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ROUGHNESS AND SIMILARITY MEASURE OF ROUGH NEUTROSOPHIC MULTISETS USING VECTORIAL MODEL OF INFORMATION

(Ukuran Kekasaran dan Keserupaan bagi Multiset Neutrosopik Kasar Menggunakan Model Maklumat Vektoran)

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ABSTRACT

The roughness and similarity measure for two different information in the same universal set is useful in explaining the strength and completeness of the information given. Then, for rough neutrosophic multisets environment, the lower and upper approximation was a concerned property to study in explaining the roughness of the information needed. Meanwhile, the vectorial models of information which are cosine measure and dice measure represent the result for the similarity measure of rough neutrosophic multisets. The finding of this set theory gives a new generalization about similarity measure for multiple information involving indeterminacy information in the same environment. Besides that, the rough neutrosophic multisets theory also applicable set-in decision making for medical diagnosis. The comparison result showed that the roughness approximation of information is essential to get the best result in a close similarity measure.

Keywords: rough neutrosophic multisets; roughness; similarity measure

ABSTRAK

Pengukuran kekasaran dan keserupaan untuk dua maklumat yang berbeza dalam set sejagat yang sama adalah penting untuk menjelaskan kekuatan dan kesempurnaan maklumat yang diberikan. Bagi multiset neutrosopik kasar, nilai penghampiran bawah dan atas adalah sifat yang bersangkutan dalam menjelaskan kekasaran maklumat yang diperlukan. Sementara itu, model maklumat vektoran, iaitu ukuran kosinus dan ukuran dadu mewakili keputusan untuk ukuran keserupaan multiset neutrosopik kasar. Penemuan teori set ini memberikan penjelmaan baharu mengenai ukuran keserupaan untuk pelbagai maklumat yang melibatkan maklumat ketidakpastian dalam set sejagat yang sama. Di samping itu, teori multiset neutrosofik kasar juga diaplikasikan dalam membuat keputusan untuk diagnosis perubatan. Hasil perbandingan menunjukkan bahawa penganggaran kekasaran maklumat adalah penting untuk mendapatkan hasil terbaik bagi ukuran keserupaan yang paling hampir.

Kata kunci: multiset neutrosopik kasar; kekasaran; ukuran keserupaan

1. Introduction

Information collected from a various factor about the generality of the object may differ by the subjectiveness of the object and its element. The generality of the object like similarity plays a vital role in discussing the object in element set. It also differs from the various type of uncertainty information collected such as Fuzzy Set (FS) introduced by Zadeh (1965) represented the information value in term of membership degree within interval value [0,1] in universal set X. Then, Atanassov (1983) extended the idea of FS by introduced Intuitionistic Fuzzy Set (IFS), where IFS theory involving two membership degree value, which is a membership and non-membership value that is more practical in representing the uncertainty

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information. Both membership degree value is within interval value [0,1] in universal set *X*. Later, Smarandache (1999) introduced the Neutrosophic Set (NS) to represent the neutral (indeterminate) opinion between the truth and false information. As a generalization of the FS and IFS, the element set of NS involve the three-membership degree value which is truth membership degree, indeterminate membership degree and falsity membership degree within non-standard interval $]^{-}0, 1^{+}[$. Next, Wang *et al.* (2012) introduced Single Valued Neutrosophic Set (SVNS) to consist of for all membership degree value is also within interval value [0,1] in universal set *X*. This element set value is more practical in solving the real problem of uncertainty information such as decision making.

Fuzzy Multiset (FM), Intuitionistic Fuzzy Multiset (IFM) and Neutrosophic Multiset (NM) are also introduced since there exist the multiple occurring information taken in a different time interval which allows repeated or same membership value more than one time (Yager 1986; Miyamoto 2001; Shinoj & John 2012; Ye & Ye 2014). Later, for the first time, Smarandache (2013) refined NS to neutrosophic refined set (NRS); $T_1, T_2, \ldots, T_m$ and $I_1, I_2, \ldots, I_n$ and $F_1, F_2, \ldots, F_r$ to overcome the refinement situation of many sub-opinion. Then, single valued neutrosophic multisets (NM) introduced by Ye and Ye (2014) inspired from Fuzzy multiset (FM) (Yager 1986) and NRS (Smarandache 1999) published an operation and properties of NS number to become repeated sequences of NS element of $(T_A(x), F_A(x), I_A(x))$. Instead of one-time occurring for each element, the NM allowed an element to occur more than once with possibly the same or different truth membership sequences, indeterminacy membership sequences, and falsity membership sequences $({T_A}^i(x), {F_A}^i(x), {I_A}^i(x))$.

