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Analytical Formulation for Optimisation of Torque Density in Magnetic Gears

(Formulasi Analitikal untuk Pengoptimuman Ketumpatan Tork dalam Gear Magnetik)

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ABSTRACT

Energy efficiency is a very important consideration in vehicle design today. Minimizing weight would go a long way in this direction, as it directly contributes up to 75% of fuel consumption. Mechanical gearboxes and transmissions are components that add significantly to vehicle weight. For example, the mass of typical automotive transmissions ranges from 40 to 200 kg, depending on the size of the vehicle. In recent years, magnetic gears (MG) have gained attention as realistic alternatives to mechanical gears. MGs operate without contact, where torque amplification, reduction, and transmission are achieved through the interaction of magnetic fields, which allows for lighter designs. For practical use in automotive vehicles, MGs must be capable of transferring sufficient amounts of torque. To maintain being compact and lightweight, the torque density of the MG must be maximized. Hence, the MG needs to be designed in such a way so as to achieve high flux concentration. Torque density analysis of MGs is not trivial, and often, some trade-offs between accuracy and computational cost is required. Several methods of analysis have been used by various researchers, including Finite Element Analysis, Reluctance Network Analysis (RNA), the Quasi 3D Analytical Method, and Genetic Algorithms (GA). In this work, we examine these methods and present an analytical formulation that relates MG torque to its physical parameters. Understanding such relationships could help optimise future MG designs such that torque density can be maximised.

Keywords: Magnetic gear (MG); Analytical method; Torque density; MG performance; Design optimization.

ABSTRAK

Kecekapan tenaga adalah pertimbangan yang sangat penting dalam reka bentuk kenderaan hari ini. Meminimumkan berat akan sangat membantu ke arah ini, kerana ia menyumbang secara langsung sehingga 75% penggunaan bahan api. Kotak gear dan transmisi mekanikal adalah komponen yang menambah berat kenderaan dengan ketara. Sebagai contoh, jisim transmisi automotif biasa adalah antara 40 hingga 200 kg, bergantung pada saiz kenderaan. Dalam beberapa tahun kebelakangan ini, gear magnetik (MG) telah mendapat perhatian sebagai alternatif yang realistik kepada gear mekanikal. MG beroperasi tanpa sentuhan, di mana penguatan tork, pengurangan, dan penghantaran dicapai melalui interaksi medan magnet, yang membolehkan reka bentuk yang lebih ringan. Untuk kegunaan praktikal dalam kenderaan automotif, MG mesti mampu memindahkan jumlah tork yang mencukupi. Untuk kekal kompak dan ringan, ketumpatan tork MG mesti dimaksimumkan. Oleh itu, MG perlu direka bentuk sedemikian untuk mencapai penumpuan fluks yang tinggi. Analisis ketumpatan tork MG tidaklah remeh, dan selalunya, beberapa saling pertukaran antara ketepatan dan kos komputeran diperlukan. Beberapa kaedah analisis telah digunakan oleh pelbagai penyelidik, termasuk Analisis Elemen Terhad, Analisis Rangkaian Keengganan (RNA), Kaedah Analisis Kuasi 3D, dan Algoritma Genetik (GA). Dalam kerja ini, kami meneliti kaedah ini dan membentangkan rumusan analitikal yang mengaitkan tork MG dengan parameter *fizikalnya. Memahami perhubungan sedemikian boleh membantu mengoptimumkan reka bentuk MG masa hadapan supaya ketumpatan tork dapat dimaksimumkan.*

Kata kunci: Gear magnet (MG); Kaedah analisis; Ketumpatan tork; Prestasi MG; Pengoptimuman reka bentuk

INTRODUCTION

In vehicle design, minimizing weight is an important consideration as it directly contributes to as much as 75% of fuel consumption. Mechanical linkages contribute significantly to the vehicle weight. For example, the mass of typical automotive transmissions ranges from 40 to 200 kg, depending on the size of the vehicle. The use of mechanical gears themselves have some drawbacks, among which are that they function through mechanical contact, causing wear and tear, as well as emitting noise. As such, they must be continuously lubricated, which increases maintenance costs. Researchers have proposed possible solutions based on magnetic gears (MG) over the last decade to address these shortcomings.

