1 &= 1 \\ X_2 &= 1.63 \\ X_3 &= 0.16 \end{aligned}$$

For control point C_{23} ;

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1.50 & 1.65 & 1.70 \\ 0.10 & 0.15 & 0.25 \end{bmatrix} \begin{bmatrix} 0.2 \\ 0.2 \\ 0.6 \end{bmatrix} \quad (28)$$

Solving Eqn. 28 gives the values for C_{23} as;

$$\begin{aligned} X_1 &= 1 \\ X_2 &= 1.65 \\ X_3 &= 0.20 \end{aligned}$$

The values for the real and its corresponding pseudo components generated for the control points which are used to validate the formulated models are presented in Table 2.

TABLE 2. Mix Design for Control Points

Runs	Actual Components			Responses	Pseudo Components		
	X1 Soil	X2 RHA	X3 Water		Z1 Soil	Z2 RHA	Z3 Water
1	1	1.5875	0.15	C_1	0.5	0.25	0.25
2	1	1.625	0.1625	C_2	0.25	0.5	0.25
3	1	1.6375	0.1875	C_3	0.25	0.25	0.5
4	1	1.57	0.14	C_{12}	0.2	0.2	0.6
5	1	1.63	0.16	C_{13}	0.2	0.6	0.2
6	1	1.65	0.20	C_{23}	0.6	0.2	0.2

Model Validation

Models generated through mathematical process are usually subjected to validation techniques, these techniques are mostly statistical approach employed to test the adequacy and accuracy of the model in prediction or representation of a system. The validation techniques employed in this study are the 'Fisher's Test (F-test) and Student T-test'. The validation also encapsulated two hypothesis.

Null Hypothesis (H_0): There is no significant difference between the experimental values and the predicted values from the model.

Alternate Hypothesis (H_1): There is a significant difference between the experimental values and the predicted values from the model.

Fisher's Test (F-test)

The Fisher's test statistically compares the variance of two distribution. This test gives off a value known as the F-statistics. The F-statistics is given as the ratio of variance between the model's predicted values and the empirical values. Also, the model developed is declared adequate and the null hypothesis accepted if the calculated F-statistics

is less than the F-critical at a confidence level of 95%. Otherwise, the alternate hypothesis is accepted and the model is considered inadequate. The computation of the various values associated with the F-test were automated and was carried out using Microsoft Excel Software version 2013. The output of the F-test is outlined in Table 6.

Student T-test

The T-test is a popular statistical techniques used to ascertain if the mean difference between two groups is statistically significant. Null hypothesis stated that both means are statistically equal, whereas alternative hypothesis stated that both means are not statistically equal i.e., they are statistically different from each other. The Paired Sample T-test was employed in this work for the model validation. The outcome of the T-test produces the T-Statistics also known as T-stat. This calculated T-Stat is then compared against a value obtained from a critical value table called the T-Distribution Table. This comparison helps to determine the effect of chance alone on the difference, and whether the difference is outside that chance range. In this study, the T-test was automated by the use of Microsoft Excel Software version 2013.

RESULTS AND DISCUSSION

The relevant index and engineering properties of the natural lateritic soil and the RHA treated soil at various mix proportions used to perform the optimization technique and model formulation were evaluated.

NATURAL SOIL

The soil, upon visual observation was considered to have a reddish brown colouration which indicates the presence of aluminium oxides which in actuality is predominant in all lateritic soil. The natural moisture content and

specific gravity computed was found to be 7.73% and 2.83 respectively, whilst the liquid limit deduced from the graph and plastic limit were observed to be relatively low, having values of 29% and 19.16% respectively with a plasticity index of 9.84% which further gives an indication of low clay presence.

Moreover, considering the particle size distribution and Atterberg limits, the soil was classified based on AASHTO and (USCS) as A-2-4 and CL respectively. The soil also has an optimum moisture content of 11.19% with a maximum dry density of 1889kg/m³. The CBR value computed was found to be 14.78%. An overview, outlining the different properties of the natural soil is shown in Table 3.