A various hybrid mathematical method for solving uncertainty information and incompleteness data were presented by many researchers after the successful development of that data represented in element set of universal especially by using membership degree (Dubois & Prade 1990; Rizvi *et al.* 2002; Maji 2013; Broumi *et al.* 2014; Mandal 2015; Abdul-baset *et al.* 2016; Al-Quran & Nasruddin 2016; Ali & Smarandache 2016; Alkhazaleh 2016, Broumi *et al.* 2016). Meanwhile, Alias *et al.* (2017) introduced Rough Neutrosophic Multisets (RNM) which is involving uncertainty information in term of NM membership degree with the boundary of Pawlak's lower and upper approximation in equivalence relation. RNM allows the multiple or repeated occurrences that any RNM membership element can be collected more than one with the possible different or same membership value. The RNM theory can be solved for incompleteness data involving roughness and vagueness information that given in various time or situation in element set boundary since Pawlak's approximation space is taken as consideration (Pawlak 1982).

The successful development of the similarity degree for the NS or SVNS is well known, and most of this finding is extended to NM environment and hybrid NS and NM. Besides that, the development of the similarity measure by vectorial models of information such as cosine measure and dice measure for NS, NM, and hybrid uncertainty set is also explored by many authors (Yao 2010; Broumi & Smarandache 2014; Ye 2015; Karaaslan 2015; Mondal & Pramanik 2015a, 2015b; Ye & Smarandache 2016; Pramanik *et al.* 2016). Therefore, it is an excellent opportunity to study the similarity measure for RNM. Since RNM involving lower and upper approximation, the roughness of this approximation is considered first to get the best result in the similarity measure. As indicated by Pawlak, the accuracy measure is expected to catch the degree of completeness information about the set universe, and the roughness is opposite to accuracy where it is representing the degree of inadequacy. This combination approach does not study yet in NM environment.

The objectives of this research paper are to approximate two patterns in the same universe *X*, at least to what degree of roughness they are identical and to determine how close the similarity information given for specific condition in RNM environment. The rest of the paper is organized by mathematical preliminaries of uncertainty set information, similarity measure by vectorial models of information, the definition of roughness and similarity measure for RNM with all the proven proposition, next is result and discussion by illustrative example in medical diagnosis and lastly the conclusion.

2. Preliminaries

This section recalled the definition of the rough neutrosophic multisets, rough set, accuracy, and roughness of rough approximation and vectorial models for similarity, which is cosine and dice measure. All the proof of propositions is referred to Pawlak (1982), Yao (2010), Ye & Ye (2014), Ye (2015) and Alias *et al.* (2017).

2.1 *Rough neutrosophic multisets*

Definition 2.1. (Alias *et al.* 2017) Let U be a non-null set with the generic elements in U denoted by x_j and R be an equivalence relation on U. Let

$$A = \{\left(T_A^1(x_j), T_A^2(x_j), \ldots T_A^p(x_j)\right), \left(I_A^1(x_j), I_A^2(x_j), \ldots I_A^p(x_j)\right), \left(F_A^1(x_j), F_A^2(x_j), \ldots F_A^p(x_j)\right) > \mid (x_j) \in U, j = 1,2, \ldots, q\},$$

be neutrosophic multisets in U with the truth-membership sequence $(T_A^1, T_A^2, \ldots T_A^p)$, indeterminacy-membership sequences $(I_A^1, I_A^2, \ldots I_A^p)$ and falsity-membership sequences $(F_A^1, F_A^2, \ldots F_A^p)$. The lower and the upper approximations of A in the approximation (U, R) denoted by $\underline{Nm}(A)$ and $\overline{Nm}(A)$ are respectively defined as follows:

$$\underline{Nm}(A) = \left\{ \left\langle x_j, \begin{pmatrix} \left(T^1_{\underline{Nm}(A)}(x_j), T^2_{\underline{Nm}(A)}(x_j), \ldots, T^p_{\underline{Nm}(A)}(x_j)\right) \\ \left(I^1_{\underline{Nm}(A)}(x_j), I^2_{\underline{Nm}(A)}(x_j), \ldots, I^p_{\underline{Nm}(A)}(x_j)\right) \\ \left(F^1_{\underline{Nm}(A)}(x_j), F^2_{\underline{Nm}(A)}(x_j), \ldots, F^p_{\underline{Nm}(A)}(x_j)\right) \end{pmatrix} \right\rangle \middle| \boldsymbol{y} \in [\boldsymbol{x_j}]_{\boldsymbol{R}}, \boldsymbol{x_j} \in \boldsymbol{U} \right\}$$

$$= \left\{ \langle x_j, \left(T^i_{\underline{Nm}(A)}(x_j), I^i_{\underline{Nm}(A)}(x_j), F^i_{\underline{Nm}(A)}(x_j)\right) \rangle \middle| y \in [x_j]_R, i, j \in \mathbb{Z}^+, x_j \in U \right\}$$

