

## UNSTEADY MAGNETOHYDRODYNAMIC MIXED CONVECTION STAGNATION POINT FLOW OF A VISCOELASTIC FLUID ON A VERTICAL SURFACE

(Aliran Titik Genangan Olakan Campuran Magnetohidrodinamik Tak Mantap  
bagi Bendalir Likat-kenyal pada Permukaan Menegak)

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### ABSTRACT

In this paper, the problem of unsteady magnetohydrodynamic (MHD) viscoelastic fluid flowing towards a stagnation point on a vertical surface is studied. The temperature of the surface is assumed to vary linearly with the distance from the stagnation point. The partial differential equations which governed the flow are transformed into a system of ordinary differential equations, which are then solved numerically by an implicit finite-difference scheme known as the Keller-box method. The effects of the viscoelastic parameter  $K$ , mixed convection parameter  $\lambda$ , magnetic parameter  $M$  and Prandtl number  $Pr$  on the flow and heat transfer characteristics are presented in this paper. The numerical solutions obtained are uniformly valid for all dimensionless time from initial unsteady-state flow to final steady-state flow in the whole spatial region.

*Keywords:* magnetohydrodynamic (MHD); mixed convection; unsteady stagnation point flow; vertical surface; viscoelastic fluid

### ABSTRAK

Dalam makalah ini, masalah aliran bendalir likat-kenyal magnetohidrodinamik (MHD) tak mantap yang mengalir ke arah suatu titik genangan pada permukaan tegak dikaji. Suhu permukaan dianggap berubah secara linear terhadap jarak daripada titik genangan. Persamaan pembezaan separa yang menakluk aliran kemudiannya dijelmakan kepada sistem persamaan pembezaan biasa, yang seterusnya diselesaikan secara berangka menggunakan skema beza terhingga tersirat yang dikenali sebagai kaedah kotak Keller. Kesan parameter likat-kenyal  $K$ , parameter olakan campuran  $\lambda$ , parameter magnetik  $M$  dan nombor Prandtl  $Pr$  terhadap ciri-ciri aliran dan pemindahan haba dipertimbangkan dalam makalah ini. Penyelesaian berangka yang diperolehi adalah sah secara seragam bagi julat masa tak bermatra bermula daripada aliran awal tak mantap kepada aliran akhir mantap.

*Kata kunci:* magnetohidrodinamik (MHD); olakan campuran; aliran titik genangan tak mantap; permukaan tegak; bendalir likat-kenyal

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