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DIFFERENT DOWNSIDE RISK APPROACHES IN PORTFOLIO OPTIMISATION

(Pendekatan Risiko Menurun yang Berlainan dalam Pengoptimuman Portfolio)

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ABSTRACT

Variance is commonly used as risk measure in portfolio optimisation to find the trade-off between the risk and return. Investors wish to minimise the risk at the given level of return. However, the mean-variance model has been criticised because of its limitations. The mean-variance model strictly relies on the assumptions that the assets returns are normally distributed and investor has quadratic utility function. This model will become inadequate when these assumptions are violated. Besides, variance not only penalises the downside deviation but also the upside deviation. Variance does not match investor's perception towards risk because upside deviation is desirable for investors. Therefore, downside risk measures such as semi-variance, below target risk and conditional value at risk have been proposed to overcome the deficiencies of variance as risk measure. These downside risk measures have better theoretical properties than variance because they are not restricted to normal distribution and quadratic utility function. The downside risk measures focus on return below a specified target return which better match investor's perception towards risk. The objective of this paper is to compare the optimal portfolio composition and performance using variance, semi-variance, below target risk and conditional value at risk as risk measure.

Keywords: Portfolio optimisation; variance; downside risk

ABSTRAK

Varians merupakan ukuran risiko yang biasa digunakan dalam pengoptimuman portfolio untuk mendapatkan penggantian di antara risiko dengan pulangan. Pelabur ingin meminimumkan risiko pada kadar pulangan tertentu. Namun begitu, model min-varians dikritik disebabkan oleh kekangannya. Model min-varians adalah sangat bergantung pada andaian bahawa pulangan aset bertaburan normal dan pelabur mempunyai fungsi utiliti kuadratik. Model ini menjadi tidak sesuai jika kedua-dua andaian tidak dipatuhi. Selain itu, varians bukan sahaja mendenda sisihan bawah tetapi juga sisihan atas. Varians tidak sepadan dengan persepsi pelabur terhadap risiko kerana sisihan atas adalah diingini oleh pelabur. Oleh itu, ukuran risiko menurun seperti semi-varians, risiko bawah sasaran dan nilai berisiko telah dicadangkan untuk mengatasi kekurangan varians sebagai ukuran risiko. Ukuran risiko menurun ini mempunyai ciri-ciri teori yang lebih baik daripada varians kerana mereka tidak terbatas kepada taburan normal dan fungsi utiliti kuadratik. Ukuran risiko menurun yang fokus kepada pulangan di bawah sasaran yang ditetapkan lebih sepadan dengan persepsi pelabur terhadap risiko. Objektif kertas ini adalah untuk membandingkan komposisi dan prestasi portfolio optimum dengan menggunakan varians, semi-varians, risiko bawah sasaran dan nilai berisiko sebagai ukuran risiko.

Kata kunci: Pengoptimuman portfolio; varians; risiko menurun

1. Introduction

The traditional Markowitz (1952) mean-variance (MV) model has been criticised by many researchers for its limitations. The MV model strictly depends on the assumptions that the assets returns follow normal distribution and investor has quadratic utility function. This model will not consistent with the maximisation of expected utility principle if the above two conditions do not hold (Tobin 1958). However, these two conditions do not hold in practical. Many researchers have showed that the assets returns distribution are asymmetry and exhibit skewness (Arditti 1971; Chunhachinda *et al*. 1997; Prakash *et al*. 2003). According to Pratt (1964), quadratic utility function is very unlikely because it implies investors prefer less wealth to more wealth. The past researchers have proposed the downside risk measures such as semivariance (SV), below target risk (BT) and conditional value at risk (CVaR) to overcome the disadvantages of MV model. The downside risk measures are more robust because they are not restricted to the MV assumptions. These downside risk measures are consistent with investor's perception towards risk as they focus on return dispersions below specified target return. The objective of this paper is to compare the optimal portfolio composition and performance using variance, semi-variance, below target risk and conditional value at risk as risk measure. The next section discusses the literature review and concepts of downside risk measures SV, BT and CVaR. Section 3 presents the mathematical models using variance, SV, BT and CVaR as risk measures. Section 4 describes the data and methodology of this study. Section 5 reports the empirical results of this study. Section 5 concludes the paper.