TABLE 3. Index properties of the soil

Properties	Results
Colour	Reddish-Brown
% passing sieve No 200	0
Liquid limit (%)	29.00
Plastic limit (%)	19.16
Plasticity index (%)	9.84
Specific Gravity	2.83
Natural Moisture Content (%)	7.73
AASHTO	A-2-4
USCS	CL
Maximum Dry Density (kg/m ³)	1889
Optimum Moisture Content (%)	11.19
CBR Value	14.78

MODIFIED SOIL

The mix ratios used for the stabilization and optimization technique are duly derived using the scheffe's simplex lattice design technique. The lateritic soil was mixed using the generated mix ratios and therefore subjected to California Bearing Ratio test as per BS 1377. The experimental results of the test showing the CBR values of the now modified soil, its corresponding mix components (Real and Pseudo) and mix ratios for the Trial mix and Control points is presented in Table 4 and Table 5 respectively.

SCHEFFE'S OPTIMIZATION

The model formulated by Scheffe's method used for predicting the CBR values of the soil stabilized with proportions of RHA is given in *Eqn 12*. Also, values for coefficients of the model are computed using *Eqn 13* and *Eqn 14* such that;

$$\begin{aligned}\beta_1 &= Y_1 = 21 \\ \beta_2 &= Y_2 = 19 \\ \beta_3 &= Y_3 = 30 \\ \beta_{12} &= 4Y_{12} - 2Y_1 - 2Y_2 \\ &= 4 \times 18 - 2 \times 21 - 2 \times 19 = -8 \\ \beta_{13} &= 4Y_{13} - 2Y_1 - 2Y_3 \\ &= 4 \times 18 - 2 \times 21 - 2 \times 30 = -30 \\ \beta_{23} &= 4Y_{23} - 2Y_2 - 2Y_3 \\ &= 4 \times 20 - 2 \times 19 - 2 \times 30 = -18\end{aligned}$$

TABLE 4. California Bearing Ratio Experimental Values for Trial Mix

Runs	Actual Components			Responses	CBR Values (%)	Pseudo Components		
	X ₁	X ₂	X ₃			Z ₁	Z ₂	Z ₃
	Soil	RHA	Water			Soil	RHA	Water
1	1	1.50	0.10	Y ₁	21	1	0	0
2	1	1.65	0.15	Y ₂	19	0	1	0
3	1	1.70	0.25	Y ₃	30	0	0	1
4	1	1.575	0.125	Y ₁₂	18	0.5	0.5	0
5	1	1.60	0.175	Y ₁₃	18	0.5	0	0.5
6	1	1.675	0.20	Y ₂₃	20	0	0.5	0.5

TABLE 5. California Bearing Ratio Experimental Values for Control Mix

Runs	Actual Components			Responses	CBR Values (%)	Pseudo Components		
	X ₁	X ₂	X ₃			Z ₁	Z ₂	Z ₃
	Soil	RHA	Water			Soil	RHA	Water
1	1	1.5875	0.15	C ₁	18	0.5	0.25	0.25
2	1	1.625	0.1625	C ₂	17	0.25	0.5	0.25
3	1	1.6375	0.1875	C ₃	18	0.25	0.25	0.5
4	1	1.65	0.2	C ₁₂	16	0.2	0.2	0.6
5	1	1.63	0.16	C ₁₃	19	0.2	0.6	0.2
6	1	1.57	0.14	C ₂₃	20	0.6	0.2	0.2

TABLE 6. Values for the Coefficients of the Model

β ₁	β ₂	β ₃	β ₁₂	β ₁₃	β ₂₃
21	19	30	-8	-30	-18

Furthermore, substituting the above values of the various model coefficients into the polynomial equation (Eqn.12), the optimization model for predicting the California bearing ratio of lateritic soil stabilized with rice husk ash is arrived at as;

$$Y=21Z_1+19Z_2+30Z_3-8Z_1Z_2-30Z_1Z_3-18Z_2Z_3 \quad (29)$$

The equation (Eqn 29) can be employed to predict the California bearing ratio of lateritic soil stabilized with rice

husk given any mix proportion within the same locality. This equation provides a futuristic, time and cost-effective approach to Engineers seeking to ascertain the CBR property of the same soil given same stabilizing agent.