In order to improve the torque densities from a drive train, mechanical gears are typically used. The same function is intended for magnetic gears, which also provide other advantages including smaller weights, increased dependability, no wear and tear, contact-free operation, intrinsic overload protection, less noise, higher efficiency, and low maintenance requirements. Magnets are installed on the driven and drive shafts in magnetic gear systems. This enables the transmission of torque between the shafts without any physical contact through interactions between the magnetic fields. The torque amplification or reduction effects are caused by the interplay of magnetic fields, which are regulated by a number of pole pieces positioned in the airgap.

The development of MGs has gained the attention of researchers due to its non-contact nature and the potential to replace conventional mechanical gearboxes, thus offering a solution to the existing mechanical problems. Mechanical gearboxes are widely used to transfer high torque, low speed (LS) input into low torque, high speed (HS) output, resulting in a more effective and lighter topology of the drive. However, the reliability of the mechanical gearbox and the costs associated with its failure make magnetic gear topologies that reduce or eliminate a number of components found in the mechanical gearbox attractive (Penzkofer & Atallah 2014; Polinder et al. 2006; Praslicka et al. 2020; Shi et al. 2019).

An MG sends torque or power through magnetic field interactions rather than through mechanical connections. Torque density and gear ratio are used to evaluate the performance of mechanical as well as magnetic gears. The topology of magnetic gear designs capable of producing high torque density has been previously studied. The torque densities and gear ratios are affected and determined by the magnetic gear parameters themselves. Magnetic gears exhibit the same basic behaviour as mechanical gears in that they increase and decrease the torque and input-output speed, but MGs do so without physical contact between moving components (Al-Qarni & El-Refaie 2021; Johnson et al. 2014; Kołodziej et al. 2021).

The MG must be able to transfer enough torque in automotive vehicle applications for it to be useful. The MG's torque density must be maximized for a geared transmission system to be small and light. Hence, the MG needs to be designed in such a way so as to achieve high flux concentration. The axial field magnetic gear (AFMG) topology is best suited for this, and an MG with a torque density of over 70 kN/m^3 has been reported (Kais Atallah et al. 2008; Mezani et al. 2006). This gear design is applied when it is important to isolate the input and output shafts.

The axial flux configuration (M. Chen et al. 2015a; Fodorean 2016; Lubin et al. 2013), such as tha shown in Figure 1, has the highest power density, due to the smaller size and weight of passive components like the shaft and housing. Most frequently, silicon steel strip is used to make the lower-speed rotor. The unusual disc-shaped design of the AFMG is its main benefit. They can be deployed in tight and challenging-to-reach areas because of their small active length relative to their diameter. Torque density analysis of MGs is not trivial, and often, some trade-offs between accuracy and computational cost are required.

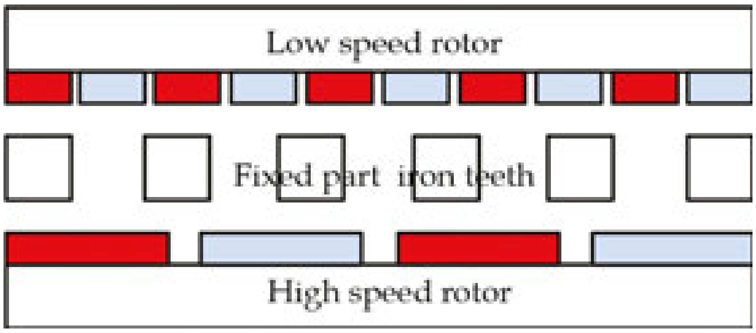

FIGURE 1. AFMG configuration (M. Chen et al. 2015b; Fodorean 2016; Lubin et al. 2013)

Calculations using three-dimensional numerical models are used to determine the torque characteristics of the examined AFMGs, which are then verified on a physical model. Studies based on concentric magnetic gear have highlighted the benefits of contactless energy conversion or transmission using magnetic fields. (K. Atallah et al. 2004; Kais Atallah et al. 2001).

