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GOODNESS OF FIT TEST FOR LOGISTIC DISTRIBUTION INVOLVING KULLBACK-LEIBLER INFORMATION

(Ujian Kebagusan Penyuaian untuk Taburan Logistik menerusi Maklumat Kullback-Leibler**)**

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ABSTRACT

In this paper, our objective is to test the statistical hypothesis $H_o : F(x) = F_o(x)$ for all x against $H_1 : F(x) \neq F_o(x)$ for some x, where $F_o(x)$ is a known distribution function. In this study, a goodness of fit test statistic for testing the logistic distribution based on Kullback-Leibler information as proposed by Song (2002) is studied. The logistic parameters are estimated by using several methods of estimation such as maximum likelihood, order statistics, moments, L-moments and LQ-moments. The critical value based on the statistics which involves the Kullback-Leibler information under the assumption that H_o is true is computed using Monte Carlo simulations. The performance of the test under simple random sampling is investigated. Ten different distributions are considered under the alternative hypothesis. Based on Monte Carlo simulations, for all the distributions considered, it is found that the test statistics based on estimators found by moment and LQ-moment methods have the highest power, except for $Weibull(2, .5)$ and Gamma distributions.

Keywords: Goodness of fit test; Kullback-Leibler information; logistic distribution

ABSTRAK

Objektif di dalam makalah ini ialah menguji hipotesis $H_o : F(x) = F_o(x)$ untuk semua x melawan $H_1 : F(x) \neq F_o(x)$ untuk sesetengah *x*, yang $F_o(x)$ suatu fungsi taburan yang diketahui. Dalam kajian ini statistik ujian kebagusan penyuaian untuk taburan logistik berdasarkan maklumat Kullback-Leibler yang dicadangkan oleh Song (2002) dikaji. Parameter-parameter logistik dianggarkan dengan menggunakan berbagai-bagai kaedah penganggaran seperti kaedah kebolehjadian maksimum, statistik tertib, kaedah momen, momen-L dan momen-LQ. Nilai genting didasarkan statistik yang melibatkan maklumat Kullback-Leibler di bawah anggapan yang H_o benar dan dihitung menerusi simulasi Monte Carlo. Prestasi ujian ini di bawah pensampelan rawak mudah dikaji. Sepuluh taburan yang berbeza dipertimbangkan sebagai hipotesis alternatif. Berdasarkan simulasi Monte Carlo telah didapati bahawa ujian statistik berdasarkan penganggar momen dan momen-LQ mempunyai kuasa yang terbesar kecuali bagi $Weibull(2, .5)$ dan taburan-taburan Gama.

Kata kunci: Ujian kebagusan penyuaian; maklumat Kullback-Leibler; taburan logistik

1. Introduction

The logistic distribution has interesting application in many different fields, such as public health, graduation of mortality statistics, survival data, income distributions, human population and biology (Balakrishnan 1992).

Arizono and Ohta (1989) proposed a test of normality based on an estimate of the Kullback-Leibler information (KLI). Song (2002) presents a general methodology for developing asymptotically distribution-free goodness of fit tests based on the Kullback-Leibler information. Also, he shows that the tests to be omnibus within an extremely large class of nonparametric global alternatives and to have good local power. Ibrahim *et al.* (2009), however, study the power of chi-square test for goodness of fit using Kullback-

Leibler information under RSS and SRS. They found that the chi-square test is more powerful under ranked set sampling (RSS) as compared to simple random sampling (SRS).

In this paper, we introduce a test statistic for goodness of fit test for logistic distribution which is based on the Kullback-Leibler information statistic. We estimate the logistic parameters by using several methods of estimation such as maximum likelihood, order statistics, moments, L-moments and LQ-moments. We compute the critical values and the power based on the statistics which involves the Kullback-Leibler information using Monte Carlo simulations.

L-moments have the theoretical advantages over the conventional moments since it is able to characterise a wider range of distributions and, when estimated from a sample it is found to be more robust to the presence of outliers in the data. Also, the parameter estimates obtained from L-moments are sometimes more accurate in small samples than even the maximum likelihood estimates.

This paper is organised as follows. In Section 2, we define the test statistic. We define the estimators of logistic distribution in Section 3. In Section 4, we define two algorithms to calculate the percentage points and the power function of the test statistic at an alternative distribution. In Section 5, a simulation study is conducted to study the power of the test statistic and we state our conclusions in Section 6.

