*Journal of Quality Measurement and Analysis* *JQMA* 12 (1-2) 2016, 1-10
*Jurnal Pengukuran Kualiti dan Analisis*

# EFFECTS OF LUBRICATED SURFACE IN THE STAGNATION POINT FLOW OF A MICROPOLAR FLUID

(Kesan Permukaan Dilicinkan Terhadap Aliran Titik Genangan dalam Bendalir Mikrokutub)

K. MAHMOOD, M. SAJID, N. ALI, A. ARSHAD & M. A. RANA

*ABSTRACT*

In this investigation, we have considered a steady, two-dimensional flow of a micropolar fluid towards a stagnation point over a lubricated plate. A power law fluid is utilized for the purpose of lubrication. To derive the slip condition in the present flow situation, continuity of shear stress and velocity has been imposed at the fluid lubricant interface. The set of nonlinear coupled ordinary differential equations subject to boundary conditions is solved by a powerful numerical technique called the Keller-box method. Some important flow features have been analyzed and discussed under the influence of slip parameter $\lambda$, material parameter $K$ and ratio of micro-rotation to the skin friction parameter $n$. The main purpose of the present article is to analyze the reduction in the shear stress and couple stress effects in the presence of lubrication as compared to the viscous fluid that may be beneficial during polymeric processing.

*Keywords:* power law lubricant; stagnation point flow; interfacial condition; micropolar fluid; Keller-box method

*ABSTRAK*

Dalam kajian ini, aliran mantap dua matra bendalir mikrokutub terhadap plat dilicinkan dipertimbangkan. Bendalir hukum kuasa digunakan bagi tujuan pelinciran. Untuk memperoleh syarat gelincir bagi keadaan aliran ini, keselanjaran tegasan ricih dan halaju dikenakan pada antara-muka bendalir pelincir. Set persamaan pembezaan biasa tertakluk kepada syarat sempadan diselesaikan secara berangka menggunakan kaedah kotak-Keller. Beberapa ciri aliran yang penting dianalisis dan dibincangkan di bawah pengaruh parameter gelincir $\lambda$, parameter bahan $K$ dan nisbah mikro putaran terhadap parameter geseran kulit $n$. Tujuan utama artikel ini adalah untuk menganalisis kesan pengurangan tegasan ricih dan tegasan gandingan dengan kehadiran pelincir berbanding dengan bendalir likat yang boleh memberi manfaat semasa pemprosesan polimer.

*Kata kunci*: pelincir hukum kuasa; aliran titik genangan; keadaan antara muka; bendalir mikrokutub; kaedah kotak-Keller

## 1. Introduction

In recent decades many researchers have been investigating flows of non-Newtonian fluids due to their significant role in applied sciences, engineering and industrial applications. The governing equations representing these fluids are highly non-linear and are complicated to solve even by a numerical approach. Among these non-Newtonian fluids, an important fluid is the micropolar one. A micropolar fluid consists of small sized particles suspended in a viscous domain with intrinsic rotational micro motion. Examples of micropolar fluids are biological fluids like blood, polymeric additives (suspensions), geomorphological sediments, colloidal, slurries and hematological suspensions etc. It has been proved experimentally by Hoyt and Fabula (1964) that the fluids containing polymeric additives show a massive reduction of polymeric concentration and shear stress as explained by Eringen (1965). Eringen (1964; 1966) also proposed the theory of micropolar fluids with which the deformation inside such fluids can be very well explained. Ahmadi (1976) investigated the boundary layer flow of a micropolar fluid past a semi-infinite surface. Some extensive applications of microploar fluid has been provided by Ariman *et al*. (1973; 1974). Guram and Smith (1980) analyzed strong and weak interaction due to stagnation point flow of a micropolar fluid. A turbulent flow of micropolar fluid has been discussed by Peddieson (1972). Nazar *et al*. (2004) presented the flow of a micropolar fluid in the vicinity of a stagnation point over a stretching surface. Heat transfer in the mixed convection flow of a micropolar fluid towards a vertical wall with conduction has been investigated by Chang (2006). Lok *et al*. (2007) carried out analysis towards an oblique stagnation point for a micropolar fluid. A problem investigating the effects of micro-rotation towards a stagnation point over a vertical surface is discussed by Ishak *et al*. (2008). They found the numerical solution by implementing an implicit finite difference scheme.