2. Literature Review

Markowitz (1952) has introduced the MV model in portfolio optimisation. Mean represents the return and variance as risk measure in this model. The objective of MV model is to minimise the portfolio variance at given level of return. Covariance matrix of assets returns need to be calculated to compute portfolio variance. Variance measures the deviation above and below the mean return. Variance is not an appropriate risk measure because it does not only penalises the downside deviation but also the upside deviation. However, upside deviation is desirable for investors while downside deviation is undesirable. Therefore, variance is not consistent with the investor's actual perception towards risk.

Downside risk is an appropriate investment risk measure because investors are more concerned about loss below the target return. Markowitz (1959) has proposed the mean semi-variance model using semi-variance as risk measure instead of variance to overcome the weaknesses of MV model. SV is defined as follows:

$$\text{SV} = \frac{1}{T}\sum_{t=1}^{T}\max[0, E(R) - R_T] \tag{1}$$

where R_T is the asset return during period *t, T* is the number of observations and $E(R)$ is the expected return of assets returns. SV is an asymmetric risk measure that focuses on squared return deviations below the mean return. The SV and MV models are equivalent when the assets returns are normally distributed. Many studies have been using SV as risk measure in portfolio optimisation (Harlow 1991; Nawrocki 1991; Markowitz *et al.*1993; Grootveld & Hallerbach 1999; Sing & Ong 2000).

Fishburn (1977) has proposed the below target risk measure with below target semi-variance as special case. Below target risk represents the expected deviation of returns falling below the target rate. Below target semi-variance is defined as follows:

$$\text{Below target semi-variance} = \frac{1}{T}\sum_{t=1}^{T}\max[0, \tau - R_T] \tag{2}$$

where R_T is the asset return during period *t, T* is the number of observations and τ is the target rate of return. Below target risk is consistent with the maximisation of expected utility principle. Grootveld and Hallerbach (1999), Nawrocki(1999), Konno *et al.* (2002) have studied this risk measure in the past.

CVaR is a quantile risk measure proposed by Rockafellar and Uryasev (2000). CVaR is the conditional expectation of loss above the amount α at specified probability level β which can be defined as follows:

$$\text{CVaR} = \frac{1}{1-\beta} E[L(X) \mid L(X) \geq VaR(X)] \tag{3}$$

where $L(X)$ is the loss function and *VaR(X)* is value at risk. According to Artzner *et al.* (1999), CVaR is a coherent risk measure. CVaR is also known as expected shortfall, mean excess loss or tail VaR and it is widely used in portfolio optimisation (Uryasev 2000; Krokhmal *et al*. 2001; Acerbi & Tasche 2002; Mansini *et al*. 2007).

3. Mathematical Models

3.1. *Variance*

$$\text{minimise} \sum_{i=1}^{n}\sum_{j=1}^{n}\sigma_{ij}x_i x_j$$

$$\text{subject to} \sum_{j=1}^{n} r_j x_j \geq \rho M_{0,}$$

$$\sum_{j=1}^{n} x_j = M_{0,}$$

$$0 \leq x_j \leq u_{j,} j = 1,\ldots,n. \tag{4}$$

where σ_{ij} is the covariance between assets i and j, x_j is the amount invested in asset j, r_j is the expected return of asset j per period, ρ is a parameter representing the minimal rate of return required by an investor, $M_{0,}$ is the total amount of fund and u_j is the maximum amount of money which can be invested in asset j.

3.2. *Semi-variance*

$$\text{minimise} \sum_{t=1}^{T} p_t z_t^{\,2}$$

subject to $z_t \geq -\sum (r_{jt} - r_j)x_j, t = 1,2,...T,$

$$\sum_{j=1}^{n} x_j = M_{0,},$$

$$0 \leq x_j \leq u_{j,} j = 1,...,n. \tag{5}$$

where x_j is the amount invested in asset j, r_j is the expected return of asset j per period, ρ is a parameter representing the minimal rate of return required by an investor, $M_{0,}$ is the total amount of fund and u_j is the maximum amount of money which can be invested in asset j, p_t is the probability that R achieves r_t.

3.3. *Below Target Semi-variance*

minimise $\frac{1}{T}\sum_{t=1}^{T} \max[0, \tau - R_T]$

$$E(R_P) = \mu,$$

$$\sum_{i=1}^{N} x_i = 1,$$

$$x_i \geq 0. \tag{6}$$

where τ is the target rate of return, x_i is the amount invested in asset i, R_T is the asset return during period t and $E(R_P)$ is the portfolio mean return.