2. Hypothesis Testing Involving KLI

Let $X_1, X_2, ..., X_n$ be a random sample with probability density function $f(x)$ and cumulative distribution function $F(x)$, with a finite mean μ and variance σ^2. Let $X_{1:n} \le X_{2:n} \le ... \le X_{n:n}$ denote the corresponding order statistics.

We are interested in testing the hypothesis

$$H_o : F(x) = F_o(x) \ \ \forall x \quad \text{vs.} \quad H_1 : F(x) \ne F_o(x) \ \text{ for some } x,$$

where $F_o(x)$ is a logistic distribution function given in the form

$$F_o(x;\alpha,\beta) = \exp\left(-\frac{x-\alpha}{\beta}\right)\left(1+\exp\left(-\frac{x-\alpha}{\beta}\right)\right)^{-1}, \tag{1}$$

and its density function is given as

$$f_o(x;\alpha,\beta) = \frac{1}{\beta}\exp\left(-\frac{x-\alpha}{\beta}\right)\left(1+\exp\left(-\frac{x-\alpha}{\beta}\right)\right)^{-2}, \tag{2}$$

where α is a location parameter, β is a scale parameter, $x, \alpha \in (-\infty, \infty)$ and $\beta > 0$.

To discriminate between null and alternative hypotheses, H_0 and H_1, the KLI is employed, and can be given by

$$I(f, f_0) = \int_{-\infty}^{\infty} f(x) \log \frac{f(x)}{f_0(x;\alpha,\beta)} dx. \tag{3}$$

From equation (3) and using logarithms, the KLI is given as

$$I(f,\, f_0) = \int_{-\infty}^{\infty} f(x)\log f(x)dx - \int_{-\infty}^{\infty} f(x)\log f_0(x;\alpha,\beta)dx$$

$$= -H(f) - \int_{-\infty}^{\infty} f(x)\log f_0(x;\alpha,\beta)dx, \tag{4}$$

where $H(f)$ is the entropy of a distribution $f(x)$, i.e. population entropy, with a density function $f(x)$ and $H(f)$ is known as Shannon's entropy. According to Vasicek (1976), the estimator of

$$H(f) = -\int_{-\infty}^{\infty} f(x)\log f(x)dx = \int_0^1 \log\left(\frac{\mathrm{d}}{\mathrm{dp}}F^{-1}(p)\right)dp,$$

where $p = F(x)$, is given by

$$\frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{n}{2w}\left(X_{i+m:n} - X_{i-m:n}\right)\right), \tag{5}$$

where w, called window size, is a positive integer $(w \le n/2)$, $X_{i:n} = X_{1:n}$ for $i < 1$ and $X_{i:n} = X_{n:n}$ for $i > n$. According to Song (2002), the estimator of $\int_{-\infty}^{\infty} f(x)\log\ f_0(x;\alpha,\beta)dx$ is given by

$$\frac{1}{n}\sum_{i=1}^{n}\log f_0(X_i;\hat{\alpha}_{SRS},\hat{\beta}_{SRS}), \tag{6}$$

where $\hat{\alpha}_{SRS}$ and $\hat{\beta}_{SRS}$ are the estimators of α and β under SRS respectively. By substituting the pdf of logistic (2) in the equation (6), the result is given by

$$\frac{1}{n}\sum_{i=1}^{n}\log f_0(X_i;\hat{\alpha}_{SRS},\hat{\beta}_{SRS})$$

$$= \frac{1}{n}\sum_{i=1}^{n}\left(-\log\hat{\beta} - \left(\frac{X - \hat{\alpha}_{SRS}}{\hat{\beta}_{SRS}}\right) - 2\log\left(1 + \exp\left(-\frac{X - \hat{\alpha}_{SRS}}{\hat{\beta}_{SRS}}\right)\right)\right)$$

$$= -\log\hat{\beta}_{SRS} - \frac{1}{n}\sum_{i=1}^{n}\left(\frac{X - \hat{\alpha}_{SRS}}{\hat{\beta}_{SRS}}\right) - \frac{2}{n}\sum_{i=1}^{n}\log\left(1 + \exp\left(-\frac{X - \hat{\alpha}_{SRS}}{\hat{\beta}_{SRS}}\right)\right). \tag{7}$$

