Journal of Quality Measurement and Analysis *JQMA 4(2) 2008, 49-56*
Journal Pengukuran Kualiti dan Analisis

ANALYTICAL APPROXIMATIONS FOR DETECTION OF A CHANGE-POINT IN CASE OF LIGHT-TAILED DISTRIBUTIONS

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ABSTRACT

We derive analytic approximations for the expectation of exit times of Exponentially Weighted Moving Average (EWMA) procedure by using the martingale technique. Based on this technique, martingale approach is able to adapt to monitoring of changes of light-tailed distributions such as Gaussian, Poisson and Bernoulli distributions. Simple procedures are addressed for obtaining the optimal design of EWMA. A comparison with Monte Carlo simulation is also presented.

Keywords: martingales; exponentially weighted moving average chart; cumulative sum; average run length; average delay time; overshoot

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