

## PERFORMANCE COMPARISON OF THE EXACT RUN-LENGTH DISTRIBUTION BETWEEN THE RUN SUM $\bar{X}$ AND EWMA $\bar{X}$ CHARTS

(Perbandingan Prestasi antara Carta Hasil Tambah Larian  $\bar{X}$  dan EWMA  $\bar{X}$   
Berdasarkan Taburan Panjang Larian Tepat)

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### ABSTRACT

The run sum (RS)  $\bar{X}$  and exponentially weighted moving average (EWMA)  $\bar{X}$  charts are very effective in detecting small and moderate process mean shifts. The design of the RS  $\bar{X}$  and EWMA  $\bar{X}$  charts based on the average run length (ARL) alone, can be misleading and confusing. This is due to the fact that the run-length distribution of a control chart is highly right-skewed when the process is in-control or slightly out-of-control; while that for the out-of-control cases, the run-length distribution is almost symmetric. On the other hand, the percentiles of the run-length distribution provide the probability of getting a signal by a certain number of samples. This will benefit practitioners as the percentiles of the run-length distribution give comprehensive information regarding the behaviour of a control chart. Accordingly, this paper provides a thorough study of the run-length properties (ARL, standard deviation of the run length and percentiles of the run-length distribution) for the RS  $\bar{X}$  and EWMA  $\bar{X}$  charts. Comparative studies show that the EWMA  $\bar{X}$  chart outperforms the RS  $\bar{X}$  charts for detecting small mean shifts when all the control charts are optimized with respect to a small shift size. However, the RS  $\bar{X}$  charts surpass the EWMA  $\bar{X}$  chart for all sizes of mean shifts when all the control charts are optimized with respect to a large shift size.

*Keywords:* average run length; EWMA  $\bar{X}$  chart; percentiles of the run-length distribution; run sum  $\bar{X}$  chart; standard deviation of the run length

### ABSTRAK

Carta  $\bar{X}$  hasil tambah larian (RS) dan  $\bar{X}$  purata bergerak berpemberat eksponen (EWMA) adalah sangat berkesan untuk mengesan anjakan min proses yang kecil dan sederhana. Reka bentuk carta  $\bar{X}$  RS dan  $\bar{X}$  EWMA berdasarkan panjang larian purata (ARL) sahaja adalah mengelirukan. Hal ini kerana taburan panjang larian untuk carta kawalan adalah sangat terpencong ke kanan apabila proses berada dalam kawalan atau hanya sedikit di luar kawalan; manakala bagi kes di luar kawalan, taburan panjang larian adalah hampir simetri. Sebaliknya, persentil taburan panjang larian memberikan kebarangkalian untuk mendapat isyarat dengan bilangan sampel yang tertentu. Hal ini dapat memanfaatkan para pengamal kerana persentil taburan panjang larian memberi maklumat yang komprehensif tentang kelakuan carta kawalan. Oleh hal yang demikian, makalah ini memberikan kajian yang menyeluruh tentang sifat-sifat panjang larian (ARL, sisihan piawai panjang larian dan persentil taburan panjang larian) untuk carta  $\bar{X}$  RS dan  $\bar{X}$  EWMA. Perbandingan dalam kajian ini menunjukkan bahawa carta  $\bar{X}$  EWMA adalah lebih baik daripada carta  $\bar{X}$  RS untuk mengesan anjakan min yang kecil apabila semua carta kawalan itu dioptimumkan berdasarkan saiz anjakan yang kecil. Walau bagaimanapun, carta  $\bar{X}$  RS adalah lebih baik daripada carta  $\bar{X}$  EWMA bagi semua saiz anjakan min apabila semua carta kawalan itu dioptimumkan berdasarkan suatu saiz anjakan yang besar.

*Kata kunci:* panjang larian purata; carta  $\bar{X}$  EWMA; persentil taburan panjang larian; carta  $\bar{X}$  hasil tambah larian; sisihan piawai panjang larian

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