

STABILITY ANALYSIS OF UNSTEADY THREE-DIMENSIONAL VISCOUS FLOW OVER A PERMEABLE STRETCHING/SHRINKING SURFACE

(Analisis Kestabilan bagi Aliran Likat Tiga Matra tak Mantap terhadap Permukaan Telap Meregang/Mengecut)

EZAD HAFIDZ HAFIDZUDDIN¹, ROSLINDA NAZAR¹, NORIHAN MD ARIFIN² & IOAN POP³

ABSTRACT

In this paper, a theoretical and numerical study on the unsteady three-dimensional boundary layer flow of a viscous fluid past a permeable stretching/shrinking sheet is considered. Similarity transformation is used to reduce the governing system of nonlinear partial differential equations into a system of ordinary differential equations. These equations are then solved numerically by using the “bvp4c” function in MATLAB. The effects of the governing parameters, namely the unsteadiness parameter, the stretching/shrinking parameter, the suction parameter and the ratio of the surface velocity gradients on the skin friction coefficients, as well as the velocity profiles are presented and discussed. Multiple solutions are found for a certain range of the governing parameters. Stability analysis has been performed to determine which solution is stable and physically realisable.

Keywords: boundary layer; dual solutions; stability analysis; stretching/shrinking sheet; three-dimensional unsteady flow

ABSTRAK

Dalam makalah ini, kajian secara teori dan berangka telah dilakukan bagi aliran lapisan sempadan tiga matra tak mantap bagi bendalir likat melepasi helaian telap meregang/mengecut. Penjelmaan keserupaan digunakan bagi menurunkan sistem menakluk persamaan pembezaan separa tak linear kepada sistem persamaan pembezaan biasa. Persamaan ini kemudiannya diselesaikan secara berangka dengan menggunakan fungsi “bvp4c” dalam perisian MATLAB. Kesan-kesan parameter menakluk, iaitu parameter ketidakmantapan, parameter meregang/mengecut, parameter sedutan dan nisbah kecerunan halaju permukaan terhadap pekali geseran kulit dan profil halaju dibentang dan dibincangkan. Penyelesaian berganda ditemui bagi julat tertentu parameter menakluk. Analisis kestabilan dilakukan bagi menentukan penyelesaian manakah yang stabil dan memberi makna secara fizikal.

Kata kunci: lapisan sempadan; penyelesaian dual; analisis kestabilan; helaian meregang/mengecut; aliran tak mantap tiga matra

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¹*School of Mathematical Sciences
Faculty of Science & Technology
Universiti Kebangsaan Malaysia
43600 UKM Bangi
Selangor DE, MALAYSIA
E-mail: ezadhafidz@gmail.com, rmn@ukm.edu.my**

²*Department of Mathematics
Universiti Putra Malaysia
43400 UPM Serdang,
Selangor DE, MALAYSIA
E-mail: norihanarifin@yahoo.com*

³*Department of Mathematics
Babeş-Bolyai University
R-400048 Cluj-Napoca
ROMANIA
E-mail: popm.ioan@yahoo.co.uk*