MARTINGALE APPROACH TO EWMA CONTROL CHARTS FOR CHANGES IN EXPONENTIAL DISTRIBUTION

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ABSTRACT

Exponentially weighted moving average (EWMA) procedure is a popular chart used for detecting small shifts of parameters of distributions in quality control and surveillance problems. The objective of this paper is to derive characteristics of a particular process such as Average Run Length (ARL) and Average Delay time (AD) under EWMA procedure assuming that observations follow an exponential distribution. Using the martingale approach we find explicit expressions for characteristics of EWMA as Average Run Length (ARL) and Average Delay time (AD). We compare the performance of the proposed expressions under EWMA and other procedures such as CUSUM based ARL and AD using some simulation studies.

Keywords: Martingale approach; stopping time; EWMA; averaged run length; average delay

References

- Borror C.M., Montgomery D.C. & Runger G.C. 1999. Robustness of the EWMA Control Chart to Non-normality. *Journal of Quality Technology* **31**: 309-316.
- Brook D. & Evans D.A. 1972. An approach to the probability distribution of Cusum run length. *Biometrika* **59**: 539-548.
- Crowder S.V. 1987. A simple method for studying run length distributions of exponentially weighted moving average charts. *Technometrics* **29**: 401-407.
- Frisén M. & Sonesson C. 2006. Optimal surveillance based on exponentially weighted moving averages. *Sequential Analysis* **25**: 379-403.
- Gan F.F. 1998. Designs of one- and two-sided exponential EWMA chart. *Journal of Quality Technology* **30**: 55-69.
- Hawkins D.M. & Olwell D.H. 1998. Cumulative sum charts and charting for quality improvement. New York: Springer Verlag.
- Hunter J.S. 1986. The exponentially weighted moving average. Journal of Quality Technology 18: 203-10.
- Lucas J.M. & Saccucci M.S. 1990. Exponentially weighted moving average control schemes: properties and enhancements. *Technometrics* **32**: 1-29.
- Novikov A.A. 1990. On the first passage time of an autoregressive process over a level and application to a disorder problem. *Theory of probability and its applications* **35**: 269-279.
- Novikov A.A. 2006. Levy-driven Ornstein-Uhlenbeck processes: survey of results on first passage times. Lecture notes presented at the conference "Stochastic Calculus with Jumps". University of Angers, May 3-9, 2006.

Page E.S. 1954. Continuous inspection schemes. *Biometrika* 41: 100–114.

- Roberts S.W. 1959. Control chart tests based on geometric moving average. Technometrics 1: 239-250.
- Sukparungsee S. & Novikov A.A. 2006. On EWMA procedure for detection of a change in observations via martingale approach. KMITL Science Journal: An International Journal of Science and Applied Science 6: 373–380.
- Woodall W.H. & Adams B.M. 1993. The statistical design of CUSUM charts. Quality Engineering 5: 559-570.

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