Using equations (5), (6) and (7), the estimator of $I(f, f_0)$, denoted as I_{mn}, is given by

$$I_{mn} = -H(f) - \int_{-\infty}^{\infty} f(x)\log f_0(x;\alpha,\beta)dx,$$

$$= -\frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{n}{2w}\left(X_{i+m:n} - X_{i-m:n}\right)\right) - \frac{1}{n}\sum_{i=1}^{n}\log f_0(X_i;\hat{\alpha}_{SRS},\hat{\beta}_{SRS})$$

$$= -\frac{1}{n}\sum_{i=1}^{n}\log\left(\frac{n}{2w}\left(X_{i+m:n} - X_{i-m:n}\right)\right) + \log\hat{\beta}_{SRS}$$

$$+ \frac{1}{n}\sum_{i=1}^{n}\left(\frac{X_i - \hat{\alpha}_{SRS}}{\hat{\beta}_{SRS}}\right) + \frac{2}{n}\sum_{i=1}^{n}\log\left(1 + \exp\left(-\frac{X_i - \hat{\alpha}_{SRS}}{\hat{\beta}_{SRS}}\right)\right). \tag{8}$$

From equation (8), it is clear that the test statistic I_{mn} is a function of $\left(X_i - \hat{\alpha}_{SRS}\right)/\hat{\beta}_{SRS}$, $i = 1,...,n$. So, the statistics $\left(X_i - \hat{\alpha}_{SRS}\right)/\hat{\beta}_{SRS}$ are location and scale invariant under the null hypothesis. Therefore, the test statistic I_{mn} is location and scale invariant. Hence, a Monte Carlo simulation is used to obtain the critical and power values for any given values of the parameters.

3. Estimators of α, β

We will introduce four different types of estimators for α and β, which are maximum likelihood estimator (mle), moment estimator (moe), order statistic estimator (ose), L-moment estimator (lm) and LQ-moment estimator (lqe). The mles are determined by taking the partial derivatives with respect to α and β, and equating the resulting quantities to zero and with some algebraic manipulation, we obtain the following equations:

$$\frac{\partial L(\alpha,\beta)}{\partial \alpha} = \frac{n}{2} - \sum_{i=1}^{n} \frac{\exp\left(-\frac{X_i - \alpha}{\beta}\right)}{\left(1+\exp\left(-\frac{X_i - \alpha}{\beta}\right)\right)} = 0,$$

and

$$\frac{\partial L(\alpha,\beta)}{\partial \beta} = \frac{n}{2} - \frac{1}{2}\sum_{i=1}^{n}\left(\frac{X_i - \alpha}{\beta}\right) + \sum_{i=1}^{n} \frac{\left(\frac{X_i - \alpha}{\beta}\right)\exp\left(-\frac{X_i - \alpha}{\beta}\right)}{\left(1+\exp\left(-\frac{X_i - \alpha}{\beta}\right)\right)} = 0. \tag{9}$$

No closed form can be found for the maximum likelihood estimators. The moment estimators for logistic distribution are given by

$$\hat{\alpha}_{moe-SRS} = \bar{X}_{SRS} \quad \text{and} \quad \hat{\beta}_{moe-SRS} = \frac{\sqrt{3}}{\pi} S, \tag{10}$$

where $\bar{X}$ and S are the mean and standard deviation for the sample of size n respectively. The p^{th} quantile for the logistic distribution is given by

$$Q(p;\alpha,\beta) = F^{-1}(p) = \alpha + \beta \log\left(\frac{p}{1-p}\right). \tag{11}$$

It is known that the lower, median, upper quartiles, denoted as $F^{-1}(.25)$, $F^{-1}(.5)$, $F^{-1}(.25)$ respectively and distributional limits, $F^{-1}(0)$, $F^{-1}(1)$, gave a feel for the spread of the distribution over the axis. Since the interquartile range (IQR) which is given by

$$IQR = F^{-1}(0.75) - F^{-1}(0.25), \tag{12}$$

is independent of the location parameter, the scale parameter can be estimated using IQR. Based on (11), if we substitute $p = 0.25$ and $p = 0.75$, corresponding to the first and third quartiles, namely lower and upper quartile, we will have

$$F^{-1}(0.25)=\alpha+\beta\log\left(\frac{0.25}{1-0.25}\right)=\alpha-\beta\log(3),$$

and

$$F^{-1}(0.75)=\alpha+\beta\log\left(\frac{0.75}{1-0.75}\right)=\alpha+\beta\log(3),$$

respectively. The two parameters α and β are defined as

$$\alpha=\frac{1}{2}\left(F^{-1}(0.75)+F^{-1}(0.25)\right), \tag{13}$$

and

$$\beta=\frac{1}{2\ \log 3}\left(F^{-1}(0.75)-F^{-1}(0.25)\right), \tag{14}$$

respectively. Accordingly, the two estimators of α and β, denoted as $\hat{\alpha}_{ose-SRS}$ and $\hat{\beta}_{ose-SRS}$, are

$$\hat{\alpha}_{ose-SRS}=\frac{1}{2}\left(\hat{F}^{-1}(0.75)+\hat{F}^{-1}(0.25)\right), \tag{15}$$

and

$$\hat{\beta}_{ose-SRS}=\frac{1}{2\log(3)}\left(\hat{F}^{-1}(0.75)-\hat{F}^{-1}(0.25)\right). \tag{16}$$