In the practical life, there are many situations where the no-slip conditions do not hold. Examples include foams, polish, emulsions, suspensions etc. To handle such situations, one has to use the partial slip conditions. Wang (2003) discussed the effects of slip parameter on the stagnation point flow of a viscous fluid. Labropulu *et al*. (2008) examined slip flow due to a second grade fluid impinging orthogonally or obliquely on a surface. Blyth and Pozrikidis (2005) studied stagnation point flow by introducing slip condition at the interface of two viscous fluids. Axisymmetric stagnation-point flow near a lubricated stationary disc has been carried out by Santra *et al*. (2007). They used a power-law fluid as a lubricant. Sajid *et al*. (2012) reconsidered the problem of Santra *et al*. (2007) by applying generalized slip condition at fluid-lubricant interface introduced by Thompson and Troian (1997). Recently Mahmood *et el*. (2016) discussed various flows situations on the semi-infinite domain due to a lubricated surface.

Our goal in this communication, is to investigate steady, two-dimensional stagnation point flow of a micropolar fluid on a lubricated surface. A power law fluid is used for the lubrication purpose. The flow problem consists of the set of coupled nonlinear ordinary differential equations along with nonlinear coupled boundary conditions. The Keller-box method (Keller 1970; Keller & Cebeci 1972; Bradshaw *et al*. 1981; Ahmad & Nazar 2010) has been implemented to solve the considered flow problem numerically. Influence of pertinent parameters on the flow characteristics is discussed through graphs and tables. The validity of the present study is checked by comparing results in the limiting case with those exist in the literature.

## 2. Mathematical Formulation

Consider steady, two-dimensional stagnation-point flow of a micropolar fluid past a semi-infinite lubricated plate. A power-law fluid is used as lubricant. The lubricated plate is placed along *x*-axis and the fluid flows in the region $y > 0$ as shown in Fig. 1.

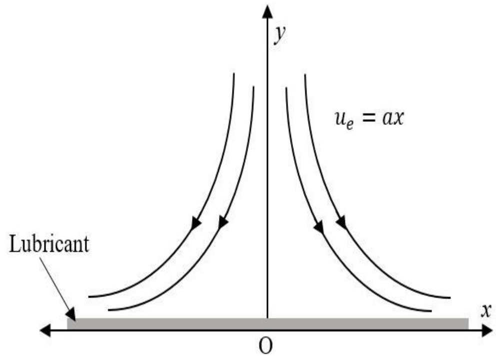



Figure 1: Physical model of flowing phenomenon

If $U$ and $V$ are respectively the horizontal and vertical velocity components of the power law fluid, then we have

$$Q = \int_0^{\delta(x)} U(x, y)\, dy, \tag{1}$$

where $Q$ is the flow rate of the lubricant and $\delta(x)$ is the variable thickness of the lubrication layer. Assuming $u$, $v$ the velocity components in $x$ and $y$ direction and $N$ as micro-rotation in $z$ direction, the boundary layer equations for the micropolar fluid are (Lok *et al*. 2007):

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{2}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -u_e\frac{du_e}{dx} + \left(\frac{\mu+k}{\rho}\right)\nabla^2 u + \frac{k}{\rho}\frac{\partial N}{\partial y}, \tag{3}$$

$$\rho j\left(u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y}\right) = \gamma\nabla^2 N - k\left(2N + \frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right), \tag{4}$$

where, $p$ is the pressure, $\rho$ is density, $k$ is vortex viscosity, $\gamma$ is spin gradient viscosity and $j$ is micro inertia density. The wall shear stress is given as

$$\tau_w = (\mu + k)\left(\frac{\partial u}{\partial y}\right) + kN\Big|_{y=0}. \tag{5}$$