$$\overline{Nm}(A) = \left\{ \left\langle x_j, \begin{pmatrix} \left(T^1_{\overline{Nm}(A)}(x_j), T^2_{\overline{Nm}(A)}(x_j), \ldots, T^p_{\overline{Nm}(A)}(x_j)\right) \\ \left(I^1_{\overline{Nm}(A)}(x_j), I^2_{\overline{Nm}(A)}(x_j), \ldots, I^p_{\overline{Nm}(A)}(x_j)\right) \\ \left(F^1_{\overline{Nm}(A)}(x_j), F^2_{\overline{Nm}(A)}(x_j), \ldots, F^p_{\overline{Nm}(A)}(x_j)\right) \end{pmatrix} \right\rangle \middle| \boldsymbol{y} \in [\boldsymbol{x_j}]_{\boldsymbol{R}}, \boldsymbol{x_j} \in \boldsymbol{U} \right\}$$

$$= \left\{ \langle x_j, \left(T^i_{\overline{Nm}(A)}(x_j), I^i_{\overline{Nm}(A)}(x_j), F^i_{\overline{Nm}(A)}(x_j), \right) \rangle \middle| y \in [x_j]_R, i, j \in \mathbb{Z}^+, x_j \in U \right\}$$

where

$j = 1, 2, \ldots, q$ and $i = 1, 2, \ldots, p$ are a positive integer,

$T^i_{\underline{Nm}(A)}(x_j) = \wedge_{y \in [x_j]_R} T^i_A(y_j)$, $I^i_{\underline{Nm}(A)}(x_j) = \vee_{y \in [x_j]_R} I^i_A(y_j)$,

$F^i_{\underline{Nm}(A)}(x_j) = \vee_{y \in [x_j]_R} F^i_A(y_j)$,

$T^i_{\overline{Nm}(A)}(x_j) = \vee_{y \in [x_j]_R} T^i_A(y_j)$,

$I^i_{\overline{Nm}(A)}(x_j) = \wedge_{y \in [x_j]_R} I^i_A(y_j)$,

$F^i_{\overline{Nm}(A)}(x_j) = \wedge_{y \in [x_j]_R} F^i_A(y_j)$.

Here $\wedge$ and $\vee$ denote "min" and "max'' operators respectively and $[x_j]_R$ is the equivalence class of the x_j. Meanwhile, $T_A^i(y_j)$, $I_A^i(y_j)$ and $F_A^i(y_j)$ are the membership sequences, indeterminacy sequences, and non-membership sequences of y concerning A. It is easy to see that $T_{\underline{Nm}(A)}^i(x_j), I_{\underline{Nm}(A)}^i(x_j), F_{\underline{Nm}(A)}^i(x_j) \in [0,1]$ moreover, $0 \leq T_{\underline{Nm}(A)}^i(x_j) + I_{\underline{Nm}(A)}^i(x_j) + F_{\underline{Nm}(A)}^i(x_j) \leq 3$. Then, $\underline{Nm}(A)$ is a neutrosophic multisets. Similarly, we have $T_{\overline{Nm}(A)}^i(x), I_{\overline{Nm}(A)}^i(x), F_{\overline{Nm}(A)}^i(x) \in [0,1]$ and $0 \leq T_{\overline{Nm}(A)}^i(x) + I_{\overline{Nm}(A)}^i(x) + F_{\overline{Nm}(A)}^i(x) \leq 3$. Then, $\overline{Nm}(A)$ is neutrosophic multisets.

Since $\underline{Nm}(A)$ and $\overline{Nm}(A)$ are two neutrosophic multisets in U, thus the neutrosophic multisets mappings $\underline{Nm}, \overline{Nm}: Nm(U) \to Nm(U)$ are respectively referred to as lower and upper rough neutrosophic multisets approximation operators and the pair of $\left(\underline{Nm}(A), \overline{Nm}(A)\right)$ is called the rough neutrosophic multisets (RNM) in (U, R), respectively. Rough neutrosophic multisets (RNM) is denoted by:

$$\mathcal{R}NM(A) = \left(\underline{Nm}(A), \overline{Nm}(A)\right)$$
$$= \left\{ \left\langle x_j, \begin{pmatrix} \left[T_{\underline{Nm}(A)}^i(x_j), I_{\underline{Nm}(A)}^i(x_j), F_{\underline{Nm}(A)}^i(x_j)\right], \\ \left[T_{\overline{Nm}(A)}^i(x_j), I_{\overline{Nm}(A)}^i(x_j), F_{\overline{Nm}(A)}^i(x_j)\right] \end{pmatrix} \right\rangle \middle| y \in [x_j]_R, i,j \in \mathbb{Z}^+, x_j \in U \right\}.$$

The truth membership sequence $\left[T_{\underline{Nm}(A)}^i(x_j), T_{\overline{Nm}(A)}^i(x_j)\right]$, indeterminate membership sequence $\left[I_{\underline{Nm}(A)}^i(x_j), I_{\overline{Nm}(A)}^i(x_j)\right]$ and falsity membership sequence $\left[F_{\underline{Nm}(A)}^i(x_j), F_{\overline{Nm}(A)}^i(x_j)\right]$ for lower and upper approximation of RNM may be in decreasing or increasing order.

