

## A COMPARISON BETWEEN THE PERFORMANCES OF DOUBLE SAMPLING $\bar{X}$ AND VARIABLE SAMPLE SIZE $\bar{X}$ CHARTS

(Suatu Perbandingan antara Prestasi Carta-carta  $\bar{X}$  Pensampelan Berganda dan  $\bar{X}$  dengan Saiz Sampel yang Berubah-ubah)

W.L. TEOH<sup>1</sup>, K.L. LIOW<sup>1</sup>, MICHAEL B.C. KHOO<sup>2</sup>, W.C. YEONG<sup>2</sup> & S.Y. TEH<sup>3</sup>

### ABSTRACT

The double sampling (DS)  $\bar{X}$  and variable sample size (VSS)  $\bar{X}$  charts are very effective to detect small and moderate shifts in the process mean. Both charts are usually investigated under the assumption of known process parameters. However, the process parameters are commonly estimated from an in-control Phase-I dataset because they are usually unknown in practice. Therefore, both cases of known and estimated process parameters for the DS  $\bar{X}$  and VSS  $\bar{X}$  charts are considered in this paper. It is well known that the run length distribution of a control chart is highly skewed, especially when the process parameters are estimated and the process is in-control or slightly out-of-control. Interpretation based solely on a specific performance measure could be misleading. Thus, various performance measures need to be used to evaluate the properties of the control charts. Generally, the design of a control chart with estimated process parameters is proposed without comparing with other control charts. Accordingly, this paper focuses mainly on the comparison of the average run length (ARL), standard deviation of the run length (SDRL) and average sample size (ASS) between the DS  $\bar{X}$  and VSS  $\bar{X}$  charts with known and estimated process parameters. The ARL and SDRL results indicate that the DS  $\bar{X}$  chart outperforms the VSS  $\bar{X}$  chart for all ranges of shifts. However, the converse is true in terms of the ASS.

*Keywords:* double sampling (DS)  $\bar{X}$  chart; variable sample size (VSS)  $\bar{X}$  chart; average run length (ARL); standard deviation of the run length (SDRL); average sample size (ASS)

### ABSTRAK

Carta  $\bar{X}$  pensampelan berganda (DS) dan carta  $\bar{X}$  dengan saiz sampel yang berubah-ubah (VSS) adalah sangat berkesan untuk mengesan anjakan min proses yang kecil dan sederhana. Kedua-dua carta ini biasanya diasas dengan andaian bahawa parameter-parameter proses adalah diketahui. Walau bagaimanapun, parameter-parameter proses biasanya dianggarkan daripada set data Fasa-I yang berada dalam kawalan kerana parameter-parameter proses biasanya tidak diketahui dalam amalan. Oleh hal yang demikian, kedua-dua kes dengan parameter-parameter proses yang diketahui dan dianggarkan bagi carta-carta  $\bar{X}$  DS dan  $\bar{X}$  VSS dipertimbangkan dalam makalah ini. Adalah diketahui bahawa taburan panjang larian bagi suatu carta kawalan adalah sangat terpencong, terutamanya apabila parameter-parameter proses dianggarkan dan proses berada dalam kawalan atau hanya sedikit yang berada di luar kawalan. Tafsiran yang semata-mata berdasarkan satu ukuran prestasi yang spesifik adalah mengelirukan. Justeru, pelbagai ukuran prestasi perlu digunakan untuk menilai sifat-sifat carta kawalan. Secara umumnya, reka bentuk carta kawalan berdasarkan penganggaran parameter proses dicadangkan tanpa perbandingan dengan carta-carta kawalan yang lain. Makalah ini bertujuan untuk membandingkan panjang larian purata (ARL), sisihan piawai panjang larian (SDRL) dan saiz sampel purata (ASS) antara carta-carta  $\bar{X}$  DS dan  $\bar{X}$  VSS berdasarkan parameter-parameter proses yang diketahui dan dianggarkan. Keputusan ARL dan SDRL menunjukkan bahawa carta  $\bar{X}$  DS adalah lebih baik daripada carta  $\bar{X}$  VSS bagi semua julat anjakan. Namun demikian, hal

yang sebaliknya adalah benar jika dikaji dari segi ASS.

*Kata kunci:* carta  $\bar{X}$  pensampelan berganda (DS); carta  $\bar{X}$  dengan saiz sampel yang berubah-ubah (VSS); panjang larian purata (ARL); sisihan piawai panjang larian (SDRL); saiz sampel purata (ASS)

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<sup>1</sup>*Department of Physical and Mathematical Science  
Faculty of Science  
Universiti Tunku Abdul Rahman  
Kampar, Perak DR, MALAYSIA  
E-mail: teohwl@utar.edu.my\*, meganliow@yahoo.com*

<sup>2</sup>*School of Mathematical Sciences  
Universiti Sains Malaysia  
Penang, MALAYSIA  
E-mail: mkbc@usm.my, thomas\_yeong@yahoo.com*

<sup>3</sup>*School of Management  
Universiti Sains Malaysia  
Penang, MALAYSIA  
E-mail: tehsyin@usm.my*