The boundary conditions at the surface, interface and free stream are as follows:
At the surface the no-slip boundary conditions imply

$$U(x, 0) = 0\,,\;\; V(x, 0) = 0. \tag{6}$$

Assuming that the lubricant film is very thin, we have

$$V(x, y) = 0\,,\;\; \forall\, y \in [0, \delta(x)]. \tag{7}$$

The interfacial condition between the micropolar fluid and the lubricant can be obtained by imposing continuity of velocity and shear stress of both fluids. If $\mu_L$ is the apparent viscosity of the power law fluid, then the continuity of shear stress at $y = \delta(x)$ gives

$$(\mu + k)\left(\frac{\partial u}{\partial y}\right) + kN = \mu_L \frac{\partial U}{\partial y}, \tag{8}$$

where $\mu_L$ is defined as

$$\mu_L = k_1 \left(\frac{\partial U}{\partial y}\right)^{m-1}, \tag{9}$$

in which $m$ is power law index and $k_1$ is the consistency coefficient. We assume $U(x, y)$ in the following form

$$U(x, y) = \frac{\tilde{U}(x)\, y}{\delta(x)}. \tag{10}$$

Here $\tilde{U}(x)$ denotes the velocity of both fluids at the interface. Using Eq. (1), the thickness $\delta(x)$ of the lubricant can be expressed as

$$\delta(x) = \frac{2Q}{\tilde{U}(x)}. \tag{11}$$

Substituting Eqs. (9)-(11), Eq. (8) leads to the following slip boundary condition:

$$(\mu + k)\frac{\partial u}{\partial y} + kN = k_1 \left(\frac{1}{2Q}\right)^m u^{2m}. \tag{12}$$

It is to mention here that $\tilde{U} = u$ according to the continuity of horizontal velocity at $y = \delta(x)$. Moreover $N$ at the surface is

$$N = -n\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right), \tag{13}$$

in which the constant $n \in [0,1]$. The case $n = 0$ gives rise to strong concentration, implying $N = 0$ at the surface indicating that there is no micro rotation closer to the wall due to strong concentration (Guram & Smith 1980). The case $n = 1$ is utilized to describe turbulent boundary layer flow (Peddieson 1972) and the case $n = 0.5$ describes weak concentration indicating vanishing of anti-symmetrical part of stress tensor (Ahmadi 1976). The continuity of vertical velocity components of both fluids at $y = \delta(x)$ implies

$$v\left(x, \delta(x)\right) = V\left(x, \delta(x)\right), \tag{14}$$

Equations. (7) and (14) together give

$$v\left(x, \delta(x)\right) = 0. \tag{15}$$

Assuming the lubrication layer to be very thin, we can apply boundary conditions (12) and (15) at $y = 0$. The boundary conditions at free stream are

$$u_e = ax, \quad N = -\frac{b}{a}, \tag{16}$$

where $b/a$ represents the shear in the free stream. To express the set of equations into dimensionless form, we introduce

$$u = axf'(\eta), \quad v = -\sqrt{a\nu} f(\eta), \quad N = axq(\eta), \quad \eta = y\sqrt{\frac{a}{\nu}}. \tag{17}$$

In new variables governing equations subject to boundary conditions become

$$(1+K)f''' - f'^2 + ff'' + 1 + Kq' = 0, \tag{18}$$
$$(1+K/2)\, q'' + fq' - f'q - K(2q + f'') = 0, \tag{19}$$
$$f(0) = 0, \ \{1 + (1-n)K\} f''(0) = \lambda (f'(0))^{2m}, \quad f'(\infty) = 1, \tag{20}$$
$$q(0) = -nf''(0), \qquad q(\infty) = 0, \tag{21}$$

where $K = k/\mu$ is vortex viscosity parameter. The parameter $\lambda$ in Eq. (20) is given by

$$\lambda = \frac{k_1\sqrt{\nu}}{\mu} \frac{a^{2m}\, x^{2m-1}}{a^{3/2}\, (2Q)^m}. \tag{22}$$

From Eq. (22) we note that Eqs. (18)-(21) possess a similarity solution when $m = 1/2$. Furthermore $\lambda$ is the ratio between the viscous and lubrication length scales, thus

$$\lambda = \frac{\sqrt{\frac{\nu}{a}}}{\frac{\mu}{k_1}\sqrt{2Q}} = \frac{L_{visc}}{L_{lub}}. \tag{23}$$

The case when $L_{lub}$ is small, $\lambda$ becomes sufficient large and when $\lambda \to \infty$, the no-slip condition $f'(0) = 0$, is restored from Eq. (20). The case when $L_{lub}$ attains a huge value then $\lambda \to 0$ and the full slip boundary condition $f''(0) = 0$ is obtained. Thus the parameter $\lambda$ can be utilized to control the slip produced by the lubricant and is called slip parameter.

## 3. Numerical Results and Discussions

The values of $f'$, $f''$,$-q$ and $-q'$ are evaluated for various values of $n$, $\lambda$ and $K$ by solving eqs. (18) and (19) subject to boundary conditions (20) and (21) numerically by implementing Keller-Box method.

