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UNIVALENCE CRITERIA OF CERTAIN INTEGRAL OPERATOR

(Kriterium Univalen bagi Pengoperasi Kamiran Tertentu)

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ABSTRACT

In this paper, univalence criteria of certain integral operator defined by a generalised derivative operator is obtained.

Keywords: analytic function; integral operator; derivative operator

ABSTRAK

Dalam makalah ini, kriterium univalen bagi pengoperasi kamiran tertentu yang ditakrifkan oleh pengoperasi terbitan teritlak diperoleh.

Kata kunci: fungsi analisis; pengoperasi kamiran; pengoperasi terbitan

1. Introduction and preliminaries

Let S be the class of normalised analytic and univalent in a unit disc $U = \{z: |z| < 1\}$. Also let H be the class of functions analytic in $U = \{z: |z| < 1\}$ and let $H[a, n]$ be the subclasses of H consisting of functions of the form $f(z) = a_n z^n + a_{n+1} z^{n+1} + a_{n+2} z^{n+2} + \cdots$. Let A be the subclasses of H consisting of functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k. \tag{1}$$

Let $A(n)$ denote the class of functions $f(z)$ of the form

$$f(z) = z - \sum_{k=n}^{\infty} a_{k+1} z^{k+1}, \ a_{k+1} \geq 0, n \in \{1,2,3, \ldots\} \tag{2}$$

which are analytic in the open unit disc U.

For the function $f \in A$ given by (1), we define a new generalised derivative operator $D^{\alpha,n}(m, l, q, \lambda) f(z): A \to A$ as follows:

$$D^{\alpha,n}(m, l, q, \lambda) f(z) = z + \sum_{k=2}^{\infty} k^{\alpha} \left(\frac{q+\lambda(k-1)+l}{q+l}\right)^m c(n, k) a_k z^k, \tag{3}$$

where $n, \alpha \in N_0 = \{0,1,2, \ldots\}, m \in Z, \ \lambda, l, q \geq 0, l + q \neq 0$

$$c(n,k) = \binom{k+n-1}{n} = \frac{\prod_{j=1}^{k-1}(j+n)}{(k-1)!}, k \geq 2.$$

Here $D^{\alpha,n}(m,l,q,\lambda)f(z)$ can also be written in terms of convolution as follows:

If $m = 0,1,2,...$ then

$$D^{\alpha,n}(m,l,q,\lambda)f(z) = \underbrace{\phi(z) * \cdots * \phi(z)}_{(m)-times} * \left[\frac{z}{(1-z)^{n+1}}\right] * \sum_{k=1}^{\infty} k^{\alpha} z^k * f(z)$$

$$= R^n D^{\alpha,n}(m,l,q,\alpha)f(z),$$

where $\phi(z) := \left(\frac{l+q-\lambda}{l+q}\right)\frac{z}{1-z} + \left(\frac{\lambda}{l+q}\right)\frac{z}{(1-z)^2}$ for any $z \in U$, $R^n = z + \sum_{k=2}^{\infty} c(n,k)z^k$ is the Ruscheweyh derivative operator and $D^{\alpha,n}(m,l,q,\lambda)f(z)$ is given by (3).

If $m = -1,-2,...,$ then

$$D^{\alpha,n}(m,l,q,\lambda)f(z) = \underbrace{\phi(z) * \cdots * \phi(z)}_{(-m)-times} * \left[\frac{z}{(1-z)^{n+1}}\right] * \sum_{k=1}^{\infty} k^{\alpha} z^k * f(z)$$

$$= R^n * D^{\alpha,n}(m,l,q,\lambda)f(z).$$

Note that $D^{0,0}(0,l,q,\lambda)f(z)$=$D^{0,0}(1,0,0,0)f(z) = f(z)$ and $D^{0,0}(1,l,q,\lambda)f(z) = zf'(z)$.