2.2 *Rough set approximation*

Definition 2.2. (Pawlak 1982) Let E denote an equivalence relation with the induced partition U/E. For a subset of objects $X \subseteq U$, Pawlak introduces a pair of lower and upper approximation as follows:

$$\underline{apr}_E(X) = \cup\{[x]_E \subset U/E \mid [x]_E \subseteq X\}, \overline{apr}_E(X) = \cup\{[x]_E \subset U/E \mid [x]_E \cap X \neq \emptyset,$$

where X is non-empty set, $E \in X$. The pair $\left(\underline{apr}_E(X), \overline{apr}_E(X)\right)$ is referred to as the rough set approximation of X.

2.3 *Accuracy and roughness measure of pawlak's approximation*

Yao (2010) indicated that it is necessary to introduce a new measure for accuracy and roughness of Pawlak's approximation because of specific properties.

Definition 2.3. For a subset of object $X \subseteq U$, the accuracy measure is defined as:

$$\alpha_E(X) = \frac{\left|\underline{apr}_E(X)\right| + \left|(\overline{apr}_E(X))^c\right|}{|U|},$$

where X is non-empty set, $E \in X$, $\underline{apr}_E(X)$ is a lower approximation of set E, $\overline{apr}_E(X)$ is an upper approximation of set E, $|.|$ denotes the cardinality of a set E, and $0 \leq \alpha_E(X) \leq 1$. Based on the accuracy measure, the roughness measure is defined by $\rho_E(X) = 1 - \alpha_E(X)$.

2.4 *Similarity measure by vectorial models of information for neutrosophic multisets*

There are two vectorial models of information chosen for neutrosophic multisets (NM) which is improved cosine similarity measure and dice similarity measure.
Let
$A = \{< x_j, T_A^i(x_j), I_A^i(x_j), F_A^i(x_j) > (x_j) \in X, i = 1,2,\dots,q\}$ and $B = \{< x_j, T_B^i(x_j), I_B^i(x_j), F_B^i(x_j) > (x_j) \in X, i = 1,2,\dots,q\}$ be any two of NM in $X = \{x_1, x_2, \dots, x_k\}$. Then, the following is a vectorial model of information for NM.

Definition 2.4. (Pramanik *et al.* 2016) Improved Cosine similarity measure between A and B is defined to be $S_{NM}^C(A,B) = \frac{1}{k}\sum_{j=1}^{k}\left\{\frac{1}{q}\sum_{i=1}^{q}\cos\left\{\frac{\pi}{6}\left|T_A^i(x_j) - T_B^i(x_j)\right| + \left|I_A^i(x_j) - I_B^i(x_j)\right| + \left|F_A^i(x_j) - F_B^i(x_j)\right|\right\}\right\}$.

Definition 2.5. (Ye & Ye 2014) Dice similarity measure between A and B is defined to be
$S_{NM}^D(A,B) = \frac{1}{k}\sum_{j=1}^{k}\frac{1}{q}\left[\frac{\sum_{i=1}^{q}|T_A^i(x_j)T_B^i(x_j) + I_A^i(x_j)I_B^i(x_j) + F_A^i(x_j)F_B^i(x_j)|}{\sum_{i=1}^{q}|\left(T_A^i(x_j)^2 + I_A^i(x_j)^2 + F_A^i(x_j)^2\right) + \left(T_B^i(x_j)^2 + I_B^i(x_j)^2 + F_B^i(x_j)^2\right)|}\right]$.

Proposition 1. *The cosine similarity measure $S_{NM}^C(A,B)$ moreover, dice similarity measure $S_{NM}^D(A,B)$ for A and B satisfies the following properties:*
(P1) $0 \le S_{NM}^C(A,B) \le 1$; $0 \le S_{NM}^D(A,B) \le 1$;
(P2) $S_{NM}^C(A,B) = 1$ if and only if for $A = B$; $S_{NM}^D(A,B) = 1$ if and only if for $A = B$;
(P3) $S_{NM}^C(A,B) = S_{NM}^C(B,A)$; $S_{NM}^D(A,B) = S_{NM}^D(B,A)$;
(P4) $S_{NM}^C(A,C) \le S_{NM}^C(A,B)$ and $S_{NM}^C(A,C) \le S_{NM}^C(B,C)$ if C is NM in X and $A \subseteq B \subseteq C$; $S_{NM}^D(A,C) \le S_{NM}^D(A,B)$ and $S_{NM}^D(A,C) \le S_{NM}^D(B,C)$ if C is NM in X and $A \subseteq B \subseteq C$.
All the proof for the properties were discussed in (Yao 2010; Ye & Ye 2014; Pramanik *et al.* 2016).