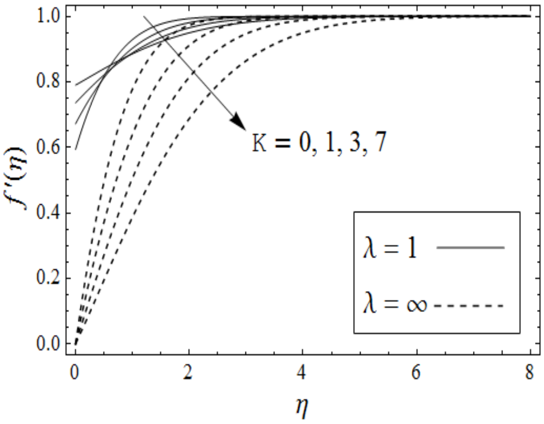
Figures 2 and 3 have been plotted to see the effects of vortex viscosity parameter $K$ and slip parameter $\lambda$ on $f'$ respectively while figs. 4 and 5 display the effects of parameters $K$ and $\lambda$ on $-q$. Table 1 shows the comparison of $f''(0)$ and $q'(0)$ for the no-slip case (Lok *et al*. 2007). Numerical values of $f''(0)$ and $q'(0)$ for various values of parameters have been shown in tables 2 and 3.

Figure 2 displays the effects of material parameter $K$ on horizontal velocity component $f'$ for the partial slip case (solid lines) and for the no-slip case (dashed lines). This figure shows that $f'$ decreases with an increase in the magnitude of $K$. This decrease is more prominent on rough surface ($\lambda \to \infty$) as compared to the lubricated surface (e.g. $\lambda = 1$) indicating that lubrication enhances the velocity of core fluid. The reason is that $K$ is the ratio between vortex and absolute viscosities. If $K$ is large, we can say that viscous effects are dominant near the wall. That is why $f'$ reduces near the surface. Moreover, this decrease is much significant when there is a strong concentration of micro-particles in the fluid i.e. when $n = 0$ (Fig. 2 (i)) as compared to the weak concentration of micro-particles i.e. when $n = 0.5$ (Fig. 2

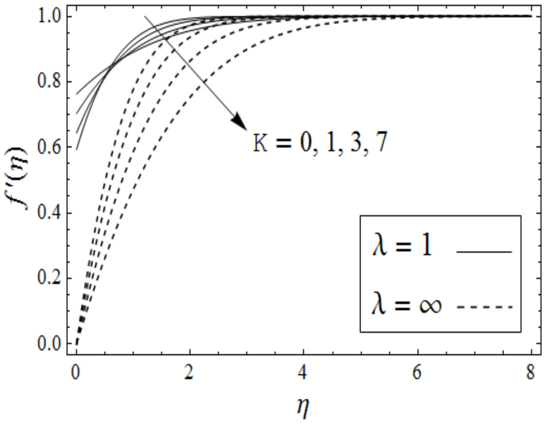
(ii)). Variation in $f'$ against slip parameter $\lambda$ is described in Fig. 3. It is evident that $f'$ decelerates by enhancing $\lambda$. It is also clear from this figure that $K$ appreciates the effects of $\lambda$. Variation in $f'$ is more effective for $n = 0$ (Fig. 3 (i)) as compared to when $n = 0.5$ (Fig. 3 (ii)).

Figure 4 indicates how micro rotation profile $-q$ behaves against $K$. According to Fig. 4(i), $-q$ shows an increasing trend against $K$ which is more significant on the rough surface ($\lambda \to \infty$). The same behaviour is observed in Fig. 4(ii) except some oscillations in the beginning. Fig. 5 is presented for the impact of $\lambda$ on $-q$. It is clear from this figure that $-q$ is an increasing function of $\lambda$ for all values of $K$. From the above discussion, it is clear that the velocity and micro rotation boundary layer thickness augment as $K$and $\lambda$ gain the values.

