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INTUITIONISTIC FUZZY PARAMETERISED FUZZY SOFT SET

(Set Lembut Kabur Berparameter Kabur Berintuisi)

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ABSTRACT

In this paper, the definition of intuitionistic fuzzy parameterised fuzzy soft set (*ifpfs*-sets) is introduced with their properties. Two operations on *ifpfs*-set, namely union and intersection are introduced. Also, some examples for these operations are given. The definition of *ifpfs*-aggregation operator which forms *ifpfs*-decision making method is introduced.

Keywords: **s**oft set; fuzzy soft set; fuzzy parameterised fuzzy soft set; *ifpfs*-sets; *ifpfs*-aggregation operator; *ifpfs*-decision making method

ABSTRAK

Dalam makalah ini, diperkenalkan takrif set lembut kabur berparameter kabur berintuisi (set-*lkbki*) beserta sifat-sifatnya. Turut diperkenalkan dua operasi ke atas set-*lkbki*, iaitu kesatuan dan persilangan. Hasil-hasil yang berkaitan dan contoh untuk operasi ini diberikan. Diperkenalkan juga takrif pengoperasi pengagregatan-*lkbki* yang membentuk kaedah pembuatan keputusan-*lkbki*.

Kata kunci: **s**et lembut; set lembut kabur; set lembut kabur berparameter kabur; set-*lkbki*; pengoperasi pengagregatan-*lkbki*; kaedah pembuatan keputusan-*lkbki*

1. Introduction

In most of the real life problems in social sciences, engineering, medical sciences, economics, the data involved are imprecise in nature. The solutions of such problems involve the use of mathematical principles based on uncertainty and imprecision. A number of theories have been proposed for dealing with uncertainties in an efficient way, such as fuzzy set (Zadeh 1965), intuitionistic fuzzy set (Atanassov 1986), vague set (Gau & Buchrer 1993) and theory of interval mathematics (Atanassov 1994). In 1999, Molodtsov initiated the theory of soft sets as a new mathematical tool for dealing with uncertainties which traditional mathematical tools cannot handle. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science and medical science.

Maji *et al.* (2003) have further studied the theory of soft sets and used this theory to solve some decision making problems. They also introduced the concept of fuzzy soft set (Maji *et al.* 2001a), a more general concepts, which is a combination of fuzzy set and soft set. Moreover, they introduced the idea of an intuitionistic fuzzy soft set (Maji *et al.* 2001b) which is a combination of intuitionistic fuzzy set and soft set. Cağman *et al.* (2010) introduced and studied the fuzzy parameterised fuzzy soft sets (*fpfs*-sets) and their operations. Alkhazaleh *et al.* (2011) introduced the notion of fuzzy parameterised interval-valued fuzzy soft set and Bashir and Salleh (2012) introduced the notion of fuzzy parameterised soft expert set. In this paper, we use the notion of fuzzy parameterised fuzzy soft sets of Cağman *et al.* to define intuitionistic fuzzy parameterised fuzzy soft sets (*ifpfs*-sets). We also define their operations namely union and intersection, and discuss their properties.

2. Preliminary

In this section we recall some definitions which are required in this paper.

Definition 2.1. (Molodtsov 1999). A pair (F, E) is called a soft set over U, where F is a mapping given by $F : E \to P(U)$.

In other words, a soft set over *U* is a parameterised family of subsets of the universe *U*.

Definition 2.2. (Maji *et al.* 2001a). Let *U* be an initial universal set and let *E* be a set of parameters. Let I^U denote the power set of all fuzzy subsets of *U*. Let $A \subseteq E$. A pair(*F, E*) is called a *fuzzy soft set* over *U* where *F* is a mapping given by $F : A \to I^U$.