Analytical approaches need significantly less computing time than FEA because they are predicated on some simplifying assumptions (linear material properties, simple boundary conditions, and simplified geometry), and thus can be useful tools as a first step in design optimization (Lubin et al. 2013). Analytical methods are essential tools for the initial assessment of magnetic gear performances and design optimization because continuous derivatives generated from the analytical solution are fundamental in the majority of optimization approaches.

In this paper, some commonly used methods for analysing torques in MGs are examined. Numerical methods are first studied, focussing on Finite Element Analysis (Y. Chen et al. 2014; Evans & Zhu 2011), Reluctance Network Analysis (M. Fukuoka et al. 2012; Michinari Fukuoka et al. 2011b), the Quasi 3D Analytical Method (Afsari et al. 2015; Azzouzi et al. 2003), and Genetic Algorithms (GA) (Li-bing Jing et al. 2016; Mac et al. 2021). The derivation of analytical equations relating MG torques to MG physical characteristics comes next. Based on a certain MG design, the analysis that is being presented. It is planned that by figuring out the connections between torques and the physical variables, optimal MG designs that maximum torque density can be accomplished in a quick and easy way.

ANALYSIS METHODS

Analysis of magnetic fields are mainly done either numerically or analytically. Due to the eccentric rotor's eccentric revolution and rotation, magnetic field analysis is challenging. The rotation of the non-uniform wave generator impacts automatic meshing and lengthens the computing process even though numerical methods utilizing FE analysis provide excellent precision. Analytical methods, on the other hand, utilise simple physical concepts, resulting in quicker calculations, and allow for free rotation without mesh constraints. As a result, analytical methods are flexible even if the air gap is not uniform due to the eccentric rotation of the rotor (Zhang et al. 2019).

Due to coercion in 3-D models, the numerical technique in axial flux designs is more challenging. Analytical techniques have been offered as a way of overcoming this drawback. These techniques are based on a number of simplifying assumptions, including the following: 1) simplified geometry; 2) linear magnetic characteristics of materials; and 3) simple boundary conditions (Lubin et al. 2010, 2013).

Alternative to using numerical approaches for magnetic field analysis, the analytical approach is a highly desirable method as it provides in-depth knowledge of the design and optimization of the MG.

Analytical approaches can be helpful tools for the first stage of an optimal design because they are based on some simplifying assumptions and take significantly fewer computations than FEM (Gysen et al. 2010; Lubin et al. 2010, 2012, 2013). By analysing the Laplace and Poisson equations for each subdomain, Lubin et al. (2010) created an analytical technique for estimating flux distribution in a radial flux MG. For forecasting the flux distribution and electromagnetic torque in an AFMG, Lubin et al. (2013) proposed a 2-D analytical model that solves Maxwell's equations in subdomains.

NUMERICAL APPROACHES

FINITE ELEMENT ANALYSIS

In the design and study of MGs, numerical techniques like the finite element method (FEM) have been extensively used since they are capable of properly simulating magnetic field distributions. One of the most efficient ways to calculate electromagnetic fields is via the FEM (Lou & Yao 2020; Zhuang & Zeng 2013).

The distribution of the air-gap magnetic field must be well understood in order to predict the performance of an axial magnetic gear. The air-gap magnetic field can be calculated using numerical methods like finite element analysis (FEA), analytical or semi-analytical procedures, or both. FEA generates accurate outcomes, when geometrical details of magnetic gears and nonlinearities caused by magnetic saturation of the iron parts are taken into account. However, for the first stage in the design of a magnetic gear, this method is extremely time demanding and inflexible (Li-bing Jing et al. 2016; Lubin et al. 2013).