The numerical values of $f''(0)$ for the no-slip case are shown in Table 1. One can observe that these values show a good agreement with (Lok *et al*. 2007) showing the proof of correctness of the provided numerical solutions. Table 2 gives $f''(0)$ for different values of $\lambda$, $K$ and $n$. It is observed through this table that $f''(0)$ increases by increasing $\lambda$ for both $n = 0$ and $n = 0.5$. However, $f''(0)$ is a decreasing function of $K$. Table 3 elucidates that couple stress $q'(0)$ shows a decreasing trend for $n = 0$ and an increasing trend for $n = 0.5$ when $\lambda$ gains the magnitude. This table also suggests that by increasing $K$, $q'(0)$ first loses and then gains the values for $n = 0$. However, $q'(0)$ reduces by increasing $K$ for $n = 0.5$.

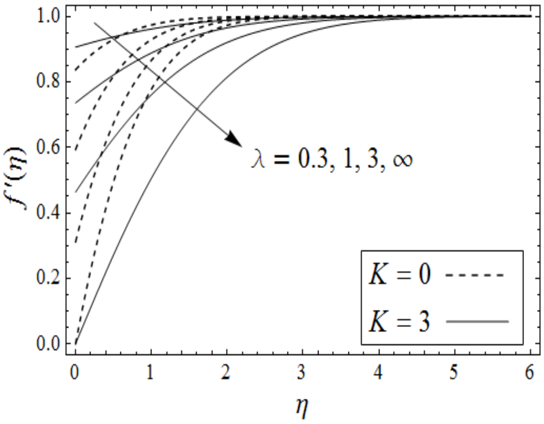



(i) $n = 0$

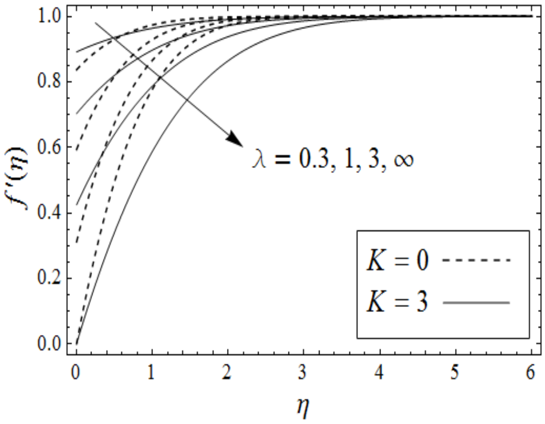



(ii) $n = 0.5$

Figure 2: Influence of $K$ on $f'$ for both no-slip (Lok *et al*. 2007) and partial slip cases when (i) $n = 0$ (ii) $n = 0.5$