By specialising the parameters of $D^{\alpha,n}(m,l,q,\lambda)f(z)$ in (3), we get the following derivative and integral operators:

a) $D^{0,n}(0,l,q,\lambda)f(z) \equiv D^{0,n}(1,0,0,0)f(z); (n \in \mathbb{N}_0) \equiv R^n = z + \sum_{k=2}^{\infty} c(n,k)z^k$, introduced by Ruscheweyh (1975).

b) $D^{\alpha,0}(0,0,q,\lambda)f(z) \equiv D_1^{0,0}(n,0,0,1)f(z); (n \in \mathbb{N}_0) \equiv D^n = z + \sum_{k=2}^{\infty} k^n z^k$ introduced by Salagean (1983).

c) $D^{0,0}(n,0,0,\lambda); (n \in \mathbb{N}_0) \equiv D_\lambda^n = z + \sum_{k=2}^{\infty}\big(1+\lambda(k-1)\big)^n a_k$ (Al-Oboudi 2004).

d) $D^{0,n}(1,0,0,\lambda); (n \in \mathbb{N}_0) \equiv R_\lambda^n = z + \sum_{k=2}^{\infty}\big(1+\lambda(k-1)\big)c(n,k)a_k z^k$ (Al-Shaqsi & Darus 2009).

e) $D^{0,\beta}(m,0,0,\lambda); (m \in \mathbb{N}_0) \equiv D_{\lambda,\beta}^n = z + \sum_{k=2}^{\infty}\big(1+\lambda(k-1)\big)^m c(\beta,k)a_k z^k$ (Darus & Al-Shaqsi 2008).

f) The derivative operator introduced by Catas (2009):
$D^{0,\beta}(m,l,1,\lambda); (m \in \mathbb{N}_0) \equiv D^m(\lambda,\beta,l) = z + \sum_{k=2}^{\infty}\left(\frac{1+\lambda(k-1)+l}{1+l}\right)^m c(\beta,k)a_k z^k$.

g) $D^{0,0}(n,0,1,1) \equiv I^m = z + \sum_{k=2}^{\infty}\left(\frac{k+1}{2}\right)^m a_k z^k$, introduced by Uralegaddi and Somanatha (1992).

h) $D^{0,0}(n,0,\lambda,1) \equiv I_\lambda^m = z + \sum_{k=2}^{\infty}\left(\frac{k+\lambda}{1+\lambda}\right)^m a_k z^k$, studied by Flett (1972).

i) $D^{0,0}(-n,0,\lambda,1) \equiv I_n^\lambda = z + \sum_{k=2}^{\infty} k\left(\frac{1+\lambda}{k+\lambda}\right)^n a_k z^k$, introduced by Cho and Kim (1983).

j) $D^{\alpha,n}(m,1,q,\lambda)f(z) \equiv D^{\alpha,n}(m,q,\lambda)f(z) = z + \sum_{k=2}^{\infty} k^{\alpha}\left(1+\frac{k-1}{q+l}\lambda\right)^m c(n,k)a_k z^k$, introduced by Mustafa and Darus (2011).

Here, we introduce a new general integral operator by using generalised derivative operator given by (3).

For $f_i \in A$ and $i = (1,2,3,\dots,s)$, $n \in \mathbb{N} \cup \{0\}$ and $\gamma_1, \gamma_2, \gamma_3, \dots, \gamma_s, \beta$ are complex numbers, we define a family of integral operator by

$$F^{\alpha,n,\beta}(m,l,q,\lambda,\gamma_i;z) = \left[\beta \int_0^z t^{\beta-1} \prod_{i=1}^{s} \left(\frac{D^{\alpha,n}(m,l,q,\lambda) f_i(t)}{t}\right)^{\frac{1}{\gamma_i}} dt\right]^{\frac{1}{\beta}}, \tag{4}$$

where $\alpha \in \mathbb{N}_0, m \in Z,\ \lambda, l, q \geq 0, l+q \neq 0$ and $D^{\alpha,n}(m,l,q,\lambda)$ defined by (3), which generalises many integral operators. In fact, if we choose suitable values of parameters, we get the following interesting operators. For example

a) For $m = l = \alpha = n = 0,\ \gamma_i = \frac{1}{\alpha-1}, \beta = n(\alpha-1)+1,$ reduces to $F_{n,\alpha}(z)$ of Breaz et al (2009)

b) Let $m = l = \alpha = n = 0,\ \gamma_i = \frac{1}{\alpha_i}, \beta = 1$. Then it reduces to $F_n(z)$ of Breaz and Breaz (2002).

In this paper, we discuss the univalence properties of the new general integral operator.

2. Preliminary Result

To discuss our problems, we need the following results, called the Schwarz lemmas due to Pascu (1987).

Lemma 1 (*Pascu* 1987). *Let $f \in A$ and β be a complex number with $R\{\beta\} > 0$.*

If f satisfies

$$\frac{1-|z|^{2R\{\beta\}}}{R\{\beta\}} \left|\frac{z\grave{f}(z)}{\acute{f}(z)}\right| \leq 1,$$

then for all $z \in U$, the integral operator

$$F_\beta(z) = \left[\beta \int_0^z t^{\beta-1} \acute{f}(t) dt\right]^{\frac{1}{\beta}}$$

is in the class S.