The CBR values for the trial mix and control points in the sixth columns of Table 7 and Table 8 respectively is obtained by substituting the values for each pseudo component (Z₁, Z₂, and Z₃) into Eqn 29.

TABLE 7. California Bearing Ratio Model Values for Trial Mix

Runs	Actual Components			Responses	CBR Values (%)	Pseudo Components		
	X ₁	X ₂	X ₃			Z ₁	Z ₂	Z ₃
	Soil	RHA	Water			Soil	RHA	Water
1	1	1.50	0.10	Y ₁	21	1	0	0
2	1	1.65	0.15	Y ₂	19	0	1	0
3	1	1.70	0.25	Y ₃	30	0	0	1
4	1	1.575	0.125	Y ₁₂	18	0.5	0.5	0
5	1	1.60	0.175	Y ₁₃	18	0.5	0	0.5
6	1	1.675	0.20	Y ₂₃	20	0	0.5	0.5

TABLE 8. California Bearing Ratio Model Values for Control Point

Runs	Actual Components			Responses	CBR Values (%)	Pseudo Components		
	X ₁	X ₂	X ₃			Z ₁	Z ₂	Z ₃
	Soil	RHA	Water			Soil	RHA	Water
1	1	1.5875	0.15	C ₁	16.875	0.5	0.25	0.25
2	1	1.625	0.1625	C ₂	17.125	0.25	0.5	0.25
3	1	1.6375	0.1875	C ₃	18.5	0.25	0.25	0.5
4	1	1.65	0.2	C ₁₂	19.92	0.2	0.2	0.6
5	1	1.63	0.16	C ₁₃	17.28	0.2	0.6	0.2
6	1	1.57	0.14	C ₂₃	17.12	0.6	0.2	0.2

VALIDATION ANALYSIS

The validation involved six control points which were used to verify the adequacy of the generated model. Table 9 and Table 10 gives a view of the results from the f-test and the student t-test which were automatically computed using Microsoft Excel program. It shows the variance of the experimental and model values, mean and probability value (p-value). Also, the f-statistics value which is the ratio of the both variance (experimental and model) is shown in Table 9 as 1.422. This value being less than the f-critical value of approximately 5.05 affirms that there is no significant difference between the experimental values of the California

bearing ratio and the model predicted values. Therefore, the null hypothesis was accepted and the model is adequate.

MANUAL CHECK

In order to further verify the accuracy of the F-test results computed by Microsoft Excel software, a manual computation was also carried out. The average mean values for the experimental and model responses were calculated. The variance for each responses (experiment and model) are also computed. The F-statistics value obtained is compared to the F-critical value corresponding to a degree of freedom value of five (5) obtained from an F-test distribution table in order to ascertain which value is greater or lesser.

TABLE 9. F-Test Two-Sample for Variances

Description	Responses	
	Experiment	Model
Mean	18	
Variance	2	17.80333333
Observations	6	1.405676667
Df	5	6
F	1.422802304	5
P(F<=f) one-tail	0.354116816	
F Critical one-tail	5.050329058	

TABLE 10. T-test: Paired Two Sample for Means

Description	Responses	
	Experiment	Model
Mean	18	
Variance	2	
Observations	6	
Pearson Correlation	-0.649486553	
Hypothesized Mean Difference	0	17.80333333
Df	5	1.405676667
t Stat	0.20386668	6
P(T<=t) one-tail	0.423247774	
t Critical one-tail	2.015048373	
P(T<=t) two-tail	0.846495549	
t Critical two-tail	2.570581836	