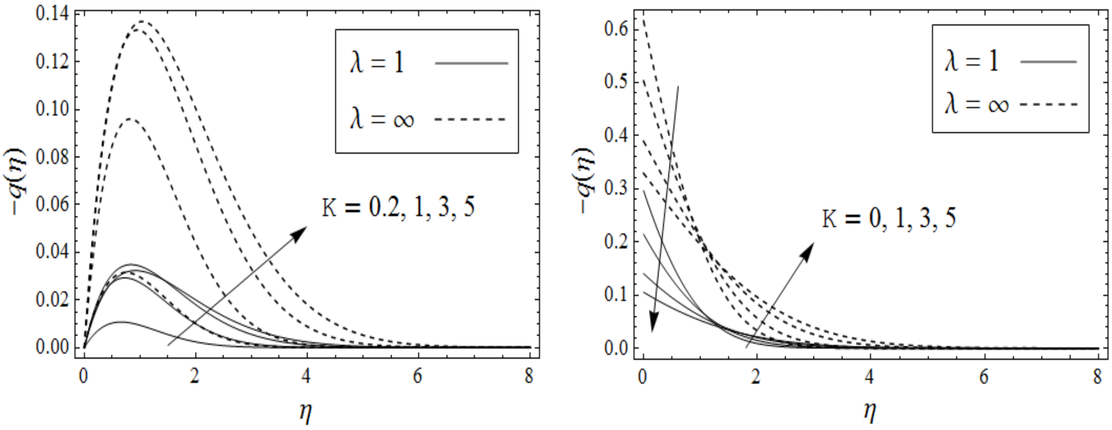



(i) $n = 0$



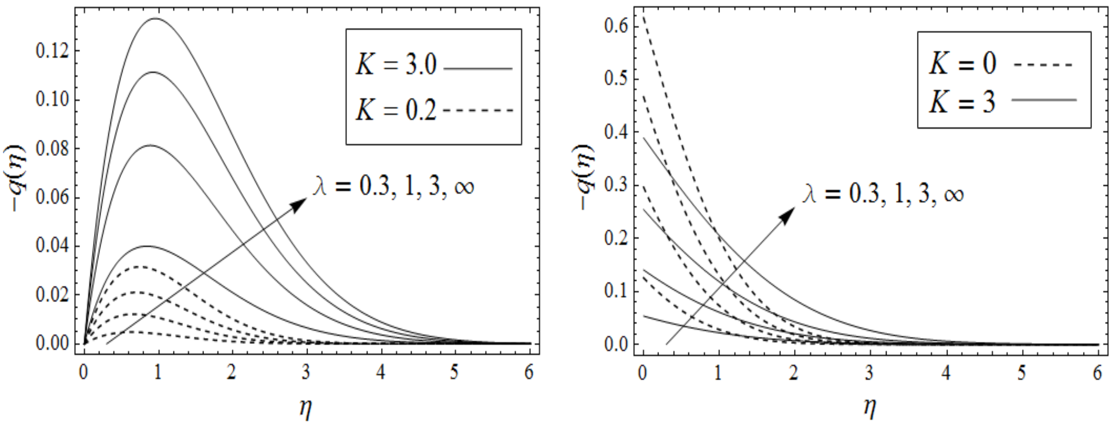

(ii) $n = 0.5$

Figure 3: Influence of $\lambda$ on $f'$ for two different values of $K$ when (i) $n = 0$ (ii) $n = 0.5$




(i) $n = 0$ (ii) $n = 0.5$

Figure 4: Influence of $K$ on $-q$ for both no-slip and partial slip cases when (i) $n = 0$ (ii) $n = 0.5$




(i) $n = 0$ (ii) $n = 0.5$

Figure 5: Influence of $\lambda$ on $-q$ for two different values of $K$ when (i) $n = 0$ (ii) $n = 0.5$

Table 1: Comparison of numerical values of $f''(0)$ with results by Lok *et al*. (2007) for different values of $K$ and $n$ when $\lambda = \infty$

| | $n = 0$ | | $n = 0.5$ | |
|---|---|---|---|---|
| | Present Results | Results by (Lok *et al*. 2007). | Present Results | Results by (Lok *et al*. 2007). |
| $K$ | | | | |
| 0 | 1.232598 | 1.232627 | 1.232598 | 1.232627 |
| 0.5 | 0.992640 | 0.992657 | 1.102467 | 1.102486 |
| 1.0 | 0.841074 | 0.841085 | 1.006409 | 1.006423 |
| 1.5 | 0.736826 | 0.736835 | 0.931753 | 0.931764 |
| 2.0 | 0.660473 | 0.660479 | 0.871574 | 0.871584 |
| 2.5 | 0.601901 | 0.601906 | 0.821728 | 0.821736 |
| 3.0 | 0.555369 | 0.555374 | 0.779559 | 0.779566 |

Table 2: Variation in $f''(0)$ against $\lambda$ and $K$ when $m = 0.5$

| | $K = 0$ | $K = 2$ | | $K = 5$ | |
|---|---|---|---|---|---|
| $\lambda$ | $-$ | $n = 0$ | $n = 0.5$ | $n = 0$ | $n = 0.5$ |
| 0.05 | 0.0484719 | 0.0163541 | 0.0244558 | 0.0082160 | 0.0140496 |
| 0.5 | 0.3758870 | 0.1391757 | 0.2031912 | 0.0726727 | 0.1218241 |
| 1.0 | 0.5934642 | 0.2369180 | 0.3390219 | 0.1281173 | 0.2110098 |
| 5.0 | 1.0425914 | 0.5090383 | 0.6882787 | 0.3110572 | 0.4834611 |
| 10 | 1.1342889 | 0.5803336 | 0.7744962 | 0.3680737 | 0.5633605 |
| 50 | 1.2126286 | 0.6441740 | 0.8516245 | 0.4218026 | 0.6389223 |
| 100 | 1.2226039 | 0.6523301 | 0.8615842 | 0.4287217 | 0.6488615 |
| 500 | 1.2305978 | 0.6588462 | 0.8695746 | 0.4342355 | 0.6568476 |
| 5000 | 1.2323976 | 0.6603101 | 0.8713743 | 0.4354716 | 0.6586472 |
| $\infty$ | 1.2325976 | 0.6604727 | 0.8715743 | 0.4356088 | 0.6588472 |