3.4. *Conditional Value at Risk*

minimise $\alpha + \frac{1}{T(1-\beta)}\sum_{t=1}^{T} z_t$

subject to $z_t \geq -\sum_{j=1}^{n} r_{jt}x_j - \alpha, t = 1,2,...,T,$

$$z_t \geq 0, t = 1,2,...,T,$$

$$\sum_{j=1}^{n} r_j x_j \geq \rho M_{0,}$$

$$\sum_{j=1}^{n} x_j = M_{0,}$$

$$0 \leq x_j \leq u_{j,} j = 1,\ldots,n. \tag{7}$$

where α is the lowest amount of loss, β is the probability that the loss will not exceed α, T is the number of periods, r_{jt} be the realisation of random variable R_j during period t, x_j is the amount invested in asset j, r_j is the expected return of asset j per period, ρ is a parameter representing the minimal rate of return required by an investor, $M_{0,}$ is the total amount of fund and u_j is the maximum amount of money which can be invested in asset j.

4. Data and Methodology

Four different portfolios have been constructed using the MV (4), SV (5), BT (6) and CVaR (7) models to compare the portfolio compositions and performances of different optimal portfolios. The data consists of monthly returns of 54 stocks included in the Kuala Lumpur Composite Index (KLCI) from January 2004 until December 2007. The minimum rate of return which is represented by ρ is set to 1% (average return of KLCI during study period) whereas β is set to 0.90 for CVaR model based on Konno et al. (2002). Portfolio mean return is calculated as follows:

$$\text{Portfolio mean return} = \sum_{j=1}^{N} x_j r_j \tag{8}$$

where x_j is the amount invested in asset j and r_j is the expected return of asset j. Portfolio performance is calculated using reward to variability ratio as follows:

$$\text{Portfolio performance} = \text{mean return/risk} \tag{9}$$

5. Empirical Results

5.1. *Portfolio Performance*

The table 1 shows the summary statistics of four different optimal portfolios. It indicates that the mean return of the CVaR model (0.0212) is the highest whereas the other three models (0.0100) give the same mean return. All models achieve the target return (1%) in the optimal portfolios. The MV model (0.0181) is the most risky portfolio while the CVaR model (0.0112) generates the less risky portfolio. The CVaR model (1.8929) gives the highest performance among the four models while MV (0.5525) shows the lowest performance. The CVaR model is useful to control downside risk (Konno *et al.* 2002). It is obvious that the three downside risk models outperform the MV model.

Table 1: Summary Statistics of Optimal Portfolios

	MV	SV	BT	CVaR
Mean Return	0.0100	0.0100	0.0100	0.0212
Risk	0.0181	0.0124	0.0166	0.0112
Performance	0.5525	0.8065	0.6028	1.8929

5.2. *Portfolio Composition*

As shown in table 2, the compositions of stocks are different among the four models. According to Byrne and Lee (2004), the difference in weight is due to the non-normality displayed by data. BAT is the largest component stock in MV (23.53%) and SV (18.93%) optimal portfolios. Affin (27.06%) dominates other stocks in BT model whereas PetGas (21.34%) is the most dominant stock in CVaR model.

Table 2: Percentage of Stocks in Optimal Portfolios

Stock	MV	SV	BT	CVaR
Affin	-	-	27.06	4.85
AMMB	-	-	4.37	-
BAT	23.53	18.93	8.47	-
Carlsbg	6.06	5.31	1.76	9.87
CCM	3.95	5.68	6.85	-
CIMB	-	-	-	1.22
DiGi	4.36	3.48	-	9.67
GAB	6.99	1.52	-	-
IOICorp	0.32	-	-	2.55
Kulim	2.66	-	-	-
Maybank	6.5	12.24	20.21	-
MPI	-	-	4.07	-
MISC	-	-	-	0.61
MMCCorp	-	-	-	8.19
Mulpha	-	2.06	-	1.65
Bernas	3.19	2.75	-	13.33
PetDag	-	-	1.34	-
PetGas	20.93	16.18	8.51	21.34
Pos	--	-	2.95	-
PPB	0.05	1.22	-	2.85
Proton	-	-	0.57	-
Puncak	2.81	-	3.18	-
RHBCap	-	6.25	0.18	3.18
Sarawak	3.22	7.59	-	-