The *m*th L-moment estimator, denoted as λ_m, of a probability distribution as explained in Hosking (1990), is given by

$$\lambda_m=\frac{1}{m}\sum_{j=0}^{m-1}(-1)^j\binom{m-1}{j}\mu_{m-j:m},\quad m=1,2,\ldots \tag{17}$$

where $\mu_{i:n}$ is defined as

$$\mu_{i:n}=E\left(X_{i:n}\right)=\int_{-\infty}^{\infty}xf_{i:n}(x), \tag{18}$$

where

$$f_{i:n}(x)=\frac{1}{B(i,n-i+1)}\left(F(x)\right)^{i-1}\left(1-F(x)\right)^{n-i}f(x). \tag{19}$$

Using (17), the first and second L-moments, respectively, are given by

$$\lambda_1=\alpha=\mu_{1:1}\quad\text{and}\quad\lambda_2=\frac{1}{2}\left(\mu_{2:2}-\mu_{1:2}\right)=\beta,$$

where

$$\mu_{2:2}=\alpha+\frac{\beta}{B(2,1)}\int_0^1\log\left(\frac{u}{1-u}\right)udu=\alpha+\beta,$$

and

$$\mu_{1:2}=\alpha+\frac{\beta}{B(1,2)}\int_0^1\log\left(\frac{u}{1-u}\right)(1-u)du=\alpha-\beta.$$

Hence, $\mu_{2:2}-\mu_{1:2}=2\beta$ implies that $\beta=\frac{1}{2}\left(\mu_{2:2}-\mu_{1:2}\right)=\lambda_2$.

Now we calculate the estimated value of $\mu_{1:2}$ and $\mu_{2:2}$ by using

$$M_{s:d;n}=\sum_{i=s}^{s+n-d} a_i\left(s,d,n\right)X_{i:n}, \qquad 1\le s\le d\le n, \tag{20}$$

where $a_i=\binom{i-1}{s-1}\binom{n-i}{d-s}\Big/\binom{n}{d}$. Here, a_i is the probability that in a subsample $X_{1:d},\ldots,X_{d:d}$, drawn without replacement from $X_{1:n},\ldots,X_{n:n}$, as given in David and Nagaraja (2003). For $d=2$ and from (20), the estimated value of $\mu_{1:2}$ and $\mu_{2:2}$, respectively, are given by

$$\hat{\mu}_{1:2}=\frac{1}{\binom{n}{2}}\sum_{i=1}^{n-1}\binom{i-1}{1-1}\binom{n-i}{2-1}X_{i:n}=\frac{2}{n\left(n-1\right)}\sum_{i=1}^{n-1}\left(n-i\right)X_{i:n},$$

and

$$\hat{\mu}_{2:2}=\frac{1}{\binom{n}{2}}\sum_{i=1}^{n-1}\binom{i-1}{1-1}\binom{n-i}{2-2}X_{i:n}=\frac{2}{n\left(n-1\right)}\sum_{i=1}^{n-1} i\ \mathrm{X}_{i+1:n}.$$

Thus,

$$\hat{\mu}_{2:2}-\hat{\mu}_{1:2}=\frac{2}{n\left(n-1\right)}\sum_{i=1}^{n-1}\left(i\right)X_{i+1:n}-\frac{2}{n(n-1)}\sum_{i=1}^{n-1}\left(n-i\right)X_{i:n}$$

$$=\frac{2}{n\left(n-1\right)}\left(\sum_{i=1}^{n-1} i\ \mathrm{X}_{i+1:n}-\sum_{i=1}^{n-1}\left(n-i\right)X_{i:n}\right).$$

The L-moment estimators, denoted as $\hat{\alpha}_{lme-SRS}$ and $\hat{\beta}_{lme-SRS}$, respectively, are given by

$$\hat{\alpha}_{lme-SRS}=\bar{X}_{SRS},$$

and

$$\hat{\beta}_{lme-SRS}=\frac{1}{2}\left(\hat{\mu}_{2:2}-\hat{\mu}_{1:2}\right)=\frac{1}{n\left(n-1\right)}\left(\sum_{i=1}^{n-1} i\ \mathrm{X}_{i+1:n}-\sum_{i=1}^{n-1}\left(n-i\right)X_{i:n}\right). \tag{21}$$