3. Proposed Roughness and Similarity Measure of Rough Neutrosophic Multisets

In this section, a roughness for the rough approximation of rough neutrosophic multisets (RNM) is defined simultaneously with similarity measure by vectorial models of information. The roughness of RNM is calculated between lower and upper approximation instead of mean value between them. An improved cosine similarity measure for neutrosophic multisets (Pramanik *et al.* 2016) and a dice similarity measure between single-valued neutrosophic multisets (Ye & Ye 2014) was used to define the similarity measure based on basis RNM theory.

In our case, we only consider the same multiplicity (l) of all NM number where $l(x_j : A) = l(x_j : B)$ moreover, $j = 1, 2, \dots, k$. Assume that A and B be any two rough neutrosophic multisets in the universe of discourse X as followed:
$A = \langle x_j, \left(T_{\underline{Nm}(A)}^i(x_j), I_{\underline{Nm}(A)}^i(x_j), F_{\underline{Nm}(A)}^i(x_j)\right), \left(T_{\overline{Nm}(A)}^i(x_j), I_{\overline{Nm}(A)}^i(x_j), F_{\overline{Nm}(A)}^i(x_j)\right) | x_j \in X, i = 1,2,\dots,q\rangle$ and
$B = \langle x_j, \left(T_{\underline{Nm}(B)}^i(x_j), I_{\underline{Nm}(B)}^i(x_j), F_{\underline{Nm}(B)}^i(x_j)\right), \left(T_{\overline{Nm}(B)}^i(x_j), I_{\overline{Nm}(B)}^i(x_j), F_{\overline{Nm}(B)}^i(x_j)\right) | x_j \in X, i = 1,2,\dots,q\rangle$
in $X = \{x_1, x_2, \dots, x_k\}$.

Then, we define the roughness and similarity measure for RNM A and B.

Definition 3.1. The improved cosine similarity measure between RNM A and B is defined as follows:

$$S_{RNM}^{C}(A,B)=\frac{1}{k}\sum_{j=1}^{k}\left\{\frac{1}{q}\sum_{i=1}^{q}\cos\left\{\frac{\pi}{6}\left(\begin{array}{c}\left|\Delta T_{Nm(A)}^{i}(x_j)-\Delta T_{Nm(B)}^{i}(x_j)\right|\\+\left|\Delta I_{Nm(A)}^{i}(x_j)-\Delta I_{Nm(B)}^{i}(x_j)\right|\\+\left|\Delta F_{Nm(A)}^{i}(x_j)-\Delta F_{Nm(B)}^{i}(x_j)\right|\end{array}\right)\right\}\right\}. \tag{1}$$

where

$\Delta T_{Nm(A)}^{i}(x_j)=1-\left(\frac{T_{\underline{Nm}(A)}^{i}(x_j)+\left(T_{\overline{Nm}(A)}^{i}(x_j)\right)^{c}}{|X|}\right)$, $\Delta T_{Nm(B)}^{i}(x_j)=1-\left(\frac{T_{\underline{Nm}(B)}^{i}(x_j)+\left(T_{\overline{Nm}(B)}^{i}(x_j)\right)^{c}}{|X|}\right)$,

$\Delta I_{Nm(A)}^{i}(x_j)=1-\left(\frac{I_{\underline{Nm}(A)}^{i}(x_j)+\left(I_{\overline{Nm}(A)}^{i}(x_j)\right)^{c}}{|X|}\right)$, $\Delta I_{Nm(B)}^{i}(x_j)=1-\left(\frac{I_{\underline{Nm}(B)}^{i}(x_j)+\left(I_{\overline{Nm}(B)}^{i}(x_j)\right)^{c}}{|X|}\right)$,

$\Delta F_{Nm(A)}^{i}(x_j)=1-\left(\frac{F_{\underline{Nm}(A)}^{i}(x_j)+\left(F_{\overline{Nm}(A)}^{i}(x_j)\right)^{c}}{|X|}\right)$, and $\Delta F_{Nm(B)}^{i}(x_j)=1-\left(\frac{F_{\underline{Nm}(B)}^{i}(x_j)+\left(F_{\overline{Nm}(B)}^{i}(x_j)\right)^{c}}{|X|}\right)$.

