

ON THE COMPUTATIONS OF SOME HOMOLOGICAL FUNCTORS OF 2-ENGEL GROUPS OF ORDER AT MOST 16

(Pengiraan Beberapa Fungtor Homologi bagi Kumpulan Engel-2 Berperingkat Tidak Melebihi 16)

HAZZIRAH IZZATI MAT HASSIM, NOR HANIZA SARMIN,
NOR MUHAINIAH MOHD ALI & MOHD SHAM MOHAMAD

ABSTRACT

The homological functors including $J(G)$, $\nabla(G)$, exterior square, the Schur multiplier, $\Delta(G)$, the symmetric square and $\tilde{J}(G)$ of a group were originated in homotopy theory. The nonabelian tensor square which is a special case of the nonabelian tensor product is vital in the computations of the homological functors of a group. It was introduced by Brown and Loday in 1987. The nonabelian tensor square $G \otimes G$ of a group G is generated by the symbols $g \otimes h$, for all $g, h \in G$ subject to the relations $gg' \otimes h = ({}^s g' \otimes {}^s h)(g \otimes h)$ and $g \otimes hh' = (g \otimes h)({}^h g \otimes {}^h h')$, for all $g, g', h, h' \in G$ where ${}^s g' = gg'g^{-1}$. In this paper, the computations of nonabelian tensor squares and some homological functors of all 2-Engel groups of order at most 16 are done. Groups, Algorithms and Programming (GAP) software has been used to assist and verify the results.

Keywords: Nonabelian tensor square; homological functors; 2-Engel groups; GAP

ABSTRAK

Fungtor homologi termasuk $J(G)$, $\nabla(G)$, kuasa dua peluaran, pendarab Schur, $\Delta(G)$, kuasa dua simetrik dan $\tilde{J}(G)$ bagi suatu kumpulan berasal daripada teori homotopi. Kuasa dua tensor tak Abelian yang merupakan satu kes istimewa bagi hasil darab tensor tak Abelian adalah sangat penting dalam pengiraan fungtor homologi bagi suatu kumpulan. Ianya telah diperkenalkan oleh Brown dan Loday dalam tahun 1987. Kuasa dua tensor tak Abelian $G \otimes G$ bagi suatu kumpulan G adalah dijanakan oleh simbol $g \otimes h$, untuk semua $g, h \in G$ tertakluk kepada hubungan $gg' \otimes h = ({}^s g' \otimes {}^s h)(g \otimes h)$ dan $g \otimes hh' = (g \otimes h)({}^h g \otimes {}^h h')$ bagi semua $g, g', h, h' \in G$ dengan ${}^s g' = gg'g^{-1}$. Dalam makalah ini, pengiraan bagi kuasa dua tensor tak Abelian dan beberapa fungtor homologi untuk semua kumpulan Engel-2 berperingkat paling tinggi 16 telah dilakukan. Perisian *Groups, Algorithms and Programming* (GAP) digunakan untuk membantu dan mengesahkan keputusan yang diperolehi.

Kata kunci: Kuasa dua tensor tak Abelian; fungtor homologi; kumpulan Engel-2; GAP

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*Department of Mathematics, Faculty of Science
and Ibnu Sina Institute for Fundamental Science Studies
Universiti Teknologi Malaysia, 81310 UTM Johor Bahru, MALAYSIA
E-mails: hazzirah_hassim@yahoo.com, nhs@fs.utm.my**

*Corresponding author