Analogous to L-moments, the *m*th LQ-moments, denoted as η_m, is given by

$$\eta_m=\frac{1}{m}\sum_{j=0}^{m-1}\left(-1\right)^j\binom{m-1}{j}\tilde{\mu}_{m-j:m}, \quad m=1,2,\ldots \tag{22}$$

where $\tilde{\mu}_{i:m}$ is defined by

$$\tilde{\mu}_{i:n}=F^{-1}\left(qbeta(0.50;i,n-i+1)\right). \tag{23}$$

Using (22), the first and second LQ-moments, respectively, are given by

$$\eta_1=\tilde{\mu}_{1:1}=median,$$

and

$$\eta_2=\frac{1}{2}(\tilde{\mu}_{2:2}-\tilde{\mu}_{1:2}). \tag{24}$$

Using (23), $\tilde{\mu}_{1:2}$ and $\tilde{\mu}_{2:2}$, respectively, are given by

$$\tilde{\mu}_{2:2} = F^{-1}\left(qbeta(0.50;2,1)\right)$$
$$= \alpha + \beta \log\left[\frac{qbeta(0.50;2,1)}{1-qbeta(0.50;2,1)}\right] = \alpha + 0.881\beta,$$

and

$$\tilde{\mu}_{1:2} = F^{-1}\left(qbeta(0.50;1,2)\right)$$
$$= \alpha + \beta \log\left[\frac{qbeta(0.50;1,2)}{1-qbeta(0.50;1,2)}\right] = \alpha - 0.881\beta.$$

Hence, $\tilde{\mu}_{2:2} - \tilde{\mu}_{1:2} = 1.762\beta$ implies that $\hat{\beta} = \dfrac{\hat{\mu}_{2:2} - \hat{\mu}_{1:2}}{1.762}$.

The LQ-moments can be estimated in a straightforward manner by estimating the quantiles of order statistics. The estimator of the parameters based on LQ-moments, denoted as $\hat{\alpha}_{lqe-SRS}$ and $\hat{\beta}_{lqe-SRS}$, respectively, are given by

$$\hat{\alpha}_{lqe-SRS} = median, \tag{25}$$

and

$$\hat{\beta}_{lqe-SRS} = \frac{1}{2}\left(\hat{\hat{\mu}}_{2:2} - \hat{\hat{\mu}}_{1:2}\right) = \frac{1}{2}\left(\hat{F}^{-1}\left(qbeta\left(0.50;2,1\right)\right) - \hat{F}^{-1}\left(qbeta\left(0.50;1,2\right)\right)\right)$$
$$= \frac{1}{2}\left(\hat{F}^{-1}(0.7071) - \hat{F}^{-1}(0.2929)\right), \tag{26}$$

where $\hat{F}^{-1}(0.7071)$ and $\hat{F}^{-1}(0.2929)$ are the empirical estimates from data.

4. Algorithm for Power Comparison

Under SRS, to compare the power of I_{mn} when different estimators are considered, the following algorithm is designed to calculate the critical values via Monte Carlo simulation:

a) Let $X_1,\ldots,X_n$ be a random sample from logistic distribution with $\alpha = 0$, $\beta = 1$, and $F_o(x;0,1)$.

b) Given significance level $\alpha^* = 0.05$, random sample of sizes $n = 12$, 18, 24, 36, and window sizes $w = 1,2,3,4$.

c) Estimate the parameters α and β from the sample by maximum likelihood estimators in equation (9), method of moment estimators in equation (10), order statistic estimators in equations (15) and (16), L-moment estimators in equation (21) and LQ-moment estimators in equations (25) and (26).

d) Calculate the test statistic $T = I_{mn}$ using equation (8).

e) Repeat the steps (1)-(4) 20, 000 times to get $T_1, \ldots, T_{20,000}$.

f) Determine the critical value d_{α^*} of T which is given by the $(1-\alpha^*)100^{th}$ quantile of the distribution of $T_1, \ldots, T_{20,000}$.