Table 3: Variation in couple stress $q'(0)$ against $\lambda$, $K$ and $n$ when $m = 0.5$

| | $K = 0.2$ | | $K = 2$ | | $K = 5$ | |
|---|---|---|---|---|---|---|
| $\lambda$ | $n = 0$ | $n = 0.5$ | $n = 0$ | $n = 0.5$ | $n = 0$ | $n = 0.5$ |
| 0.05 | $-0.0029175$ | 0.0261352 | $-0.0073188$ | 0.0107658 | $-0.0057509$ | 0.0046827 |
| 0.5 | $-0.0239872$ | 0.1913592 | $-0.0635261$ | 0.0848540 | $-0.0514761$ | 0.0389696 |
| 1.0 | $-0.0396762$ | 0.2878422 | $-0.1100672$ | 0.1350654 | $-0.0917711$ | 0.0649385 |
| 5.0 | $-0.0792375$ | 0.4331651 | $-0.2522400$ | 0.2310534 | $-0.2336033$ | 0.1264966 |
| 10 | $-0.0892695$ | 0.4481728 | $-0.2944711$ | 0.2440043 | $-0.2821674$ | 0.1373041 |
| 50 | $-0.0987187$ | 0.4542601 | $-0.3355861$ | 0.2497130 | $-0.3315334$ | 0.1425725 |
| 100 | $-0.0999937$ | 0.4544791 | $-0.3411567$ | 0.2499289 | $-0.3382738$ | 0.1427846 |
| 500 | $-0.1010287$ | 0.4545508 | $-0.3456726$ | 0.2500001 | $-0.3437297$ | 0.1428553 |
| 5000 | $-0.1012634$ | 0.4545538 | $-0.3466956$ | 0.2500031 | $-0.3449640$ | 0.1428583 |
| $\infty$ | $-0.1012896$ | 0.4545538 | $-0.3468094$ | 0.2500032 | $-0.3451013$ | 0.1428583 |

## 4. Concluding Remarks

In this paper stagnation point flow of a micropolar fluid over a plate is investigated. A power-law fluid is introduced on the surface for the lubrication purpose. The flow problem is numerically solved using Keller-box method for $m = 1/2$ to obtain similar solution. The impact of slip parameter $\lambda$ and material parameter $K$ on the flow characteristics has been investigated. Some findings are listed below.

(i) The velocity of micropolar fluid $f'$ reduces by increasing $K$ and $\lambda$. This reduction is more prominent when $K > 0$ and $\lambda = \infty$ for both $n = 0$ and $n = 0.5$.

(ii) $-q$ increases by increasing K and $\lambda$. This increase is more significant for $K > 0$ and $\lambda = \infty$.

(iii) The velocity and micro rotation boundary layer thickness augment as $K$and $\lambda$ increase.

(iv) Numerical values of $f''(0)$ increase by increasing $\lambda$ for both $n = 0$ and $n = 0.5$. However, $f''(0)$ is a decreasing function of $K$.

(v) The couple stress $q'(0)$ shows a decreasing trend for $n = 0$ and an increasing trend for $n = 05$ when $\lambda$ gains the magnitude. Moreover, by increasing $K$, $q'(0)$ first loses and then gains the values for $n = 0$. However, $q'(0)$ reduces by increasing $K$ for $n = 0.5$.

## References


Ahmad K & Nazar R. 2010. Unsteady magnetohydrodynamic mixed convection stagnation point flow of a viscoelastic fluid on a vertical surface. *Journal of Quality Measurement and Analysis* **6**(2): 105-117.

Ahmadi G. 1976. Self-similar solution of incompressible micro polar boundary-layer flow over a semi-infinite plate. *Int. J. Eng. Sci.* **14**: 639-646.

Ariman T., Turk M.A. & Sylvester N.D. 1973. Microcontinuum fluid mechanics-A review. *Int. J. Eng. Sci.* **11**: 905-930.

Ariman T., Turk M.A. & Sylvester N.D. 1974. Application of microcontinuum fluid mechanics. *Int. J. Eng. Sci*. **12**: 273-293.

Blyth M.G. & Pozrikidis C. 2005. Stagnation-point flow against a liquid film on a plane wall. *Acta Mech.* **180**: 203-219.