Table 2: (Continued)

Shang	1.75	-	1.51	-
Shell	6.22	5.39	-	11.77
STAR	-	1.18	-	-
TA	1.01	-	5.15	-
TChong	0.16	-	3.82	-
Tenaga	0.49	7.22	-	1.53
UMW	5.82	1.19	-	-
YTL	-	1.82	-	7.38

6. Conclusion

This paper discusses the portfolio optimisation models and compares the performance as well as portfolio composition of the mean-variance model with other downside risk models which are semi-variance, below target semi-variance and conditional value at risk models. Different models give different optimal portfolio composition. The results show that the three downside risk models outperform the mean-variance model. The conditional value at risk model gives the highest performance optimal portfolio. It indicates that the conditional value at risk is useful to control downside risk and is a better choice for risk adverse investors.

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References

Aiditti F. 1971. Another look at mutual fund performance. *Journal of Financial and Quantitative Analysis* **6**: 909-912.

Acerbi C. & Tasche D. 2002. On the coherence of expected shortfall. *Journal of Banking and Finance* **26**: 278-287.

Artzner P., Delbaen F., Eber J.-M. & Heath D. 1999. Coherent measures of risk. *Mathematical Finance* **9**: 203-228.

Byrne P. & Lee S. 2004. Different risk measures: different portfolio compositions? *Journal of Property Investment & Finance* **22**(6): 501-511.

Chunhachinda P., Dandapani K., Hamid S. & Prakash A.J. 1997. Portfolio selection and skewness: Evidence from international stock markets. *Journal of Banking & Finance* **21**: 143-167.

Fishburn P.C. 1977. Mean-risk analysis with risk associated with below-target returns. *The American Economic Review* **67**: 116-126.

Grootveld H. & Hallerbach W. 1999. Variance vs downside risk: Is there really that much difference? *European Journal of Operational Research* **114**(2): 304-319.

Harlow W.V. 1991. Asset allocation in a downside-risk framework. *Financial Analysts Journal* September-October: 28-40.

Konno H., Waki H. & Yuuki A. 2002. Portfolio optimization under lower partial risk measures. *Asia-Pacific Financial Markets* **9**: 127-140.

Krokhmal P., Uryasev S. & Palmquist J. 2001. Portfolio optimization with conditional value-at-risk objective and constraints. *Journal of Risk* **4**(2): 43-68.

Mansini R., Ogryczak W. & Speranza M.G. 2007. Conditional value at risk and related linear programming models for portfolio optimization. *Annals of Operations Research* **152**: 227-256.

Markowitz H. 1952. Portfolio selection. *Journal of Finance* **7**(1): 77-91.
Markowitz H. 1959. *Portfolio Selection: Efficient Diversification of Investments*. New York: John Wiley & Sons.
Markowitz H., Todd P., Xu G. & Yamane Y. 1993. Computation of mean-semivariance efficient sets by the critical line algorithm. *Annals of Operations Research* **45**: 307-317.
Nawrocki D.N. 1991. Optimal algorithms and lower partial moment: ex post results. *Applied Economics* **23**: 465-470.
Nawrocki D.N. 1999. A brief history of downside risk measures. *Journal of Investing* **8**: 9-25.
Prakash A.J., Chang C. & Pactwa T.E. 2003. Selecting a portfolio with skewness: Recent evidence from US, European, and Latin American equity markets. *Journal of Banking and Finance* **27:** 1375-1390.
Pratt J.W. 1964. Risk aversion in the small and in the large. *Econometrica* **32** *(*1/2): 122-136.
Rockafellar R.T. & Uryasev S. 2000. Optimization of conditional value-at- risk. *Journal of Risk* **2**: 21-41.
Sing T.F. & Ong S.E. 2000. Asset allocation in a downside risk framework. *Journal of Real Estate Portfolio Management* **6**(3): 213-223.
Tobin J. 1958. Liquidity preference as behavior towards risk. *Review of Economic Studies* **25** (6/7): 65-86.
Uryasev S. 2000. Conditional value-at-risk: optimization algorithms and applications. *Financial Engineering News* **14**: 1-5.

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