

NOTES ON CONJUGACIES AND RENORMALISATIONS OF CIRCLE DIFFEOMORPHISMS WITH BREAKS

(Catatan mengenai Kekonjugatan dan Penormalan Semula
bagi Difeomorfisma Bulatan dengan Titik Putus-putus)

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ABSTRACT

Let f be an orientation-preserving circle diffeomorphism with irrational “rotation number” of bounded type and finite number of break points, that is, the derivative f' has discontinuities of first kind at these points. Suppose f' satisfies a certain Zygmund condition which be dependent on parameter $\gamma > 0$ on each continuity intervals. We prove that the Rauzy-Veech renormalisations of f are approximated by Mobius transformations in C^1 -norm if $\gamma \in (0,1]$ and in C^2 -norm if $\gamma \in (1,\infty)$. In particular, we show that if f has zero mean nonlinearity, renormalisation of such maps approximated by piecewise affine interval exchange maps. Further, we consider two circle homeomorphisms with the same irrational “rotation number” of bounded type and finite number of break points. We prove that if they are not break equivalent then the conjugating map between these two maps is singular.

Keywords: conjugacy; circle diffeomorphism; break point; renormalisation; interval exchange transformation; Mobius transformation; Rauzy-Veech induction

ABSTRAK

Andaikan f suatu difeomorfisma bulatan mengawet orientasi dengan “nombor putaran” tak nisbah jenis terbatas dan dengan bilangan titik putus yang terhingga, iaitu terbitan f' mempunyai ketakselajaran jenis pertama pada titik-titik tersebut. Andaikan juga f' memenuhi syarat Zygmund yang bersandar kepada parameter $\gamma > 0$ atas setiap selang keselajaran. Dibuktikan bahawa penormalan semula Rauzy-Veech f di hampirkan oleh penjelmaan Mobius dalam norma- C^1 jika $\gamma \in (0,1]$ dan di hampirkan dalam norma- C^2 jika $\gamma \in (1,\infty)$. Khususnya, ditunjukkan bahawa jika f mempunyai penormalan semula ketaklinearan min sifar, penghampiran berkenaan merupakan pemetaan pertukaran linear selang afin cebis demi cebis. Tambahan kami turut mempertimbangkan dua homeomorfisma bulatan yang mempunyai “nombor putaran” tak nisbah yang sama jenis terbatas dan bilangan titik putus yang terhingga. Kami buktikan jika kedua-dua pemetaan tersebut tidak setara terputus, maka pemetaan berkonjugat di antara mereka adalah singular.

Kata kunci: konjugasi; difeomorfisma bulatan; titik putus; penormalan semula; penjelmaan pertukaran selang; penjelmaan Mobius; aruhan Rauzy-Veech

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