Pescar (1996) obtained univalence criterion of univalence and is given in the folowing lemma.

Lemma 2 (*Pescar* 1996). *Let $f \in A$, and $\beta, c \in C$where $Re\{\beta\} > 0$ and $|c| \leq 1,\ c \neq -1$. If*

$$\left|c|z|^{2\beta} + \left(1-|z|^{2\beta}\right)\frac{z\grave{f}(z)}{\acute{f}(z)}\right| \leq 1,$$

then for all $z \in U$, the function

$$F_\beta(z) = \left[\beta \int_0^z t^{\beta-1} \acute{f}(t)dt\right]^{\frac{1}{\beta}}$$

is analytic and univalent in U.

On the other hand, the following result due to Ozaki and Nunokawa (1972) is useful in studying the univalence of integral operators for certain subclass of S.

Theorem 1 (*Ozaki and Nunokawa* 1972). *Let* $f \in A$ *satisfies the following inequality*

$$\left|\frac{z^2 f\acute{(}z)}{f^2(z)} - 1\right| \leq 1, \text{ for all } z \in U.$$

Then the function f *is univalent in* U.

Next theorem provides the univalence conditions for the functions $F^{\alpha,n,\beta}(m,l,q,\lambda,\gamma_i;z)$ given by (4) as follows:

Theorem 2. *Let the functions* $D^{\alpha,n}(m,l,q,\lambda)f_i(t) \in S(p)$, *for* $i = (1,2,3,\ldots,s)$, *satisfy the conditions* $R\left\{\frac{zf''(z)}{f'(z)} + 1\right\} > 0$, $z \in U$ *and* $|D^{\alpha,n}(m,l,q,\lambda)f_i(t)| \leq M|z|, M \geq 1$. *If* β, γ_i *and* c *are complex numbers such that* $R\{\beta\} > \sum_{i=1}^{s} \frac{(1-p)M+1}{|\gamma_i|}$ *and*

$$|c| \leq 1 - \frac{1}{R\{\beta\}} \sum_{i=1}^{s} \frac{(1+p)M+1}{|\gamma_i|},$$

then the functions $F^{\alpha,n,\beta}(m,l,q,\lambda,\gamma_i;z)$ *is univalent.*

Proof. Since $f_i \in A(i \in 1,\ldots,s)$ by (3), we get

$$\frac{D^{\alpha,n}(m,l,q,\alpha)f(z)}{z} = 1 + \sum_{k=2}^{\infty} k^\alpha \left(1 + \frac{K-1}{q+l}\lambda\right)^m c(n,k)a_k z^{k-1}.$$

Let F be defined by

$$F(z) = \int_0^z \prod_{i=1}^{s} \left(\frac{D^{\alpha,n}(m,l,q,\alpha)f_i(z)}{t}\right)^{\frac{1}{\gamma_i}} dt.$$

Then

$$F'(z) = \left(\frac{D^{\alpha,n}(m,l,q,\lambda)f_1(z)}{z}\right)^{\frac{1}{\gamma_i}} \ldots \left(\frac{D^{\alpha,n}(m,l,q,\lambda)f_s(z)}{z}\right)^{\frac{1}{\gamma_i}},$$

and we have $F(0) = F'(0) - 1$. Also, a simple computation yields

$$lnF'(z) = \frac{1}{\gamma_i} ln\left(\frac{D^{\alpha,n}(m,l,q,\lambda)f_1(z)}{z}\right) + \cdots + \frac{1}{\gamma_s} ln\left(\frac{D^{\alpha,n}(m,l,q,\lambda)f_s(z)}{z}\right).$$

