

APPROXIMATE ANALYTICAL SOLUTIONS OF THE KLEIN-GORDON EQUATION BY MEANS OF THE HOMOTOPY ANALYSIS METHOD

(Penyelesaian Analisis Penghampiran bagi Persamaan Klein-Gordon
Menggunakan Kaedah Analisis Homotopi)

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ABSTRACT

In this paper, the homotopy analysis method (HAM) is implemented to give approximate and analytical solutions for the Klein–Gordon equation. The auxiliary parameter \hbar in the HAM solutions has provided a convenient way of controlling the convergent region of series solutions. This problem shows rapid convergence of the sequence constructed by this method to the exact solution. Moreover, this technique reduces the volume of calculations by avoiding discretization of the variables, linearization or small perturbations.

Keywords: Klein–Gordon equation; homotopy analysis method; analytical solutions

ABSTRAK

Dalam makalah ini, kaedah analisis homotopi telah digunakan untuk menghasilkan penyelesaian analisis dan hampiran bagi persamaan Klein-Gordon. Parameter bantu \hbar dalam penyelesaian kaedah analisis homotopi memberikan satu cara yang mudah untuk mengawal rantau penumpuan bagi penyelesaian siri tersebut. Masalah yang dipertimbangkan ini menunjukkan penumpuan yang pantas kepada penyelesaian tepat bagi jujukan yang dibentuk oleh kaedah ini. Di samping itu, teknik ini juga mengurangkan jumlah pengiraan yang banyak tanpa melakukan pendiskretan pemboleh ubah, pelinearan atau usikan kecil.

Kata kunci: persamaan Klein-Gordon; kaedah analisis homotopi; penyelesaian analisis

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