Here, Δ denote "roughness approximation" operator by rough approximation between the lower and upper approximation of RNM, and $|.|$ is a cardinality of the universal X, respectively. Such that,

$\Delta T_{Nm(A)}^{i}(x_j),\Delta I_{Nm(A)}^{i}(x_j),\Delta F_{Nm(A)}^{i}(x_j)\in[0,1]$, $\Delta T_{Nm(B)}^{i}(x_j),\Delta I_{Nm(B)}^{i}(x_j),\Delta F_{Nm(B)}^{i}(x_j)\in[0,1]$,
$0\le\Delta T_{Nm(A)}^{i}(x_j)+\Delta I_{Nm(A)}^{i}(x_j)+\Delta F_{Nm(A)}^{i}(x_j)\le 3$,
$0\le\Delta T_{Nm(B)}^{i}(x_j)+\Delta I_{Nm(B)}^{i}(x_j)+\Delta F_{Nm(B)}^{i}(x_j)\le 3$,
for $i=1,2,\dots,q$ and $j=1,2,\dots,k$.

Definition 3.2. The dice similarity measure between RNM A and B is defined as follows:

$$S_{RNM}^{D}(A,B)=\frac{2}{k}\sum_{j=1}^{k}\frac{1}{q}\left[\sum_{i=1}^{q}\frac{U}{V}\right] \tag{2}$$

where,

$U=\left|\Delta T_{Nm(A)}^{i}(x_j)\Delta T_{Nm(B)}^{i}(x_j)+\Delta I_{Nm(A)}^{i}(x_j)\Delta I_{Nm(B)}^{i}(x_j)+\Delta F_{Nm(A)}^{i}(x_j)\Delta F_{Nm(B)}^{i}(x_j)\right|$

$V=\left(\left(\Delta T_{Nm(A)}^{i}(x_j)\right)^2+\left(\Delta I_{Nm(A)}^{i}(x_j)\right)^2+\left(\Delta F_{Nm(A)}^{i}(x_j)\right)^2\right)+\left(\left(\Delta T_{Nm(B)}^{i}(x_j)\right)^2+\left(\Delta I_{Nm(B)}^{i}(x_j)\right)^2+\left(\Delta F_{Nm(B)}^{i}(x_j)\right)^2\right)$

The operator uses for dice similarity measure is the same as an improved cosine similarity measure. Each similarity measure between two RNM A and B are undefined when: $T_{\underline{Nm}(A)}^{i}(x_j)=I_{\underline{Nm}(A)}^{i}(x_j)=F_{\underline{Nm}(A)}^{i}(x_j)=0; T_{\overline{Nm}(A)}^{i}(x_j)=I_{\overline{Nm}(A)}^{i}(x_j)=F_{\overline{Nm}(A)}^{i}(x_j)=0$, and $T_{\underline{Nm}(B)}^{i}(x_j)=I_{\underline{Nm}(B)}^{i}(x_j)=F_{\underline{Nm}(B)}^{i}(x_j)=0; T_{\overline{Nm}(B)}^{i}(x_j)=I_{\overline{Nm}(B)}^{i}(x_j)=F_{\overline{Nm}(B)}^{i}(x_j)=0$ for all $x_j\in X$.

Proposition 2. *Let A and B be two RNM-sets. Then, each similarity measure $S_{RNM}^{C}(A,B)$ and $S_{RNM}^{D}(A,B)$ satisfies the following properties:*

(P1) $0\le S_{RNM}^{C}(A,B)\le 1$; $0\le S_{RNM}^{D}(A,B)\le 1$
(P2) $S_{RNM}^{C}(A,B)=1$ if and only if for $A=B$; $S_{RNM}^{D}(A,B)=1$ if and only if for $A=B$;
(P3) $S_{RNM}^{C}(A,B)=S_{RNM}^{C}(B,A)$; $S_{RNM}^{D}(A,B)=S_{RNM}^{D}(B,A)$;
(P4) $S_{RNM}^{C}(A,C)\le S_{RNM}^{C}(A,B)$ and $S_{RNM}^{C}(A,C)\le S_{RNM}^{C}(B,C)$ if C is RNM in X and $A\subseteq$

$B \subseteq C$;
$S_{RNM}^{D}(A,C) \leq S_{RNM}^{D}(A,B)$ and $S_{RNM}^{D}(A,C) \leq S_{RNM}^{D}(B,C)$ if C is RNM in X and $A \subseteq B \subseteq C$.

Proof.

(P1) As $\Delta T_{Nm(A)}^{i}(x_j), \Delta I_{Nm(A)}^{i}(x_j), \Delta F_{Nm(A)}^{i}(x_j) \in [0,1]$, $\Delta T_{Nm(B)}^{i}(x_j), \Delta I_{Nm(B)}^{i}(x_j), \Delta F_{Nm(B)}^{i}(x_j) \in [0,1]$ for all $A, B \in RNM$, the similarity measure $S_{RNM}^{C}(A,B)$ based on cosine function also lies between $[0,1]$. Hence, $0 \leq S_{RNM}^{C}(A,B) \leq 1$. It is also true for $S_{RNM}^{D}(A,B)$ according to the inequality $a^2 + b^2 \geq 2ab$ for equation 2. Hence, $0 \leq S_{RNM}^{D}(A,B) \leq 1$.