TABLE 11. Values for Manual Statistical F-test Computation

Response	Y_E	Y_M	$Y_E - Y_{EA}$	$Y_M - Y_{MA}$	$(Y_E - Y_{EA})^2$	$(Y_M - Y_{MA})^2$
C_1	18	16.875	0	-0.92833	0	0.861802778
C_2	17	17.125	-1	-0.67833	1	0.460136111
C_3	18	18.5	0	0.696667	0	0.485344444
C_{12}	16	19.92	-2	2.116667	4	4.480277778
C_{13}	19	17.28	1	-0.52333	1	0.273877778
C_{23}	20	17.12	2	-0.68333	4	0.466944444
	$\Sigma = 108$	$\Sigma = 106.82$			$\Sigma = 10$	$\Sigma = 7.028383333$
	$Y_{EA} = 18$	$Y_{MA} = 17.80333$				

From Table 11 Where,

- Y_E = Experimental Values
- Y_M = Model Values
- Y_{EA} = Average Experimental Value
- Y_{MA} = Average Model Value

Mathematically, the F-test is given as;

$$F = \frac{S_1^2}{S_2^2} \tag{30}$$

Where;

- S_1^2 = Greater value of both Variances
- S_2^2 = Lesser value of both Variances

The value of Variance, S^2 for the Experimental values is obtained from the expression thus as;

$$S^2 = \frac{\Sigma(Y_E - Y_{EA})^2}{N-1} \tag{31}$$

Where,

$N = 6$ (Number of Observation or runs),

$$S^2 = \frac{10}{6-1} = 2$$

Also, the value of Variance, S^2 for the Model values is obtained from the expression thus as;

$$S^2 = \frac{\Sigma(Y_M - Y_{MA})^2}{N-1} \tag{32}$$

$$S^2 = \frac{7.028383333}{6-1} = 1.405676667$$

Since, Experimental variance is greater than Model variance. Therefore, the value of ‘‘F’’ is given as;

$$F = \frac{2}{1.405676667} = 1.4228023$$

The Manual calculation gives a value for F-statistics corresponding to the F-Statistics values computed by the software, hence, the Microsoft Excel value is accurate and valid.

CONCLUSION

1. In line with all Engineering standards and empirical procedures employed in the optimization of the California bearing ratio of lateritic soil stabilized with rice husk ash, the following conclusions were drawn.
2. The optimal mix was observed to be of ratio 1: 1.70: 0.25 for lateritic soil, rice husk and water with a CBR value of 30%.
3. The optimization model developed for the CBR of Rice Husk Ash stabilized lateritic soil can be employed reliably in the prediction of the CBR given any arbitrary mix ratio and vice versa.
4. The F-test and T-test employed in verifying the adequacy of the model at a 0.05% probability confirmed the adequacy of the model.

RECOMMENDATION

In line with different property variation present in soils, the results and observations from this study are exclusively limited to future works to be conducted within the same locality. More so, to reduce the impact of agricultural wastes in the environments, extensive studies should also be conducted on other available agricultural waste materials such as (bean husk, bone ash, coconut husk) to mention a few, so as to encourage its optimal usage in pavement construction.

Furthermore, other properties of the soil from this work such as the unconfined compressive strength, swelling potentials, shear strength, should be determined as this study only considered the California Bearing Ratio. In addition, available optimization techniques such as the Classical Optimization method should also be considered for use and its flexibility in incorporating more than one soil property at a time should also be exploited.

Contribution to Knowledge

Model constants for predicting the California Bearing Ratio of lateritic soil stabilized with Rice Husk Ash was developed for soils in this locality using scheffe’s method for the first time. These model when applied would save other researchers, young Engineers and road contractors from rigorous and time consuming efforts.

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DECLARATION OF COMPETING INTEREST

None

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