The FEM has proved to be more accurate and efficient compared to traditional transmission and reflection methods. Concentric MGs often have no periodic symmetry, necessitating full FE model simulations, in contrast to the conventional practise in analysing electrical machines, which uses 2D FE modelling and geometrical symmetry to reduce complexity in the FE models. This increases the computational cost of 2D FE modelling, making it similar to 3D FE modelling (Tlali et al. 2014).

The design of electrical devices utilizing FEM has proved successful. In the analysis of electromagnetic device performance, both static and dynamic, it is currently an essential tool. The advantages of FEM over analytical methods are its high accuracy and reliability. It can also be applied to intricate geometric structures. However, numerical algorithms are extremely time consuming when dealing with optimization problems because new nets must be generated repeatedly based on variations in geometric parameters from the proposed design (Niu et al. 2012). There are also some other shortcomings of using FEM for permanent magnet gear analysis, including (Ge et al. 2012; Jian et al. 2009);

1. The geometric measurements and the material characteristics of the calculated objects must be well determined in advance as they cannot be changed while the FEM program is calculating,
2. In addition to being conservative in parametric optimization, 3D FEM is unable to provide either closed-form solutions or specific model parameters.

RELUCTANCE NETWORK ANALYSIS (RNA)

A single reluctance network expresses an analytical object in an RNA, as seen in Figure 2 below. Finite element modelling, which takes time, especially for 3D structures, was commonly necessary for accurate modelling of electrical machinery and magnetic gear. In order to search a wide searching space and examine as many choices as possible, lightweight models are chosen in the predesign stages of the design.

Numerous academics are examining the use of alternative techniques to determine the best criteria, while simultaneously emphasising the importance of having higher performance models (Barrière et al. 2010; Huang et al. 2012; Jin et al. 2011, 2014; Lubin & Rezzoug 2015; Tiegna et al. 2012, 2014). As a result, some authors (Amrhein & Krein 2009b, 2009a; Hur et al. 2001) concentrate on using 3D reluctance network models for the analysis of various gear magnet and electromagnetic designs.

The RNA provides a number of benefits, including an easy-to-use analytical model, excellent computation accuracy, and simplicity in coupling simulation with rotor spinning (M. Fukuoka et al. 2012; Michinari Fukuoka et al. 2011b). Numerous electric equipment, including transformers and motors, have had their characteristics calculated using the RNA (Michinari Fukuoka et al. 2011a, 2011b). The RNA contains one resistance network that expresses an analytical item. It is possible to determine all reluctances using the B-H curve for the material and size (Hane et al. 2019; Nakamura & Ichinokura 2008). However, an approach for expressing magnetic hysteresis for RNA has not yet been developed (Michinari Fukuoka et al. 2011a; Hane & Nakamura 2018).

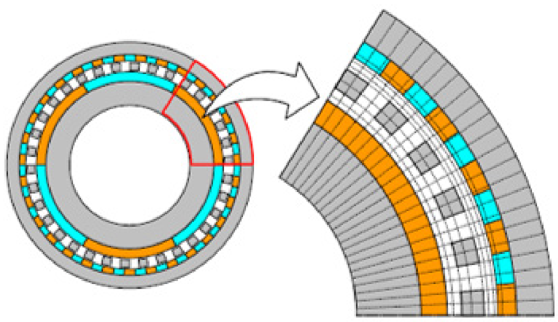

FIGURE 2. Division of the magnetic gear based on RNA (Michinari Fukuoka et al. 2011b; Hane et al. 2019)

QUASI 3D ANALYSIS

This method of analysis provides an adequate overview of the effects of geometric parameters and materials on magnetic properties because they are based on reasonable hypotheses and simplified mathematical models and are more time-saving than numerical methods.

By using an intuitive and efficient radial dependence modelling of the axial and tangential magnetic field extracted from a 3D finite element method, this quasi 3D analytical method combines the accuracy and precision of a 3D FEM with the simplicity and speed of an analytical method. It is derived from an exact 2D magnetic field analytical method that has been extended to the 3D case (Afsari et al. 2015).