Bradshaw V., Cebeci T. & Whitelaw I.H. 1981. *Engineering Calculation Methods for Turbulent Flows*. London: Academic.

Chang C.L. 2006. Numerical simulation of micropolar fluid flow along a flat plate with wall conduction and boundary effects. *J. Phys. D: App. Phys*. **39**: 1132-1140.

Eringen A.C. 1964. Simple micro fluids. *Int. J. Eng. Sci.* **2**: 205-217.

Eringen A.C. 1965. Theory of micro polar continua. *Proceedings, 9th Midwestern Conference, University of Wisconsin*, Madison, Wis., USA.

Eringen A. C. 1966. Theory of micro polar fluids. *J. Math. Mech*. **16**: 1-18.

Guram G.S. & Smith A.C. 1980. Stagnation flows of micropolar fluids with strong and weak interactions. *Comp. Math. App*. **6**: 213-233.

Hoyt J.W., Fabula A.G. 1964. The effect of additives on fluid friction. *U. S., Naval Ordinance Test Station Report*, Inyokern, Cal., USA.

Ishak A., Nazar R. & Pop I. 2008. MHD flow of a micro polar fluid towards a stagnation point on a vertical surface. *Comp. Math. App.* **56**: 3188-3194.

Keller H.B. & Cebeci T. 1972. Accurate numerical methods for boundary layer flows II: two dimensional turbulent flows. *AIAA J.* **10**: 1193-1199.

Keller H.B. 1970. A new difference scheme for parabolic problems, in *Numerical Solution of Partial Differential Equations*, Bramble J. (ed.), Vol. II. New York: Academic.

Labropulu F. & Li D. 2008. Stagnation-point flow of a second grade fluid with slip. *Int. J. Non-Linear Mech.* **43**: 941-947.

Lok Y.Y., Pop I. & Chamkha A.J. 2007. Non-orthogonal stagnation point flow of a micropolar fluid. *Int. J. Eng. Sci.* **45**: 173-184.

Mahmood K., Sajid M. & Ali N. 2016. Non-orthogonal stagnation-point flow of a second-grade fluid past a lubricated surface. *Zeitschrift für Naturforschung A* **71**: 273-280.

Mahmood K., Sajid M., Ali N. & Javed T. 2016. Heat transfer analysis in the time-dependent slip flow over a lubricated surface. *Eng. Sci. & Tech., An Int. J.* **19**: 1949-1957.

Mahmood K., Sajid M., Ali N. & Javed T. 2016. Heat transfer analysis in the time-dependent axisymmetric stagnation point flow over a lubricated surface. *Th. Sci.* DOI: 10.2298/TSCI160203257M.

Mahmood K., Sajid M., Ali N. & Javed T. 2016. Slip flow of a second grade fluid past a lubricated rotating disc. *Int. J. Phys. Sci.* **11**: 96-103.

Nazar R., Amin N., Filip D & Pop I. 2004. Stagnation point flow of a micropolar fluid towards a stretching sheet. *Int. J. Non-Linear Mech*. **39**: 1227-1235.

Peddieson J. 1972. An application of micropolar fluid model to the calculation of turbulent shear flow. *Int. J. Eng. Sci*. **10**: 23-32.

Sajid M., Mahmood K. & Abbas Z. 2012. Axisymmetric stagnation-point flow with a general slip boundary condition over a lubricated surface. *Chin. Phys. Letters* **29**: 024702.

Santra B., Dandapat B.S. & Andersson H.I. 2007. Axisymmetric stagnation-point flow over a lubricated surface. *Acta Mech.* **194**: 1-10.

Thompson P.A. & Troian S.M. 1997. A general boundary condition for liquid flow at solid surfaces. *Nature* **389**: 360-362.

Wang C.Y. 2003. Stagnation flows with slip: Exact solution of the Navier-Stokes equations. *Z. Angew Math. Phys.* **54**: 184-189.

*Department of Mathematics and Statistics*
*International Islamic University*
*Islamabad 44000*
*PAKISTAN*
*E-mail: khalidmeh2012@gmail.com*, muhammad.sajid@iiu.edu.pk, nasir.ali@iiu.edu.pk*

*Department of Mathematics*
*Riphah International University*
*Islamabad 44000*
*PAKISTAN*
*E-mail: ambreen.arshad@riphah.edu.pk, muhammad.afzal@riphah.edu.pk*

*Corresponding author