By differentiating the above equality, we get

$$\frac{zF''(z)}{F'(z)} = \sum_{i=1}^{s}\frac{1}{\gamma_i}\left(\frac{z(D^{\alpha,n}(m,l,q,\lambda)f_i(z))'}{D^{\alpha,n}(m,l,q,\lambda)f_i(z)} - 1\right), \tag{5}$$

and from (5), we have

$$\left|\frac{zF''(z)}{F'(z)}\right| \leq \sum_{i=1}^{s}\frac{1}{|\gamma_I|}\left(\left|\frac{z(D^{\alpha,n}(m,l,q,\lambda)f_i(z))'}{D^{\alpha,n}(m,l,q,\lambda)f_i(z)} + 1\right|\right) \tag{6}$$

$$= \sum_{i=1}^{s}\frac{1}{|\gamma_I|}\left(\left|\frac{Z^2(D^{\alpha,n}(m,l,q,\lambda)f_i(z))'}{\left(D^{\alpha,n}(m,l,q,\lambda)f_i(z)\right)^2}\right|\frac{|D^{\alpha,n}(m,l,q,\lambda)f_i(z)|}{|z|} + 1\right).$$

We have $z \in U$ and $|D^{\alpha,n}(m,l,q,\lambda)f_i(z)| \leq M|z|$, $z \in U$, $i = (1,2,3,\dots,s)$, then by Lemma 2, we obtain:

$$|D^{\alpha,n}(m,l,q,\lambda)f_i(z)| \leq M|z|.$$

We apply this result in inequality (6) to obtain:

$$\left|\frac{zF''(z)}{F'(z)}\right| \leq \sum_{i=1}^{s}\frac{1}{|\gamma_I|}\left(\left|\frac{z^2(D^{\alpha,n}(m,l,q,\lambda)f_i(z))'}{\left(D^{\alpha,n}(m,l,q,\lambda)f_i(z)\right)^2}\right| M + 1\right)$$

$$\leq \sum_{i=1}^{s}\frac{1}{|\gamma_I|}\left(\left|\frac{z^2(D^{\alpha,n}(m,l,q,\lambda)f_i(z))'}{\left(D^{\alpha,n}(m,l,q,\lambda)f_i(z)\right)^2} - 1\right| M + M + 1\right)$$

$$=\sum_{i=1}^{s}\frac{1}{|\gamma_I|}(pM|z|^2 + M + 1)=\sum_{i=1}^{s}\frac{1}{|\gamma_I|}\left((p+1)M + 1\right).$$

We have

$$\left|c|z|^{2\beta} + \left(1-|z|^{2\beta}\right)\frac{1}{R\{\beta\}}\left(\frac{zF''(z)}{F'(z)}\right)\right| = \left|c|z|^{2\beta} + \left(1-|z|^{2\beta}\right)\frac{1}{R\{\beta\}}\left(\frac{z(D^{\alpha,n}(m,l,q,\lambda)f_i(z))'}{D^{\alpha,n}(m,l,q,\lambda)f_i(z)} - 1\right)\right|$$

$$\leq |c| + \frac{1}{|\beta|}\sum_{i=1}^{s}\frac{1}{|\gamma_I|}\left(\left|\left|\frac{z^2(D^{\alpha,n}(m,l,q,\lambda)f_i(z))'}{\left(D^{\alpha,n}(m,l,q,\lambda)f_i(z)\right)^2}\right|\right|\frac{|D^{\alpha,n}(m,l,q,\lambda)f_i(z)|}{|z|} + 1\right)$$

and obtain

$$\left|c|z|^{2\beta} + \left(1-|z|^{2\beta}\right)\frac{zF''(z)}{R\{\beta\}F'(z)}\right| \leq |c| + \frac{1}{R\{\beta\}}\sum_{i=1}^{s}\frac{1}{|\gamma_I|}((p+1)M+1).$$

So, from there, we have

$$\left|c|z|^{2\beta} + \left(1-|z|^{2\beta}\right)\frac{zF''(z)}{R\{\beta\}F'(z)}\right| \leq 1.$$

Hence the theorem is proved.

□

Corollary 1. *Let the functions* $D^{\alpha,n}(m,l,q,\lambda)f_i(t) \in S(p)$, *for* $i = (1,2,3,\dots,s)$, *satisfy the condition* $Re\left\{\frac{zf''(z)}{f'(z)} + 1\right\} > 0$, $z \in U$ *and* $|D^{\alpha,n}(m,l,q,\lambda)f_i(t)| \leq M|z|, M \geq 1$. *If* β, γ_i *and* c *are complex numbers such that* $R\{\beta\} > \frac{n(1-p)M+1}{|\gamma|}$ *and* $|c| \leq 1 - \frac{(1+p)M+1}{R\{\beta\}|\gamma|}$, *then the functions* $F^{\alpha,n,\beta}(m,l,q,\lambda,\gamma_i;z)$ *is univalent.*