The fundamental concept is to split the 3D problem into numerous 2D problems because it is challenging to calculate the 3D magnetic field. A number of radial slices can be combined to form an AFMG, with each slice's magnetic field being seen as being invariant in the radial direction. (Tiegna et al. 2012; Zhao et al. 2021). The precision of this equivalence method is seen in Figure 3 below.

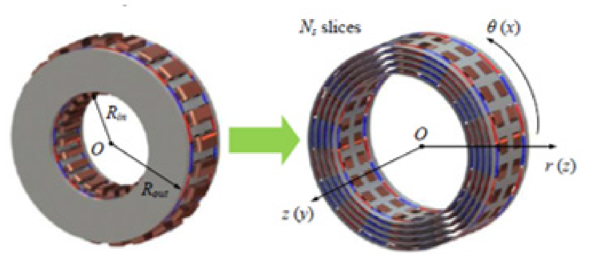

FIGURE 3. The dimension reduction process of an axial flux motor (Zhao et al. 2021)

The following presumptions and hypotheses are used in the analytical model for forecasting the magnetic field produced by the permanent magnet(PM), to aid in the analytical modeling of magnetic fields (Afsari et al. 2015; Choi et al. 2011; Lorraine & Recherche 2013; Wu & Wang 2015):

1. there is only one dimension in the magnetic field;
2. PM and coil permeability same with air gap. So, the coil region is a part of the air gap region;
3. infinite stator core and rotor permeability;
4. the magnet remains magnetized axis with relative retraction, μr=1;
5. there is no magnetic saturation on the ring-shaped iron yokes, and it has high permeability. Ferromagnetic region (back and pillar pieces) is considered perfect (that is, infinite permeability), as well as being considered a boundary condition for the problem being studied (homogeneous Neumann boundary conditions);
6. the impact of slots is ignored, and the air gap is considered to be a continuous area;
7. at certain times, a third of the driver's permanent magnets are magnetically combined with the following for the magnetic gear set.

GENETIC ALGORITHMS

John Holland put forth the idea for a genetic algorithm (GA), in the 1970s. GA developed as a result of genetic predisposition and the law of the strongest. Recently, GA has become a crucial tool in contemporary optimization theory. GA is a global optimization algorithm with implicit parallelism. (Qi et al. 2006).

In computational mathematics, the outer diameter, ratio of inner and outer diameter, permanent magnet height, and air-gap length are some examples of optimisation problems that are resolved using the GA which a random search algorithm based on natural selection and genetic principles. (Libing Jing et al. 2022; Xu et al. 2017).

Ramachandran & Prakash (Ramachandran & Prakash 2017) used GA to optimize the width and height of the magnetic adjusting block, resulting in the optimized magnetic gear having a higher torque density. According to Xu et al. (Xu et al. 2017), machine optimization is a complex nonlinear problem that can be solved with GA. GA optimization is used in conjunction with the initial design to find the best solution while taking machine specificity into consideration.

In order to enhance the performance of motors with magnetic gears, many researchers are investigating the optimisation of parameters using GA. The design of a permanent magnet brushless dc motor by Upadhyay and Rajagopal (Upadhyay & Rajagopal 2005) was optimised using GA, taking into account aspects like slot electric loading, magnet-fraction, slot-fraction, airgap, and airgap flux density.

GA has been used by Ho et al. (Ho et al. 2010) to optimise the distribution of permanent magnets and reduce the motor cogging torque. In (Mahmoudi et al. 2013), the authors reported an axial-flux permanent-magnet motor optimised by a GA-based sizing equation. The results of FEA and GA simulations were compared and were found to agree well with the flux density at different locations of the developed motor under no-load conditions.

Numerical methods such as those that have been presented in this section provide approximate solutions but are not able to provide the closed form solutions. Analysis and optimization of parameters require time consuming repeated iterations. If the closed form solution could be obtained through analytical methods, analysis and parameter optimisation could be significantly expedited as only parameter values need to be changed without having to perform iterations. In the following, we present an analytical solution for an axial field magnetic gear with cylindrically shaped magnets.

