

HARMONIC FUNCTIONS DEFINED BY GENERALISED RUSCHEWEYH OPERATOR AND HADAMARD PRODUCTS

(Fungsi Harmonik Tertakrif oleh Pengoperasi Terbitan Ruscheweyh Teritlak dan Hasil Darab Hadamard)

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ABSTRACT

In this paper, we study the generalised Ruscheweyh derivatives operator D_{λ}^n for harmonic function f . We introduce a class $\mathcal{M}_{\lambda}^n(\mathcal{A}, \mathcal{B}, \alpha)$ of all functions f, \mathcal{A} and \mathcal{B} for which

$$\operatorname{Re} \left\{ \frac{D_{\lambda}^{n+1}(f * \mathcal{A})(z)}{D_{\lambda}^n(f * \mathcal{B})(z)} \right\} > \alpha, \text{ for } n \in \mathbb{N}_0, \lambda \geq 0, 0 \leq \alpha < 1,$$

where $*$ denotes the Hadamard product (or convolution). We give sufficient coefficients conditions for normalised harmonic functions to be in $\mathcal{M}_{\lambda}^n(\mathcal{A}, \mathcal{B}, \alpha)$. These conditions are also shown to be necessary when the coefficients are negative. Distortion theorems, extreme points, closure theorems and neighbourhood property are obtained.

Keywords: Ruscheweyh derivatives; Hadamard product; Harmonic functions

ABSTRAK

Dalam makalah ini, pengoperasi terbitan Ruscheweyh teritlak D_{λ}^n dikaji untuk fungsi harmonik f . Kelas $\mathcal{M}_{\lambda}^n(\mathcal{A}, \mathcal{B}, \alpha)$ diperkenalkan bagi semua fungsi f, \mathcal{A} dan \mathcal{B} yang memenuhi

$$\operatorname{Re} \left\{ \frac{D_{\lambda}^{n+1}(f * \mathcal{A})(z)}{D_{\lambda}^n(f * \mathcal{B})(z)} \right\} > \alpha, \text{ untuk } n \in \mathbb{N}_0, \lambda \geq 0, 0 \leq \alpha < 1,$$

yang $*$ melambangkan hasil darab Hadamard (atau konvolusi). Syarat cukup pekali bagi fungsi harmonik ternormal untuk berada dalam kelas $\mathcal{M}_{\lambda}^n(\mathcal{A}, \mathcal{B}, \alpha)$ diberikan. Ditunjukkan bahawa syarat pekali ini adalah perlu apabila fungsi berpekali negatif. Teorem erotan, titik ekstrim, teorem tutupan dan sifat kejiranahan juga diperoleh.

Kata kunci: terbitan Ruscheweyh; hasil darab Hadamard; fungsi harmonik

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