Next, to calculate the power of T at H, the alternative distribution H_1, the simulation is needed. So, the following algorithm is designed:

a) Let $X_1,...,X_n$ be a random sample from H.

b) Given significance level $\alpha^* = 0.05$, random samples of size $n = 12, 18, 24, 36,$ and window sizes $w = 1, 2, 3, 4$.

c) Estimate the parameters α and β from the sample by maximum likelihood estimators in equation (9), method of moment estimators in equation (10), order statistic estimators in equations (15) and (16), L-moment estimators in equation (21) and LQ-moment estimators in equations (25) and (26).

d) Calculate the test statistic $T = I_{mn}$ using equation (8).

e) Repeat the steps (1)-(4) 20, 000 times to get $T_1, ..., T_{20,000}$.

f) To calculate the power, determine $\pi(T) \approx \frac{1}{20,000}\sum_{t=1}^{20,000} I(T_t > d_{\alpha^*})$, where $I(.)$ stands for indicator function.

5. Simulation Results

In order to assess the performance of the tests statistics under simple random sampling, many alternative distributions are studied. The distributions selected are five symmetric distributions, namely, normal, Laplace, Cauchy, Student *t* and logistic, and also three asymmetric distributions, namely, lognormal, exponential and Weibull.

In this section, to calculate the critical points and power of the test statistics, a Monte Carlo simulation of 20,000 iterations according to the algorithms of Section 4 is conducted. The powers of the tests are computed and compared for different sample sizes, i.e. $n = 12, 18, 24, 36,$ and different alternative distributions, i.e. $Normal(0, 1)$, $Logistic(0, .7)$, $Laplace(0, 1)$, $StudentT(12)$, $StudentT(4)$, $Cauchy(0, 1)$, $Exponential(1)$, $Weibull(\Gamma(1.5), 2)$, $Weibull(2, .5)$ and $Lognormal(-0.2, \sqrt{.4})$. The comparisons are made for the cases when the data are quantified via SRS and RSS.

Based on the simulation, critical values for the level of significance $\alpha^* = 0.05$ are determined and given in Table 7.6. Also, the power and the efficiency of the test given that $\alpha^* = 0.05$ when different distributions are considered under H_1 are reported in Tables 1- 10 and Tables 11 – 21 respectively.

Table 1: Critical values for the test statistics I_{mn} for different sample sizes $n = 12,\ 18,\ 24,\ 36,$ window sizes $w = 1,2,3,4$ and $\alpha^* = 0.05$

		SRS				
n	*w*	*mle*	*moe*	*ose*	*lme*	*lqe*
12	1	.582	.604	.501	.679	.868
	2	.601	.588	.712	.692	.852
	3	.567	.574	.685	.667	.814
	4	.582	.593	.701	.691	.846
18	1	.627	.637	.691	.669	.893
	2	.456	.459	.534	.498	.751
	3	.415	.432	.502	.456	.725
	4	.432	.443	.503	.474	.717
24	1	.553	.561	.606	.582	.835
	2	.382	.382	.442	.403	.695
	3	.339	.353	.408	.365	.643
	4	.340	.343	.402	.364	.653
36	1	.482	.488	.516	.491	.569
	2	.303	.315	.348	.327	.405
	3	.268	.270	.305	.279	.366
	4	.257	.258	.288	.267	.346

Table 2: Power estimates of the I_{mn} test statistic under SRS for Normal and Logistic alternative distributions with different sample sizes $n = 12,\ 18,\ 24,\ 36,$ window sizes $w = 1,2,3,4$ and $\alpha^* = 0.05$

		$Normal(0,\ 1)$					$Logistic(0,\ .7)$				
n	*w*	*mle*	*moe*	*ose*	*lme*	*lqe*	*mle*	*moe*	*ose*	*lme*	*lqe*
12	1	.050	.051	.045	.050	.038	.049	.048	.050	.048	.044
	2	.054	.060	.044	.051	.033	.051	.047	.047	.050	.049
	3	.061	.060	.050	.051	.033	.051	.049	.050	.049	.044
	4	.071	.057	.048	.051	.032	.051	.048	.065	.049	.047
18	1	.052	.052	.050	.052	.030	.048	.050	.052	.046	.045
	2	.052	.060	.048	.057	.031	.051	.050	.046	.047	.049
	3	.075	.068	.051	.060	.031	.046	.047	.049	.048	.051
	4	.068	.072	.058	.068	.031	.052	.051	.068	.047	.048
24	1	.060	.060	.052	.058	.031	.050	.044	.051	.044	.048
	2	.050	.062	.058	.058	.030	.050	.045	.048	.043	.049
	3	.079	.065	.062	.067	.028	.051	.045	.044	.046	.049
	4	.084	.080	.066	.067	.025	.047	.045	.082	.044	.044
36	1	.060	.061	.063	.056	.038	.045	.050	.049	.046	.048
	2	.055	.071	.062	.068	.032	.048	.047	.050	.043	.048
	3	.076	.082	.070	.076	.033	.048	.048	.046	.048	.044
	4	.085	.092	.080	.085	.037	.044	.058	.117	.047	.046