AN ANALYTICAL APPROACH

The tangential and radial components of air-gap magnetic fields are used to compute the torque of magnetic gears. The magnetic flux density B is described as follows in free space:

$$B = \mu_0 H \quad (1)$$

where B is expressed in weber per square metre (Wb/m2) or Tesla (T), and $\mu_0\mu_0$ is the permeability of empty space. H is the strength of the magnetic field. B represents the entire magnetic field in Equation (1), which may also include the contribution M from the magnetic characteristics of the materials in the field. The magnetic field strength of a single magnetic source may therefore be expressed as

$$H = \frac{B}{\mu_0} - M \quad (2)$$

The defined magnetic permeability value for empty space, expressed in henrys per metre (H/m), is provided by the constant $\mu_0\mu_0$, which is not dimensionless.

$$\mu_0 = 4\pi \times 10^{-7}\,\mathrm{H/m} \quad (3)$$

The designation "weber per square metre" denotes that the magnetic flux density vector of B is a member of the flux density family of vector fields. The rules of Biot-Savart and Coulomb can be compared, leading to an analogy between H and E, as one possible comparison between electric and magnetic fields. B and D can be compared thanks to the equations $B = \mu_0 H$ $B = \mu_0 H$ and $D = \epsilon_0 E D = \epsilon_0 E$. If B is calculated in teslas or Weber per square metre, magnetic flux should be calculated in Weber.
Then, the relationship between magnetic field strength and torque can be written as;

$$\tau = m \times B \quad (4)$$

where, respectively, $\tau, m, and\ B$ $\tau, m, and\ B$ represent the torque acting on the dipole, the external magnetic field, and the magnetic dipole moment.

The magnetic strength and orientation of a magnet or any item that produces a magnetic field can be characterised as the magnetic dipole moment, according to equation (4) above. In addition, the alignment torque on the item caused by an externally applied magnetic field and the field vector itself can also be characterised as a vector using the term "magnetic dipole moment."

Then, the equation for magnetic dipole moment we can stated as;

$$m = \frac{1}{\mu_0} B_r V \quad (5)$$

Where, correspondingly, $m, \mu_0, B_r\ and\ V$ $m, \mu_0, B_r\ and\ V$ stand for the dipole moment (in Am^2 = Nm/T = J/T), vacuum permeability ($4\pi \times 10^{-7} 4\pi \times 10^{-7}$ H/m), residual flux density (in T), and magnet volume (in m^3).

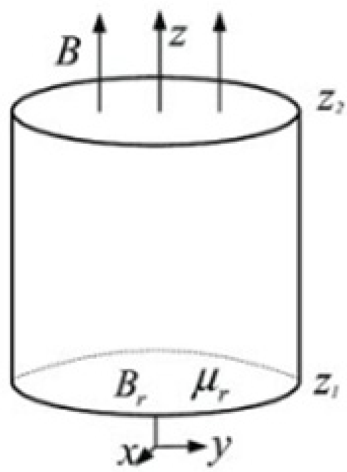

FIGURE 4. Calculation model of a cylindrical magnet

In this study, we derive an easy analytical formula for calculating the magnetic field generated by a ring-shaped magnet. The magnet is positioned with its upper surface at $z_2 z_2$ and lower surface at $z_1 z_1$, as shown in Figure 4 above, and is magnetised along the z-direction.