Definition 2.3. (Cağman *et al.* 2010**).** Let *U* be an initial universe, *E* the set of all parameters and *X* a fuzzy set over *E* with membership function : $\mu_X : E \to [0,1]$. and let γ_X be a fuzzy set over *U* for all $x \in E$. Then, a fuzzy parameterised fuzzy soft set (*fpfs*-set) Γ_X over *U* is a set defined by a function γ_X representing a mapping $\gamma_X : E \to F(U)$ such that $\gamma_X(x) = \varnothing$ if $\mu_X(x) = 0$. Here, γ_X is called a fuzzy approximate function of the *fpfs*-set Γ_X, and the value $\gamma_X(x)$ is a set called *x*-element of the *fpfs*-set for all $x \in E$. Thus, an *fpfs-s*et Γ_X over *U* can be represented by the set of ordered pairs

$$\Gamma_X = \left\{ \left(\mu_X(x)/x, \gamma_X(x) \right) : x \in E, \gamma_X(x) \in F(U), \mu_X(x) \in [0,1] \right\}.$$

3. Intuitionistic fuzzy parameterised fuzzy soft sets

In this section, we introduce the definition of an intuitionistic fuzzy parameterised fuzzy soft set (*ifpfs*-set). Also, we introduce some operations on the intuitionitstic fuzzy parameterised fuzzy soft sets, namely union and intersection, and discuss their properties.

Definition 3.1. Let *U* be an initial universe, *E* the set of all parameters and *X* an intuitionistic fuzzy set over *E* with membership function $\mu_X : E \to [0,1]$, non-membership function $\nu_X : E \to [0,1]$ and let $\gamma_X(x)$ be a fuzzy set over *U* for all $x \in E$. Then an intuitionistic fuzzy parameterised fuzzy soft set (*ifpfs*-set) Γ_X over *U* is a set defined by a function $\gamma_X : E \to F(U)$, where *F*(U) is a set of fuzzy sets over *U,* such that $\gamma_X(x) = \varnothing$ if $\mu_X(x) = 0$ and $\nu_X(x) = 0$. Here, γ_X is called a fuzzy approximate function of the *ifpfs*-set Γ_X, and the value $\gamma_X(x)$ is a fuzzy set called an *x*-element of the *ifpfs*-set, for all $x \in E$. Thus, an *ifpfs*-set Γ_X over *U* can be represented by the set of ordered pairs

$$\Gamma_X = \left\{ \left(\frac{x}{(\mu_X(x), \nu_X(x))}, \gamma_X(x) \right) : x \in E, \gamma_X(x) \in F(U), \mu_X(x), \nu_X(x) \in [0,1] \right\}.$$

It must be noted that the set of all *ifpfs*-sets over *U,* will be denoted by $IFPFS(U)$.

Example 3.1. Let $U=\{u_1, u_2, u_3\}$ be a universal set and $E=\{x_1, x_2, x_3\}$ be a set of parameters. Suppose $X=\left\{\frac{x_1}{(0.2, 0.6)}, \frac{x_2}{(0.5, 0.3)}, \frac{x_3}{(1, 0)}\right\}$ and

$$\gamma_X(x_1)=\left\{\frac{u_1}{0.5}, \frac{u_2}{0}, \frac{u_3}{0.3}\right\}, \gamma_X(x_2)=\left\{\frac{u_1}{0.1}, \frac{u_2}{0.3}, \frac{u_3}{0.6}\right\}, \gamma_X(x_3)=\left\{\frac{u_1}{0}, \frac{u_2}{0.1}, \frac{u_3}{0.7}\right\}.$$

Then the *ifpfs*-set Γ_X is given as follows:

$$\Gamma_X=\left\{\left(\frac{x_1}{(0.2, 0.6)}, \left\{\frac{u_1}{0.5}, \frac{u_2}{0}, \frac{u_3}{0.3}\right\}\right), \left(\frac{x_2}{(0.5, 0.3)}, \left\{\frac{u_1}{0.1}, \frac{u_2}{0.3}, \frac{u_3}{0.6}\right\}\right),\right.$$
$$\left.\left(\frac{x_3}{(1, 0)}, \left\{\frac{u_1}{0}, \frac{u_2}{0.1}, \frac{u_3}{0.7}\right\}\right)\right\}.$$

Definition 3.2. Let $\Gamma_X \in IFPFS(U)$. If $\gamma_X(x)=\varnothing$ for all $x \in E$, then Γ_X is called an *X*-empty *ifpfs*-set, denoted by Γ_{ϕ_X}.

If $X=\varnothing$, then the *X*-empty *ifpfs*-set (Γ_{ϕ_X}) is called an empty *ifpfs*-set, denoted by Γ_ϕ.

