*Journal of Quality Measurement and Analysis* *JQMA 5(2) 2009, 73-80*
*Jurnal Pengukuran Kualiti dan Analisis*

# A STUDY ON THE EFFECTS OF TRENDS DUE TO INERTIA ON EWMA AND CUSUM CHARTS

(Suatu Kajian Kesan Trend Akibat Inertia ke atas Carta EWMA dan CUSUM)

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*ABSTRACT*

Unlike a Shewhart chart, the exponentially weighted moving average (EWMA) and cumulative sum (CUSUM) charts are memory control charts (also known as time weighted control charts) that are used for a quick detection of small shifts in the process mean. Control charts that combine information from present and past samples, like the EWMA and CUSUM charts have the ability to detect process changes based on past information. A trend could exist on the EWMA and CUSUM charts, where all the sample points fall in one particular direction on the charts, for example, only above or below the center line. For this case, if a shift in the process mean occurs in the opposite sides of the chart, then such a shift cannot be detected quickly. This phenomenon is known as the inertia problem. A simulation study is conducted using Statistical Analysis System (SAS) to study and compare the effects of inertia on EWMA and CUSUM charts. It is found that the EWMA chart is affected by the inertia problem but not the CUSUM chart.

*Keywords*: EWMA chart; CUSUM chart; inertia; average run length (ARL)

*ABSTRAK*

Berbeza daripada carta Shewhart, carta purata bergerak berpemberat eksponen (EWMA) dan hasil tambah longgokan (CUSUM) adalah carta kawalan beringatan (juga dikenali sebagai carta kawalan berpemberatkan masa) yang digunakan untuk pengesanan cepat anjakan kecil dalam min proses. Carta kawalan yang menggabungkan maklumat daripada sampel baru dan lama, seperti carta EWMA dan CUSUM mempunyai kebolehan untuk mengesan anjakan dalam proses berdasarkan maklumat lama. Suatu tren mungkin wujud pada carta-carta EWMA dan CUSUM, yang mana semua titik sampel jatuh pada suatu arah tertentu di atas carta, misalnya, hanya di atas atau di bawah garisan tengah. Untuk kes ini, jika anjakan dalam min proses berlaku dalam arah yang bertentangan di atas carta, maka anjakan sedemikian tidak dapat dikesan dengan cepat. Fenomena ini dikenali sebagai masalah inersia. Suatu kajian simulasi dijalankan dengan menggunakan Sistem Analisis Berstatistik (SAS) untuk mengkaji dan membandingkan kesan inersia ke atas carta EWMA dan CUSUM. Adalah didapati bahawa carta EWMA dipengaruhi oleh masalah inersia tetapi bukan carta CUSUM.

*Kata kunci*: carta EWMA; carta CUSUM; inersia; purata panjang larian (ARL)

## References

Apley D.W. & Lee H.C. 2008. Robustness comparison of exponentially weighted moving-average charts on autocorrelated data and on residuals. *Journal of Quality Technology* **40**(4): 428-447.

Castagliola P., Celano G., Fichera S. & Giuffrida F. 2007. A variable sampling interval $S^2$ EWMA control chart for monitoring the process variance. *International Journal of Technology Management* **37**(1-2): 125-146.

Chou C.Y., Chen C.H. & Liu H.R. 2006. Economic design of EWMA charts with variable sampling intervals. *Quality and Quantity* **40**(6): 879-896.

Crowder S.V. 1987. A simple method for studying run-length distributions of exponentially weighted moving average charts. *Technometrics* **29**(4): 401-407.

Crowder S.V. 1989. Design of exponentially weighted moving average schemes. *Journal of Quality Technology* **21**(3): 155-162.

Eyvazian M., Naini S.G.J. & Vaghefi A. 2008. Monitoring process variability using exponentially weighted moving sample variance control charts. *International Journal of Advanced Manufacturing Technology* **39**(3-4): 261-270.

Gan F.F. 1991. An optimal design of CUSUM quality control charts. *Journal of Quality Technology* **23**(4): 279-286.

Hawkins D.M. 1993. Cumulative sum control charting: an underutilized SPC tool. *Quality Engineering* **5**(3): 463-477.

Lucas J.M. & Saccucci M.S. 1990. Exponentially weighted moving average control schemes : properties and enhancements. *Technometrics* **32**(1): 1-29.

Luo Y., Li Z. & Wang Z. 2009. Adaptive CUSUM control chart with variable sampling intervals. *Computational Statistics and Data Analysis* **53**(7): 2693-2701.

MacEachern S.N., Rao Y. & Wu C. 2007. A robust-likelihood cumulative sum chart. *Journal of the American Statistical Association* **102**(480): 1440-1447.

Montgomery D. C. 2005. *Introduction to Statistical Quality Control.* 5th Ed. New York: John Wiley & Sons.

Page E.S. 1954. Continuous inspection schemes. *Biometrika* **41**(1-2): 100-115.

Roberts S.W. 1959. Control chart tests based on geometric moving averages. *Technometrics* **1**(3): 239-250.

Serel D.A. & Moskowitz H. 2008. Joint economic design of EWMA control charts for mean and variance. *European Journal of Operational Research* **184**(1): 157-168.

Sheu S., Lin T. & Tai S. 2007. Extended Poisson exponentially weighted moving average control chart for monitoring process quality. *International Journal of Industrial Engineering - Theory Applications and Practice* **14**(4): 382-391.

Shu L. 2008. An adaptive exponentially weighted moving average control chart for monitoring process variances. *Journal of Statistical Computation and Simulation* **78**(4): 367-384.

Shu L., Jiang W. & Wu Z. (2008). Adaptive CUSUM procedure with Markovian mean estimation. *Computational Statistics and Data Analysis* **52**(9): 4395-4409.

Testik M.C. 2007. Conditional and marginal performance of the Poisson CUSUM control chart with parameter estimation. *International Journal of Production Research* **45**(23): 5621-5638.

Thaga K. & Yadavalli V. 2007. Max-EWMA chart for autocorrelated process (MEWMAP chart). *South African Journal of Industrial Engineering* **18**(2): 131-152.

Wu Y., Zhu Y. & Wang W. 2009. Effect of inspection error on out-of-control average run length of CUSUM charts. *Communications in Statistics–Simulation and Computation* **38**(7): 1435-1445.

Wu Z., Zhang S. & Wang P. 2007. A CUSUM scheme with variable sample sizes and sampling intervals for monitoring the process mean and variance. *Quality and Reliability Engineering International* **23**(2): 157-170.

Zantek P.F. 2008. A Markov-chain method for computing the run-length distribution of the self-starting cumulative sum scheme. *Journal of Statistical Computation and Simulation* **78**(5): 463-473.

Zhang S. & Wu Z. 2007. A CUSUM scheme with variable sample sizes for monitoring process shifts. *International Journal of Advanced Manufacturing Technology* **33**(9-10): 977-987.

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