Proof. Take $\gamma_{1=}\gamma_{2=} \dots = \gamma_{s=}\gamma$ in Theorem 2. □

Corollary 2. *Let the functions* $D^{\alpha,n}(m,l,q,\lambda)f_i(t) \in S(p)$, *for* $i = (1,2,\dots,s)$, *satisfy the condition* (7) *and* $|D^{\alpha,n}(m,l,q,\lambda)f_i(t)| \leq 1$. *If* β, γ_i *and c are complex numbers such that* $Re\{\beta\} > \sum_{i=1}^{s} \frac{p+2}{|\gamma_i|}$ *and* $|c| \leq 1 - \frac{2(p+1)}{R\{\beta\}|\gamma|}$,
then the functions $F^{\alpha,n,\beta}(m,l,q,\lambda,\gamma_i;z)$ *is univalent.*

Proof. Put $M = 1$ in Theorem 2. □

We will need the following definition for further results.

Definition 1 (Nunokawa and Obradovi`c 1989). For some real p with $0 < p \leq 2$, we define a subclass $S(p)$ of A consisting of all functions f which satisfy

$$\left|\left(\frac{z}{f(z)}\right)''\right| \leq p, \text{ for all}(z \in U).$$

Singh (2000) has shown that if $f \in S(p)$, then f satisfies

$$\left|\frac{z^2 f''(z)}{f^2(z)}\right| \leq p|z|^2, \text{ for all } (z \in U). \tag{7}$$

Theorem 3. *Let the functions* $D^{\alpha,n}(m,l,q,\lambda)f_i(t) \in S(p)$, *for* $i = (1,2,\dots,s)$, *satisfy the condition* $\left|\frac{z^2(D^{\alpha,n}(m,l,q,\lambda)f_i(z))'}{\left(D^{\alpha,n}(m,l,q,\lambda)f_i(z)\right)^2} - 1\right| \leq 1$. *If* β, γ_i *and c are complex numbers such that* $Re\{\beta\} > \sum_{i=1}^{s} \frac{2M+1}{|\gamma_i|}$ *and*

$$|c| \leq 1 - \frac{1}{R\{\beta\}} \sum_{i=1}^{s} \frac{(1+p)M+1}{|\gamma_i|}, \tag{8}$$

then the functions $F^{\alpha,n,\beta}(m,l,q,\lambda,\gamma_i;z)$ *is univalent.*

Proof. We know from the proof of Theorem 2 that

$$\frac{zF'(z)}{F(z)} = \sum_{i=1}^{s} \frac{1}{\gamma_i}\left(\frac{z(D^{\alpha,n}(m,l,q,\lambda)f_i(z))'}{D^{\alpha,n}(m,l,q,\lambda)f_i(z)} - 1\right)$$

and

$$\left|c|z|^{2\beta} + \left(1 - |z|^{2\beta}\right)\frac{1}{R\{\beta\}}\left(\frac{z(D^{\alpha,n}(m,l,q,\lambda)f_i(z))'}{D^{\alpha,n}(m,l,q,\lambda)f_i(z)} - 1\right)\right|$$

$$\leq |c| + \frac{1}{|\beta|}\sum_{i=1}^{s}\frac{1}{|\gamma_I|}\left(\left|\frac{z^2(D^{\alpha,n}(m,l,q,\lambda)f_i(z))'}{\left(D^{\alpha,n}(m,l,q,\lambda)f_i(z)\right)^2}\right|\frac{|D^{\alpha,n}(m,l,q,\lambda)f_i(z)|}{|z|} + 1\right)$$

where

$$\left|\frac{z^2(D^{\alpha,n}(m,l,q,\lambda)f_i(z))'}{\left(D^{\alpha,n}(m,l,q,\lambda)f_i(z)\right)^2}\right| \leq 2 \quad \text{and} \quad |D^{\alpha,n}(m,l,q,\lambda)f_i(t)| \leq M|z|,$$

so

$$\left|c|z|^{2\beta} + \left(1-|z|^{2\beta}\right)\frac{zF\grave{(}z)}{R\{\beta\}F(z)}\right| \leq \frac{1}{|\beta|}\sum_{i=1}^{s}\frac{2M+1}{|\gamma_i|}.$$

Using (7) we have

$$\left|c|z|^{2\beta} + \left(1-|z|^{2\beta}\right)\frac{zF\grave{(}z)}{R\{\beta\}F(z)}\right| \leq 1.$$

Applying Lemma 2, we obtain $F^{\alpha,n,\beta}(m,l,q,\lambda,\gamma_i;z)$ is belong to S. □