Definition 3.3. Let $\Gamma_X \in IFPFS(U)$. If $\gamma_X(x)=U$ for all $x \in X$, then Γ_X is called an *X*-universal *ifpfs*-set, denoted by $\Gamma_{\tilde{X}}$.

If $X=E$, then the *X*-universal *ifpfs*-set $(\Gamma_{\tilde{X}})$ is called a universal *ifpfs*-set, denoted by $\Gamma_{\tilde{E}}$.

Example 3.2. Assume that $U=\{u_1, u_2, u_3\}$ is a universal set and $E=\{x_1, x_2, x_3\}$ is a set of all parameters.

Let $X=\left\{\frac{x_1}{(1, 0)}, \frac{x_2}{(0.2, 0.6)}, \frac{x_3}{(0.7, 0.2)}\right\}$ and $\gamma_X(x_1)=\gamma_X(x_2)=\gamma_X(x_3)=\left\{\frac{u_1}{0}, \frac{u_2}{0}, \frac{u_3}{0}\right\}$.

Then the *X*-empty *ifpfs*-set

$$\Gamma_{\phi_X}=\left\{\left(\frac{x_1}{(1, 0)}, \left\{\frac{u_1}{0}, \frac{u_2}{0}, \frac{u_3}{0}\right\}\right), \left(\frac{x_2}{(0.2, 0.6)}, \left\{\frac{u_1}{0}, \frac{u_2}{0}, \frac{u_3}{0}\right\}\right), \left(\frac{x_3}{(0.7, 0.2)}, \left\{\frac{u_1}{0}, \frac{u_2}{0}, \frac{u_3}{0}\right\}\right)\right\}.$$

Suppose $X=\left\{\frac{x_1}{(0, 0)}, \frac{x_2}{(0, 0)}, \frac{x_3}{(0, 0)}\right\}$ and $\gamma_X(x_1)=\gamma_X(x_2)=\gamma_X(x_3)=\left\{\frac{u_1}{0}, \frac{u_2}{0}, \frac{u_3}{0}\right\}$.

Then the empty *ifpfs*-set is

$$\Gamma_\phi=\left\{\left(\frac{x_1}{(0, 0)}, \left\{\frac{u_1}{0}, \frac{u_2}{0}, \frac{u_3}{0}\right\}\right), \left(\frac{x_2}{(0, 0)}, \left\{\frac{u_1}{0}, \frac{u_2}{0}, \frac{u_3}{0}\right\}\right), \left(\frac{x_3}{(0, 0)}, \left\{\frac{u_1}{0}, \frac{u_2}{0}, \frac{u_3}{0}\right\}\right)\right\}.$$

Let $X=\left\{\frac{x_1}{(1, 0)}, \frac{x_2}{(0.2, 0.6)}, \frac{x_3}{(0.7, 0.2)}\right\}$ and $\gamma_X(x_1)=\gamma_X(x_2)=\gamma_X(x_3)=\left\{\frac{u_1}{1}, \frac{u_2}{1}, \frac{u_3}{1}\right\}$.

Then an X-universal *ifpfs*-set

$$\Gamma_{\tilde{X}} = \left\{ \left(\frac{x_1}{(1,0)}, \left\{ \frac{u_1}{1}, \frac{u_2}{1}, \frac{u_3}{1} \right\} \right), \left(\frac{x_2}{(0.2, 0.6)}, \left\{ \frac{u_1}{1}, \frac{u_2}{1}, \frac{u_3}{1} \right\} \right), \left(\frac{x_3}{(0.7, 0.2)}, \left\{ \frac{u_1}{1}, \frac{u_2}{1}, \frac{u_3}{1} \right\} \right) \right\}.$$

Suppose $X = \left\{ \frac{x_1}{(1,0)}, \frac{x_2}{(1,0)}, \frac{x_3}{(1,0)} \right\}$ and $\gamma_X(x_1) = \gamma_X(x_2) = \gamma_X(x_3) = \left\{ \frac{u_1}{1}, \frac{u_2}{1}, \frac{u_3}{1} \right\}$.