Then, the magnetic field strength at a point of distance X from the surface of a cylindrical magnet, as shown in Figure 5, can be calculated using Equation (6). Here, B_r is the residual flux density, R is the cylinder radius, r is length of the inner radius and L is its axial length.

$$B(X) = \frac{B_r}{2}\left[\left(\frac{L+X}{\sqrt{R^2+(L+X)^2}} - \frac{X}{\sqrt{r^2+(L+X)^2}}\right) - \left(\frac{X}{\sqrt{X^2+R^2}} - \frac{X}{\sqrt{X^2+r^2}}\right)\right] \quad (6)$$

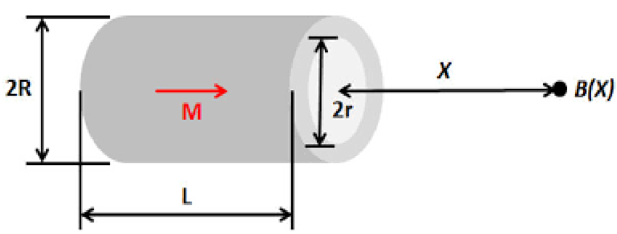

FIGURE 5. The magnetic field strength of cylindrical magnet

Consequently, the magnet's material, size, shape, and orientation, together with its distance from the magnetic surface, all affect the magnetic field's strength. First and foremost, we used neodymium magnets for our study. These magnets, sometimes referred to as super magnets or NdFeB, NIB, or Neo magnets, are the strongest permanent magnets and have been used frequently in earlier research and studies. Due to the fact that a larger magnet may produce a stronger magnetic field, the size of magnet that was used was the largest. The magnetic field is then magnetized along the magnet's axis, resulting in an axial form of magnet orientation. The magnet should be placed as close to the object as possible to produce the strongest magnetic field. The field magnets on the high-speed and low-speed rotors are necessary due to the magnetic flux of the MG. The permeance of stationary pole pieces modulates the magnetomotive forces of the high- and low-speed rotors.

If a charge q is moving at a speed of v in a direction that forms an angle $\theta\theta$ with the magnetic field's direction, the magnetic force *F* that the magnetic field exerts on it is given by

$$F = qvB \sin\theta \tag{7}$$

According to (Aiso et al. 2019),the following equations result from adapting equation (7) to account for two magnetic fields rotating within one another's fields. With reference to Figure 6, the magnetomotive forces of the high-speed rotor, $F_h F_h$, and the low-speed rotor, $F_l F_l$, can be written as Equations (8) and (9), where $F_h F_h$ and $F_l F_l$ are defined as the magnetomotive force of magnets on the high-speed rotor and magnets on the low-speed rotor, respectively. The magnetomotive force amplitudes of the high-speed and low-speed rotors in the MG system are designated $F_{sh} F_{sh}$ and $F_{sl} F_{sl}$, respectively.

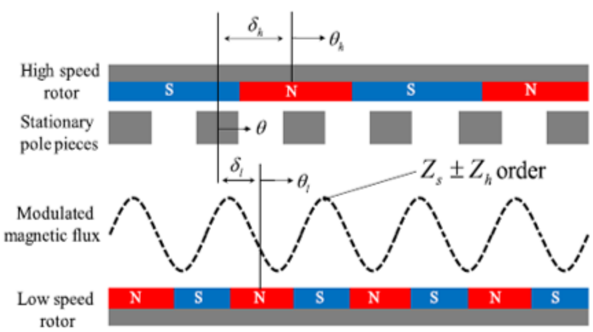

FIGURE 6. The fundamentals of magnetic flux modulation in permanent magnet gear (Aiso et al. 2019)

$$F_h = F_{sh} \cos(Z_h \theta_h - Z_h \omega_h t) \tag{8}$$

$$F_l = F_{sl} \cos(Z_l \theta_l - Z_l \omega_l t) \tag{9}$$

The following is how stationary pole piece permeance is expressed:

$$P_p = P_{s0} + P_{sa} \cos(Z_s \theta) \tag{10}$$

$P_{s0} P_{s0}$ and $P_{sa} P_{sa}$ stand for the average components of the permeance of the stationary pole pieces and the amplitude of the permeance of the stationary pole pieces in the MG, respectively, where $P_p P_p$ is defined as the permeance of the stationary pole pieces. The stationary pole pieces are identified by their . Z_s Z_s number.