Theorem 4. *Let the functions $D^{\alpha,n}(m,l,q,\lambda)f_i(t) \in S(p)$, for $i=(1,\dots,s)$, satisfy the condition (7) and $|D^{\alpha,n}(m,l,q,\lambda)f_i(t)| \leq M|z|, M \geq 1$. If β,γ_i and c are complex numbers such that $Re\{\beta\} > \frac{(1+p)M+1}{|\gamma_i|}$, then the functions $F^{\alpha,n,\beta}(m,l,q,\lambda,\gamma_i;z)$ is univalent.*

Proof . We know from the proof of theorem 3 that

$$\left|\frac{zF''(z)}{F'(z)}\right| \leq \sum_{i=1}^{s}\frac{1}{\gamma_i}\left|\frac{z(D^{\alpha,n}(m,l,q,\lambda)f_i(z))\acute{}}{D^{\alpha,n}(m,l,q,\lambda)f_i(z)} - 1\right|.$$

So, by the imposed conditions, we find that

$$\frac{(1-|z|^{2\beta})}{R\{\beta\}}\left|\frac{zF''(z)}{F'(z)}\right| \leq \frac{(1-|z|^{2\beta})}{R\{\beta\}}\sum_{i=1}^{s}\frac{1}{\gamma_i}\left|\frac{z(D^{\alpha,n}(m,l,q,\lambda)f_i(z))\acute{}}{D^{\alpha,n}(m,l,q,\lambda)f_i(z)} + 1\right|$$

$$\leq \frac{(1-|z|^{2\beta})}{R\{\beta\}}\sum_{i=1}^{s}\frac{1}{\gamma_i}\left(\left|\frac{Z^2(D^{\alpha,n}(m,l,q,\lambda)f_i(z))\acute{}}{\left(D^{\alpha,n}(m,l,q,\lambda)f_i(z)\right)^2}\right|\frac{|D^{\alpha,n}(m,l,q,\lambda)f_i(z)|}{|z|} + 1\right)$$

$$\leq \frac{(1-|z|^{2\beta})}{R\{\beta\}}\sum_{i=1}^{s}\frac{1}{\gamma_i}\left(\left|\frac{Z^2(D^{\alpha,n}(m,l,q,\lambda)f_i(z))\acute{}}{\left(D^{\alpha,n}(m,l,q,\lambda)f_i(z)\right)^2}\right| M + M + 1\right)$$

$$\leq \frac{1}{R\{\beta\}}\sum_{i=1}^{s}\frac{1}{\gamma_i}\left((1+p) + M + 1\right) \leq 1$$

By applying Lemma 2, we prove that $F^{\alpha,n,\beta}(m,l,q,\lambda,\gamma_i;z) \in S$. □

Corollary 3. *Let the functions $D^{\alpha,n}(m,l,q,\lambda)f_i(t) \in S(p)$, for $i=(1,\dots,s)$, satisfy the condition $R\left\{\frac{zf\acute{(}z)}{f\acute{(}z)} + 1\right\} > 0$, $z \in U$ and $|D^{\alpha,n}(m,l,q,\lambda)f_i(t)| \leq M|z|, M \geq 1$. If β,γ and c are complex numbers such that $Re\{\beta\} > \frac{n(1-p)M+1}{|\gamma|}$, and $|c| \leq 1 - \frac{(1+p)M+1}{R\{\beta\}|\gamma|}$, then the functions $F^{\alpha,n,\beta}(m,l,q,\lambda,\gamma_i;z)$ is univalent.*

Proof. By taking $\gamma_{1=}\gamma_{2=} \dots = \gamma_{s=}\gamma$ in Theorem 4. □

Corollary 4. *Let the functions $D^{\alpha,n}(m,l,q,\lambda)f_i(t) \in S(p)$, for $i=(1,2,\dots,s)$, satisfy the condition (7) and $|D^{\alpha,n}(m,l,q,\lambda)f_i(t)| \leq 1$. If β,γ_i and c are complex numbers such that $R\{\beta\} \geq \sum_{i=1}^{s}\frac{p+2}{|\gamma_i|}$ and$|c| \leq 1 - \frac{2(p+1)}{R\{\beta\}|\gamma|}$, thenthe functions $F^{\alpha,n,\beta}(m,l,q,\lambda,\gamma_i;z)$ is univalent.*

Proof. Put $M=1$ in Theorem 4. □

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