Then the universal *ifpfs*-set is

$$\Gamma_{\tilde{E}} = \left\{ \left(\frac{x_1}{(1,0)}, \left\{ \frac{u_1}{1}, \frac{u_2}{1}, \frac{u_3}{1} \right\} \right), \left(\frac{x_2}{(1,0)}, \left\{ \frac{u_1}{1}, \frac{u_2}{1}, \frac{u_3}{1} \right\} \right), \left(\frac{x_3}{(1,0)}, \left\{ \frac{u_1}{1}, \frac{u_2}{1}, \frac{u_3}{1} \right\} \right) \right\}.$$

Definition 3.4. Let $\Gamma_X, \Gamma_Y \in IFPFS(U)$. Γ_X is called an intuitionistic fuzzy parameterised fuzzy soft subset of Γ_Y, denoted by $\Gamma_X \subseteq \Gamma_Y$, if:

i. is an intuitionistic fuzzy subset of Y.
ii. $\gamma_x(x)$ is a fuzzy subset of $\gamma_Y(x)$.

Proposition 3.1. *Let* $\Gamma_X, \Gamma_Y \in IFPFS(U)$. *Then*

i. $\Gamma_X \tilde{\subseteq} \Gamma_{\tilde{E}}$
ii. $\Gamma_{\phi_X} \tilde{\subseteq} \Gamma_X$
iii. $\Gamma_\phi \tilde{\subseteq} \Gamma_X$
iv. $\Gamma_\phi \tilde{\subseteq} \Gamma_X$
v. $\Gamma_X \tilde{\subseteq} \Gamma_X$
vi. $\Gamma_X \tilde{\subseteq} \Gamma_Y$ *and* $\Gamma_Y \tilde{\subseteq} \Gamma_Z \Rightarrow \Gamma_X \tilde{\subseteq} \Gamma_Z$

Proof. The proof is straightforward.

Definition 3.5. Let $\Gamma_X, \Gamma_Y \in IFPFS(U)$. The union of Γ_X and Γ_Y denoted by $\Gamma_X \tilde{\cup} \Gamma_Y$, is an *ifpfs*-set Γ_Z given by

$$Z = X \underset{S-norm}{\cup} Y,$$

$$\gamma_Z(x) = \gamma_X(x) \underset{s-norm}{\cup} \gamma_Y(x), \forall x \in E.$$

Here, S is an S-norm (Fathi 2007) and s is an s-norm (Lowen 1996; Wang 1997).

Example 3.3. Let $U = \{u_1, u_2, u_3\}$ be a universal set and let $E = \{x_1, x_2, x_3\}$ be a set of parameters. Consider the *ifpfs*-set

$$\Gamma_X = \left\{ \left(\frac{x_1}{(0.3, 0.4)}, \left\{ \frac{u_1}{0.1}, \frac{u_2}{0.3}, \frac{u_3}{0.5} \right\} \right), \left(\frac{x_2}{(0.7, 0.1)}, \left\{ \frac{u_1}{0.3}, \frac{u_2}{0}, \frac{u_3}{0.4} \right\} \right), \right.$$
$$\left. \left(\frac{x_3}{(0.2, 0.6)}, \left\{ \frac{u_1}{0.6}, \frac{u_2}{0.2}, \frac{u_3}{0} \right\} \right) \right\},$$

and

$$\Gamma_Y = \left\{ \left(\frac{x_1}{(0.1, 0.4)}, \left\{ \frac{u_1}{0.4}, \frac{u_2}{0}, \frac{u_3}{0.3} \right\} \right), \left(\frac{x_2}{(0.3, 0.3)}, \left\{ \frac{u_1}{0.5}, \frac{u_2}{0.1}, \frac{u_3}{0.3} \right\} \right), \left(\frac{x_3}{(0.1, 0.5)}, \left\{ \frac{u_1}{0.2}, \frac{u_2}{0.6}, \frac{u_3}{0} \right\} \right) \right\}.$$

By applying the Atanassov union (basic intuitionistic fuzzy union) for S-norm and the basic fuzzy union, max, for s-norm, we have

$$\Gamma_X \underset{\substack{\text{Atan}\\ \text{max}}}{\cup} \Gamma_Y = \left\{ \left(\frac{x_1}{(0.3, 0.4)}, \left\{ \frac{u_1}{0.4}, \frac{u_2}{0.3}, \frac{u_3}{0.5} \right\} \right), \left(\frac{x_2}{(0.7, 0.1)}, \left\{ \frac{u_1}{0.5}, \frac{u_2}{0.1}, \frac{u_3}{0.4} \right\} \right), \left(\frac{x_3}{(0.2, 0.5)}, \left\{ \frac{u_1}{0.6}, \frac{u_2}{0.6}, \frac{u_3}{0} \right\} \right) \right\}.$$