Table 3: Power estimates of the I_{mn} statistics under SRS for Laplace and Student t alternative distributions with different sample sizes $n = 12,\ 18,\ 24,\ 36,$ window sizes $w = 1, 2, 3, 4$ and $\alpha^* = 0.05$

		Laplace(0, 1)					*StudentT*(12)				
n	*w*	*mle*	*moe*	*ose*	*lme*	*lqe*	*mle*	*moe*	*ose*	*lme*	*lqe*
12	1	.051	.051	.065	.047	.101	.050	.046	.050	.044	.032
	2	.051	.043	.076	.051	.102	.051	.047	.042	.043	.036
	3	.051	.048	.076	.050	.101	.052	.049	.044	.038	.033
	4	.049	.046	.074	.041	.109	.057	.048	.044	.052	.038
18	1	.048	.056	.088	.051	.137	.048	.046	.047	.041	.037
	2	.045	.051	.099	.051	.136	.051	.049	.047	.049	.034
	3	.041	.046	.084	.048	.138	.048	.050	.048	.051	.033
	4	.031	.038	.076	.038	.130	.056	.053	.049	.058	.038
24	1	.056	.061	.101	.050	.161	.048	.046	.046	.047	.031
	2	.052	.057	.105	.050	.162	.051	.052	.047	.049	.030
	3	.051	.052	.102	.046	.170	.050	.049	.047	.052	.032
	4	.032	.040	.085	.040	.162	.057	.052	.050	.051	.032
36	1	.074	.075	.123	.046	.199	.051	.050	.047	.051	.033
	2	.075	.085	.141	.052	.210	.055	.050	.048	.049	.033
	3	.066	.080	.148	.046	.211	.051	.053	.046	.052	.032
	4	.052	.062	.128	.046	.206	.065	.054	.052	.060	.030

Table 4: Power estimates of the I_{mn} statistics under SRS for Student t and Cauchy alternative distributions with different sample sizes $n = 12,\ 18,\ 24,\ 36,$ window sizes $w = 1, 2, 3, 4$ and $\alpha^* = 0.05$

		StudentT(4)					*Cauchy*(0, 1)				
n	*w*	*mle*	*moe*	*ose*	*lme*	*lqe*	*mle*	*moe*	*ose*	*lme*	*lqe*
12	1	.052	.058	.061	.053	.067	.612	.401	.470	.310	.510
	2	.059	.052	.068	.055	.075	.326	.402	.490	.330	.523
	3	.058	.060	.068	.054	.072	.309	.362	.491	.325	.537
	4	.056	.054	.062	.054	.074	.270	.312	.480	.321	.523
18	1	.050	.063	.070	.055	.081	.612	.562	.645	.460	.672
	2	.057	.061	.076	.061	.082	.305	.582	.663	.470	.681
	3	.056	.061	.071	.060	.083	.471	.530	.658	.462	.682
	4	.046	.053	.062	.055	.081	.381	.465	.621	.448	.685
24	1	.058	.068	.070	.052	.084	.608	.685	.743	.630	.778
	2	.060	.070	.078	.056	.088	.630	.704	.772	.650	.785
	3	.056	.067	.076	.055	.090	.632	.688	.753	.645	.776
	4	.051	.054	.063	.048	.091	.534	.632	.730	.602	.774
36	1	.062	.072	.076	.072	.101	.790	.835	.870	.820	.886
	2	.063	.081	.083	.073	.108	.623	.855	.890	.840	.894
	3	.063	.076	.082	.075	.111	.822	.845	.893	.842	.893
	4	.051	.061	.071	.060	.095	.785	.824	.872	.827	.885

Table 5: Power estimates of the I_{mn} statistics under SRS for Gamma and Weibull alternative distributions with different sample sizes $n = 12,\ 18,\ 24,\ 36,$ window sizes $w = 1, 2, 3, 4$ and $\alpha^* = 0.05$

