ORDER-4 SYMMETRIZED RUNGE-KUTTA METHODS
FOR STIFF PROBLEMS
(Kaedah Runge-Kutta Tersimetri Peringkat-4 untuk Masalah Kaku)

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ABSTRACT

If a Runge-Kutta method having an asymptotic error expansion in the stepsize h is symmetric then it is characterised by an h^2-expansion. Since elimination of the leading error terms in succession results in an increase in the order by two at a time, a symmetric method could therefore be suitable for the construction of extrapolation methods. However, when order reduction occurs for stiff problems it needs to be suppressed before an appropriate extrapolation formula can be applied. This can be achieved by a process called symmetrization which is a composition of the symmetric method with an L-stable method known as a symmetrizer. The symmetrizer is constructed so as to preserve the h^2-asymptotic error expansion. In this paper we consider symmetrization of the 2-stage Gauss and the 3-stage Lobatto IIIA methods of order 4. We show that these methods are more efficient when used with symmetrization. Extrapolation based on the symmetrized methods is therefore expected to give greater accuracy. We also show that the method with a higher stage order is more advantageous than one with a lower stage order for solving stiff problems.

Keywords: Order reduction; symmetric methods; stiff problems; symmetrizers

ABSTRAK

Jika kaedah Runge-Kutta yang mempunyai kembangan ralat asimptot dengan saiz langkah h adalah simetri, maka ia dicirikan oleh kembangan-h^2. Oleh sebab penghapusan sebutan ralat utama dalam hasil yang berturutan adalah dalam bentuk peningkatan dua peringkat pada suatu masa, suatu kaedah simetri adalah sesuai untuk pembinaan kaedah ekstrapolasi. Walau bagaimanapun apabila penurunan peringkat bagi masalah kaku berlaku ia perlu dikurangkan sebelum suatu rumus ekstrapolasi yang sesuai boleh digunakan. Ini boleh dicapai melalui proses pensimetrian yang merupakan komposisi di antara kaedah simetri dengan suatu kaedah yang L-stabil yang dikenali sebagai pensimetri. Pensimetri tersebut dibina agar mengekalkan kembangan ralat asimptot-h^2. Dalam makalah ini, dipertimbangkan pensimetrian bagi kaedah Gauss tahap-2 dan Lobatto IIIA tahap-3 dengan peringkat-4. Dapat ditunjukkan bahawa kedua-dua kaedah ini adalah lebih cekap apabila digunakan dengan pensimetri. Oleh itu, ekstrapolasi yang berasaskan kaedah tersimetri diharapkan memberikan kejituan yang lebih tinggi. Turut ditunjukkan bahawa kaedah dengan peringkat tahap yang lebih tinggi mempunyai kelebihan berbanding kaedah yang berperingkat tahap lebih rendah bagi menyelesaikan masalah kaku.

Kata kunci: Penurunan peringkat; kaedah simetri; masalah kaku; pensimetri

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