Proposition 3.2. *Let* $\Gamma_X, \Gamma_Y, \Gamma_Z \in IFPFS(U)$. *Then*

i. $\Gamma_X \tilde{\cup} \Gamma_X = \Gamma_X$

ii. $\Gamma_{\phi_X} \tilde{\cup} \Gamma_X = \Gamma_X$

iii. $\Gamma_X \tilde{\cup} \Gamma_\phi = \Gamma_X$

iv. $\Gamma_X \tilde{\cup} \Gamma_{\tilde{E}} = \Gamma_{\tilde{E}}$

v. $\Gamma_X \tilde{\cup} \Gamma_Y = \Gamma_Y \tilde{\cup} \Gamma_X$

vi. $(\Gamma_X \tilde{\cup} \Gamma_Y) \tilde{\cup} \Gamma_Z = \Gamma_X \tilde{\cup} (\Gamma_Y \tilde{\cup} \Gamma_Z)$

Proof. The proof can be easily obtained from Definition 3.5.

Definition 3.6. Let $\Gamma_X, \Gamma_Y \in IFPFS(U)$. The intersection of Γ_X and Γ_Y denoted by $\Gamma_X \tilde{\cap} \Gamma_Y$, is an *ifpfs*-set Γ_Z, where

$$Z = X \underset{T-norm}{\cap} Y,$$

$$\gamma_Z(x) = \gamma_X(x) \underset{t-norm}{\cap} \gamma_Y(x), \forall x \in E,$$

Here, T is a T-norm (Fathi 2007) and t is a t-norm (Lowen 1996; Wang 1997).

Example 3.4. Let $U = \{u_1, u_2, u_3\}$ be a universal set and let $E = \{x_1, x_2, x_3\}$ be a set of parameters. Consider the *ifpfs*-set

$$\Gamma_X = \left\{ \left(\frac{x_1}{(0.7, 0.1)}, \left\{ \frac{u_1}{0.1}, \frac{u_2}{0}, \frac{u_3}{0.4} \right\} \right), \left(\frac{x_2}{(0.1, 0.2)}, \left\{ \frac{u_1}{0.5}, \frac{u_2}{0.2}, \frac{u_3}{0} \right\} \right), \left(\frac{x_3}{(0.4, 0.1)}, \left\{ \frac{u_1}{0.5}, \frac{u_2}{0.1}, \frac{u_3}{0.3} \right\} \right) \right\},$$

$$\Gamma_Y = \left\{ \left(\frac{x_1}{(0.1, 0.6)}, \left\{ \frac{u_1}{0}, \frac{u_2}{0.1}, \frac{u_3}{0.4} \right\} \right), \left(\frac{x_2}{(0.3, 0.2)}, \left\{ \frac{u_1}{0.2}, \frac{u_2}{0.5}, \frac{u_3}{0.1} \right\} \right), \right.$$
$$\left. \left(\frac{x_3}{(0.8, 0)}, \left\{ \frac{u_1}{0.6}, \frac{u_2}{0.2}, \frac{u_3}{0.1} \right\} \right) \right\}.$$

By applying the Atanassov intersection (basic intuitionistic fuzzy intersection) for T-norm and the basic fuzzy intersection, min, for t-norm, we have

$$\Gamma_X \underset{\substack{\text{Atan} \\ \text{min}}}{\cap} \Gamma_Y = \left\{ \left(\frac{x_1}{(0.1, 0.6)}, \left\{ \frac{u_1}{0}, \frac{u_2}{0}, \frac{u_3}{0.4} \right\} \right), \left(\frac{x_2}{(0.1, 0.2)}, \left\{ \frac{u_1}{0.2}, \frac{u_2}{0.2}, \frac{u_3}{0} \right\} \right), \right.$$
$$\left. \left(\frac{x_3}{(0.4, 0.1)}, \left\{ \frac{u_1}{0.5}, \frac{u_2}{0.1}, \frac{u_3}{0.1} \right\} \right) \right\}.$$