		$Exponential(1)$					$Weibull(\Gamma(1.5),\ 2)$				
n	*w*	*mle*	*moe*	*ose*	*lme*	*lqe*	*mle*	*moe*	*ose*	*lme*	*lqe*
12	1	.313	.326	.272	.372	.169	.460	.441	.361	.511	.272
	2	.420	.413	.301	.390	.192	.522	.552	.421	.524	.272
	3	.301	.439	.319	.420	.182	.540	.575	.427	.552	.283
	4	.492	.476	.322	.453	.183	.581	.598	.425	.580	.281
18	1	.518	.518	.418	.555	.227	.648	.725	.575	.680	.355
	2	.652	.713	.565	.649	.314	.829	.831	.723	.774	.426
	3	.766	.701	.550	.675	.237	.811	.813	.704	.801	.402
	4	.710	.719	.576	.711	.271	.812	.821	.719	.813	.393
24	1	.654	.669	.590	.691	.301	.792	.806	.738	.782	.487
	2	.781	.802	.701	.740	.285	.882	.881	.819	.852	.472
	3	.831	.841	.765	.790	.311	.903	.902	.851	.903	.490
	4	.862	.861	.782	.863	.314	.917	.926	.872	.923	.532
36	1	.836	.855	.802	.821	.711	.916	.918	.893	.891	.889
	2	.903	.905	.877	.872	.792	.935	.935	.928	.922	.893
	3	.928	.926	.916	.922	.830	.939	.938	.935	.937	.909
	4	.911	.961	.923	.949	.864	.958	.952	.951	.957	.924

Table 6: Power estimates of the I_{mn} statistics under SRS for Weibull and Lognormal alternative distributions with different sample sizes $n = 12,\ 18,\ 24,\ 36,$ window sizes $w = 1, 2, 3, 4$ and $\alpha^* = 0.05$

		$Weibull(2,\ .5)$					$Lognormal(-0.2,\ \sqrt{.4})$				
n	*w*	*mle*	*moe*	*ose*	*lme*	*lqe*	*mle*	*moe*	*ose*	*lme*	*lqe*
12	1	.191	.059	.058	.058	.050	.173	.183	.154	.220	.119
	2	.273	.076	.066	.065	.041	.310	.236	.183	.244	.115
	3	.318	.099	.063	.074	.043	.327	.256	.201	.253	.126
	4	.410	.096	.060	.086	.041	.329	.293	.207	.285	.123
18	1	.220	.092	.081	.111	.040	.284	.275	.202	.292	.147
	2	.377	.112	.086	.115	.041	.291	.372	.262	.311	.138
	3	.378	.119	.101	.115	.037	.426	.405	.301	386	.123
	4	.380	.125	.102	.122	.035	.423	.432	.292	.413	.137
24	1	.251	.102	.093	.155	.032	.330	.371	.333	.402	.152
	2	.393	.135	.105	.175	.035	.501	.515	.382	.521	.152
	3	.429	.144	.109	.181	.034	.584	.572	.432	.551	.155
	4	.474	.193	.126	.186	.033	.582	.597	.444	.601	.171
36	1	.327	.139	.122	.240	.058	.481	.504	.414	.573	.362
	2	.461	.199	.168	.240	.073	.665	.717	.609	.670	.565
	3	.589	.241	.195	.263	.084	.746	.772	.642	.751	.513
	4	.610	.280	.225	.287	.087	.726	.755	.641	.736	.478

From the above tables, we make the following remarks:

a) The percentage points for the test decrease as the sample size n increases.

b) The power increases as the sample size n increases.

c) The power increases as the window size m increases.

d) In the case of moment estimator, the test has the highest power for Normal, Logistic, Student $T(12)$, and Exponential distributions.

e) In the case of L-Moment estimator, the test has the highest power for $Weibull(\Gamma(1.5),\ 2)$ and lognormal distributions.

f) In the case of Maximum Likelihood estimator, the test has the highest power for $Weibull(2,\ .5)$ distribution.

g) When the estimators are compared, the test considered has the lowest power for $Weibull(2,\ .5)$, and highest power for Cauchy.

6. **Conclusion**

An accurate estimation of parameters of the logistic distribution in statistical analysis is importance. In this paper, we have introduced a test statistic of goodness of fit for logistic distribution based on Kullback-Leibler information measure. We considered ten different distributions under the alternative hypothesis. It is found that the test statistics based on estimators found by moment and order statistic methods have the highest power, except for Weibull and Lognormal distributions. In the case of Cauchy, the test is found to have the highest power for all estimators but the test has the lowest power for $Weibull(2,\ .5)$. The theory developed could be extended easily to other distributions. Also, we can apply the test statistics considered under RSS, extreme RSS and median RSS.

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