(P2) For any two RNM A and B, if $A = B$, then the following relations hold for any

$$\Delta T_{Nm(A)}^{i}(x_j) = \Delta T_{Nm(B)}^{i}(x_j),\ \Delta I_{Nm(A)}^{i}(x_j) = \Delta I_{Nm(B)}^{i}(x_j),\ \Delta F_{Nm(A)}^{i}(x_j) = \Delta F_{Nm(B)}^{i}(x_j),$$

i.e.,

$$T_{\underline{Nm}(A)}^{i}(x_j) = T_{\underline{Nm}(B)}^{i}(x_j), T_{\overline{Nm}(A)}^{i}(x_j) = T_{\overline{Nm}(B)}^{i}(x_j),$$
$$I_{\underline{Nm}(A)}^{i}(x_j) = I_{\underline{Nm}(B)}^{i}(x_j), I_{\overline{Nm}(A)}^{i}(x_j) = I_{\overline{Nm}(B)}^{i}(x_j),$$
$$F_{\underline{Nm}(A)}^{i}(x_j) = F_{\underline{Nm}(B)}^{i}(x_j), F_{\overline{Nm}(A)}^{i}(x_j) = F_{\overline{Nm}(B)}^{i}(x_j),$$ for $i = 1,2,\dots,q$ and $j = 1,2,\dots,k$.

which states that

$$\left|T_{\underline{Nm}(A)}^{i}(x_j) - T_{\underline{Nm}(B)}^{i}(x_j)\right| = 0, \left|T_{\overline{Nm}(A)}^{i}(x_j) - T_{\overline{Nm}(B)}^{i}(x_j)\right| = 0,$$
$$\left|I_{\underline{Nm}(A)}^{i}(x_j) - I_{\underline{Nm}(B)}^{i}(x_j)\right| = 0, \left|I_{\overline{Nm}(A)}^{i}(x_j) - I_{\overline{Nm}(B)}^{i}(x_j)\right| = 0,$$
$$\left|F_{\underline{Nm}(A)}^{i}(x_j) - F_{\underline{Nm}(B)}^{i}(x_j)\right| = 0, \text{ and } \left|F_{\overline{Nm}(A)}^{i}(x_j) - F_{\overline{Nm}(B)}^{i}(x_j)\right| = 0.$$

Thus, $\cos(0) = 1$. Hence, $S_{RNM}^{C}(A,B) = 1$. The proof is clear for $S_{RNM}^{D}(A,B)$. Hence, $S_{RNM}^{D}(A,B) = 1$.

Conversely, If $S_{RNM}^{C}(A,B) = 1$, this implies

$$\left|\Delta T_{Nm(A)}^{i}(x_j) - \Delta T_{Nm(B)}^{i}(x_j),\right| = 0, \left|\Delta I_{Nm(A)}^{i}(x_j) - \Delta I_{Nm(B)}^{i}(x_j),\right| = 0$$
$$\left|\Delta F_{Nm(A)}^{i}(x_j) - \Delta F_{Nm(B)}^{i}(x_j),\right| = 0,$$

since $\cos(0) = 1$. This resulted from that

$$\Delta T_{Nm(A)}^{i}(x_j) = \Delta T_{Nm(B)}^{i}(x_j),\ \Delta I_{Nm(A)}^{i}(x_j) = \Delta I_{Nm(B)}^{i}(x_j), \text{ and}$$
$$\Delta F_{Nm(A)}^{i}(x_j) = \Delta F_{Nm(B)}^{i}(x_j), \text{ for all } i, j \text{ values.}$$

Similar to $S_{RNM}^{D}(A,B) = 1$. Hence $A = B$.

(P3) It is obvious that:

$$\Delta T_{Nm(A)}^{i}(x_j) - \Delta T_{Nm(B)}^{i}(x_j) \neq \Delta T_{Nm(B)}^{i}(x_j) - \Delta T_{Nm(A)}^{i}(x_j),$$
$$\Delta I_{Nm(A)}^{i}(x_j) - \Delta I_{Nm(B)}^{i}(x_j) \neq \Delta I_{Nm(B)}^{i}(x_j) - \Delta I_{Nm(A)}^{i}(x_j)$$

and $$\Delta F_{Nm(A)}^{i}(x_j) - \Delta F_{Nm(B)}^{i}(x_j) \neq \Delta F_{Nm(B)}^{i}(x_j) - \Delta F_{Nm(A)}^{i}(x_j).$$