Proposition 3.3. *Let* $\Gamma_X, \Gamma_Y, \Gamma_Z \in IFPFS(U)$*. Then the following results hold:*

i. $\Gamma_X \tilde{\cap} \Gamma_X = \Gamma_X$

ii. $\Gamma_{\phi_X} \tilde{\cap} \Gamma_X = \Gamma_X$

iii. $\Gamma_X \tilde{\cap} \Gamma_\phi = \Gamma_X$

iv. $\Gamma_X \tilde{\cap} \Gamma_{\tilde{E}} = \Gamma_{\tilde{E}}$

v. $\Gamma_X \tilde{\cap} \Gamma_Y = \Gamma_Y \tilde{\cap} \Gamma_X$

vi. $(\Gamma_X \tilde{\cap} \Gamma_Y) \tilde{\cap} \Gamma_Z = \Gamma_X \tilde{\cap} (\Gamma_Y \tilde{\cap} \Gamma_Z)$

Proof. The proof can be easily obtained from Definition 3.6.

Proposition 3.4. *Let* $\Gamma_X, \Gamma_Y, \Gamma_Z \in IFPFS(U)$*. Then the following results hold:*

i $\Gamma_X \tilde{\cup} (\Gamma_Y \tilde{\cap} \Gamma_Z) = (\Gamma_X \tilde{\cup} \Gamma_Y) \tilde{\cap} (\Gamma_X \tilde{\cup} \Gamma_Z)$

ii $\Gamma_X \tilde{\cap} (\Gamma_Y \tilde{\cup} \Gamma_Z) = (\Gamma_X \tilde{\cap} \Gamma_Y) \tilde{\cup} (\Gamma_X \tilde{\cap} \Gamma_Z)$

Proof. (i) From the fact that the union and the intersection of fuzzy set and intuitionistic fuzzy set satisfy distributive law, we have

$$X \underset{A\tan}{\cup} (Y \underset{A\tan}{\cap} Z) = (X \underset{A\tan}{\cup} Y) \underset{A\tan}{\cap} (X \underset{A\tan}{\cup} Z),$$
$$\gamma_{X \tilde{\cup} (Y \tilde{\cap} Z)}(x) = \gamma_{(X \tilde{\cup} Y) \tilde{\cap} (X \tilde{\cup} Z)}(x).$$

Therefore, $\Gamma_X \tilde{\cup} (\Gamma_Y \tilde{\cap} \Gamma_Z) = \Gamma_{X \cup (Y \cap Z)} = \Gamma_{(X \cup Y) \cap (X \cup Z)} = (\Gamma_X \tilde{\cup} \Gamma_Y) \tilde{\cap} (\Gamma_X \tilde{\cup} \Gamma_Z).$

Likewise, the proof of (ii) can be made similarly.

4. *ifpfs*-Aggregation Operator

In this section, we define an aggregate fuzzy set of an *ifpfs*-set. We also define *ifpfs*-aggregation operator that produces an aggregate fuzzy set from an *ifpfs*-set and its fuzzy parameter set.

Definition 4.1. Let $\Gamma_X \in IFPFSS(U)$. Then an *ifpfs*-aggregation operator, denoted by $IFPFS_{agg}$, is defined by

$$IFPFS_{agg} : F(E) \times IFPFSS(U) \to F(U),$$
$$IFPFS_{agg}\left(X, \Gamma_X\right) = \Gamma^*_X,$$

where $\Gamma^*_X = \left\{ \frac{u}{\mu_{\Gamma^*_X}(u)} : u \in U \right\}$

which is a fuzzy set over *U*. The value Γ^*_X is called an aggregate fuzzy set of the Γ_X. Here, the membership degree $\mu_{\Gamma^*_X}(u)$ of *u* is defined as follows:

$$\mu_{\Gamma^*_X}(u) = \frac{1}{|E|} \sum_{x \in E} \left(\mu_X(x) - \nu_X(x)\right) \mu_{\gamma_X(x)}(u),$$

where $|E|$ is the cardinality of *E*.