The magnetic flux distribution in the air gap is calculated using equations (8) through (10).

$$\begin{aligned}
\phi(\theta, t) &= (F_h + F_l) P_p \\
&= F_{sh} P_{s0} \cos(Z_h \theta - Z_h \omega_h t - Z_h \delta_h) \\
&+ F_{sl} P_{s0} \cos(Z_l \theta - Z_l \omega_l t - Z_l \delta_l) \\
&+ \frac{1}{2} F_{sh} P_{sa} [\cos\{(Z_s + Z_h)\theta - Z_h \omega_h t - Z_h \delta_h\} \\
&+ \cos\{(Z_s - Z_h)\theta + Z_h \omega_h t + Z_h \delta_h\}] \\
&+ \frac{1}{2} F_{sl} P_{sa} [\cos\{(Z_s + Z_l)\theta - Z_l \omega_l t - Z_l \delta_l\} + \\
&\cos\{(Z_s - Z_l)\theta + Z_l \omega_l t + Z_l \delta_l\}]
\end{aligned} \tag{11}$$

The pole pairs of the input and output rotors, respectively, are represented here as $Z_h Z_h$ and $Z_l Z_l$. The values of $\omega_h \omega_h$, $\omega_l \omega_l$, θ_h θ_h, $\theta_l \theta_l$, $\delta_h \delta_h$, and $\delta_l \delta_l$ correspond to the angular speeds of the input and output rotors, the mechanical angles of the input and output rotors, the deviation angle between the centre of the pole pieces and the centre of the input rotor magnet, and the deviation angle between the censer of the pole pieces and the centre of the output rotor magnet, respectively.

The high-speed and low-speed rotors must have the following conditions for the pole pairs to determine the gear ratio:

$$Z_l = Z_s \pm Z_h \tag{12}$$

Due to the fixed position of the pole pieces, the rotational velocity of the spatial flux is given by

$$\omega_l = \frac{Z_h}{Z_s \pm Z_h} \omega_h \tag{13}$$

In light of this, the gear ratio between the high-speed and low-speed rotors is provided by

$$G_s = \pm \frac{Z_h}{Z_l} \tag{14}$$

The average torque is then represented as follows:

$$T = \frac{\partial W_m}{\partial \theta} = \frac{1}{2\pi}\int_0^{2\pi} \phi \times \frac{\partial F}{\partial \theta} d\theta \tag{15}$$

The following torque equations are produced for the high-speed and low-speed rotors when (8), (9) and (11) are substituted into (15) if the pole pair criteria of (12) are met.

$$T_h = \frac{1}{4} F_{sl} F_{sh} P_{sa} Z_h \sin\{-(Z_l\omega_l \mp Z_h\omega_h)t - Z_l\delta_l \pm Z_h\delta_h\} \tag{16}$$

$$T_l = \frac{1}{4} F_{sl} F_{sh} P_{sa} Z_l \sin\{-(Z_l\omega_l \mp Z_h\omega_h)t - Z_l\delta_l \pm Z_h\delta_h\} \tag{17}$$

The average torque is determined by the amplitudes of the magnetomotive forces of the high-speed and low-speed rotors, the amplitude of the permeance of the stationary pole pieces, and the pole pairs of the high-speed and low-speed rotors, according to the equation above. The condition where the average torque is generated is $Z_l\omega_l \mp Z_h\omega_h = 0 Z_l\omega_l \mp Z_h\omega_h = 0$, while the condition where the maximum torque is obtained is $-Z_l\delta_l \pm Z_h\delta_h = \pi/2 - Z_l\delta_l \pm Z_h\delta_h = \pi/2$.

CONCLUSION

Equations (8) - (17) as presented by Aiso (Aiso et al. 2019) provides the relationship between the torques of an MG and the magnetomotive forces involved, the number of magnetic pole pairs, the speeds and the relative positions of the magnets and modulator pieces. We have extended this by showing that the magnetomotive forces are dependent on the magnetic material, the shape and size of the magnets, and the distance away from the magnetic surface. These relationships provide us with an insight into factors that influence torque in MGs. Such knowledge should be useful in optimising future MG designs such that torque density could be maximised.

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