5. *ifpfs*-decision Making Method

The approximate functions of an *ifpfs*-set are fuzzy. The $IFPFS_{agg}$ on the fuzzy sets is an operation by which several approximate functions of an *ifpfs*-set are combined to produce a single fuzzy set that is the aggregate fuzzy set of the *ifpfs*-set. Then, it may be necessary to choose the best single crisp alternative from this set. Therefore, we can construct an *ifpfs*-decision making method by the following algorithm.

Step 1: Construct an *ifpfs*-set Γ_X over *U*.

Step 2: Find the aggregate fuzzy set Γ^*_X of Γ_X.

Step 3: Find the largest membership grade max $\mu_{\Gamma^*_X}(u)$.

Example 5.1. Assume that a company wants to fill a position. There are seven candidates who form the set of alternatives, $U = \{u_1, u_2, u_3, u_4, u_5, u_6, u_7\}$. The hiring committee considers a set of parameters, $E = \{x_1, x_2, x_3, x_4, x_5\}$. The parameters x_i $(i = 1, 2, 3, 4, 5)$ stand for "experience", "computer knowledge", "young age", "good speaking" and "friendly", respectively.

After a serious discussion each candidate is evaluated from the point of view of the goals and the constraint according to a chosen subset

$$X = \left\{ \frac{x_1}{(0.7, 0.1)}, \frac{x_2}{(0.3, 0)}, \frac{x_3}{(0.3, 0.1)}, \frac{x_4}{(0.6, 0.1)}, \frac{x_5}{(0.2, 0.4)} \right\} \text{ of } E.$$

Finally, the committee constructs the following *ifpfs*-set over *U*.

Step 1: Let the constructed *ifpfs*-set Γ_X, be as follows:

$$\Gamma_X = \left\{\left(\frac{x_1}{(0.7, 0.1)}, \left\{\frac{u_1}{0}, \frac{u_2}{0.3}, \frac{u_3}{0.4}, \frac{u_4}{0.1}, \frac{u_5}{0.9}, \frac{u_6}{0}, \frac{u_7}{0.7}\right\}\right),\right.$$

$$\left(\frac{x_2}{(0.3, 0)}, \left\{\frac{u_1}{0.1}, \frac{u_2}{0.2}, \frac{u_3}{0.5}, \frac{u_4}{0}, \frac{u_5}{0.3}, \frac{u_6}{0.6}, \frac{u_7}{0}\right\}\right),$$

$$\left(\frac{x_3}{(0.3, 0.1)}, \left\{\frac{u_1}{0.4}, \frac{u_2}{0.4}, \frac{u_3}{0.9}, \frac{u_4}{0.1}, \frac{u_5}{0.6}, \frac{u_6}{0}, \frac{u_7}{0.3}\right\}\right),$$

$$\left(\frac{x_4}{(0.6, 0.1)}, \left\{\frac{u_1}{0.2}, \frac{u_2}{0.5}, \frac{u_3}{0.1}, \frac{u_4}{0.7}, \frac{u_5}{0}, \frac{u_6}{1}, \frac{u_7}{0}\right\}\right),$$

$$\left.\left(\frac{x_5}{(0.2, 0.4)}, \left\{\frac{u_1}{0}, \frac{u_2}{1}, \frac{u_3}{0.6}, \frac{u_4}{0.2}, \frac{u_5}{0.3}, \frac{u_6}{0.1}, \frac{u_7}{0.4}\right\}\right)\right\}.$$

Step 2: The aggregate fuzzy set can be found as

$$\Gamma_X^* = \left\{\frac{u_1}{0.042}, \frac{u_2}{0.074}, \frac{u_3}{0.1}, \frac{u_4}{0.078}, \frac{u_5}{0.138}, \frac{u_6}{0.132}, \frac{u_7}{0.08}\right\}.$$

Step 3: Finally, the largest membership grade can be chosen by max $\mu_{\Gamma_X^*}(u_5) = 0.138$.

This means that the candidate u_5 has the largest membership grade, hence he is selected for the job.

6. Conclusion

In this paper, we have introduced the concept of intuitionistic fuzzy parameterised fuzzy soft set and their properties. Moreover, we have defined some operations on *ifpfs*-set which are union and intersection, with some propositions on *ifpfs*-set. The definition of *ifpfs*-aggregation operator is introduced with application of this operator in decision making problems.

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