However,

$$\left|\Delta T_{Nm(A)}^{i}(x_j) - \Delta T_{Nm(B)}^{i}(x_j)\right| = \left|\Delta T_{Nm(B)}^{i}(x_j) - \Delta T_{Nm(A)}^{i}(x_j)\right|,$$
$$\left|\Delta I_{Nm(A)}^{i}(x_j) - \Delta I_{Nm(B)}^{i}(x_j)\right| = \left|\Delta I_{Nm(B)}^{i}(x_j) - \Delta I_{Nm(A)}^{i}(x_j)\right|$$

and $\left|\Delta F_{Nm(A)}^{i}(x_j) - \Delta F_{Nm(B)}^{i}(x_j)\right| = \left|\Delta F_{Nm(B)}^{i}(x_j) - \Delta F_{Nm(A)}^{i}(x_j)\right|$.

Hence,

$$S_{RNM}^{C}(A,B) = \frac{1}{k}\sum_{j=1}^{k}\left\{\frac{1}{q}\sum_{i=1}^{q}\cos\left\{\frac{\pi}{6}\left|\Delta T_{Nm(A)}^{i}(x_j) - \Delta T_{Nm(B)}^{i}(x_j)\right| + \left|\Delta I_{Nm(A)}^{i}(x_j) - \Delta I_{Nm(B)}^{i}(x_j)\right| + \left|\Delta F_{Nm(A)}^{i}(x_j) - \Delta F_{Nm(B)}^{i}(x_j)\right|\right\}\right\}$$
$$= \frac{1}{k}\sum_{j=1}^{k}\left\{\frac{1}{q}\sum_{i=1}^{q}\cos\left\{\frac{\pi}{6}\left|\Delta T_{Nm(B)}^{i}(x_j) - \Delta T_{Nm(A)}^{i}(x_j)\right| + \left|\Delta I_{Nm(B)}^{i}(x_j) - \Delta I_{Nm(A)}^{i}(x_j)\right| +\right.\right.$$

$\left|\Delta F^{i}_{Nm(B)}(x_j) - \Delta F^{i}_{Nm(A)}(x_j)\right|\}\} = S^{C}_{RNM}(B, A)$.

For $S^{D}_{RNM}(A, B)$, the proof of can is made similarly.

(P4) Let $A \subseteq B \subseteq C$, implies that

$\Delta T^{i}_{Nm(A)}(x_j) \leq \Delta T^{i}_{Nm(B)}(x_j) \leq \Delta T^{i}_{Nm(C)}(x_j)$, $\Delta I^{i}_{Nm(A)}(x_j) \geq \Delta I^{i}_{Nm(B)}(x_j) \geq \Delta I^{i}_{Nm(C)}(x_j)$,
$\Delta F^{i}_{Nm(A)}(x_j) \geq \Delta F^{i}_{Nm(B)}(x_j) \geq \Delta F^{i}_{Nm(C)}(x_j)$ for every $x_j \in X$;

i.e.,

$T^{i}_{\underline{Nm}(A)}(x_j) \leq T^{i}_{\underline{Nm}(B)}(x_j) \leq T^{i}_{\underline{Nm}(C)}(x_j), T^{i}_{\overline{Nm}(A)}(x_j) \leq T^{i}_{\overline{Nm}(B)}(x_j) \leq T^{i}_{\overline{Nm}(C)}(x_j)$,
$I^{i}_{\underline{Nm}(A)}(x_j) \geq I^{i}_{\underline{Nm}(B)}(x_j) \geq I^{i}_{\underline{Nm}(C)}(x_j)$, $\quad I^{i}_{\overline{Nm}(A)}(x_j) \geq I^{i}_{\overline{Nm}(B)}(x_j) \geq I^{i}_{\overline{Nm}(C)}(x_j)$,
$F^{i}_{\underline{Nm}(A)}(x_j) \geq F^{i}_{\underline{Nm}(B)}(x_j) \geq F^{i}_{\underline{Nm}(C)}(x_j)$, $\quad F^{i}_{\overline{Nm}(A)}(x_j) \geq F^{i}_{\overline{Nm}(B)}(x_j)$
$\geq F^{i}_{\overline{Nm}(C)}(x_j)$

for every $x_j \in X$. Then, we obtain the following relation:

a) $\left|\Delta T^{i}_{Nm(A)}(x_j) - \Delta T^{i}_{Nm(B)}(x_j)\right| \leq \left|\Delta T^{i}_{Nm(A)}(x_j) - \Delta T^{i}_{Nm(C)}(x_j)\right|$,
$\left|\Delta T^{i}_{Nm(B)}(x_j) - \Delta T^{i}_{Nm(C)}(x_j)\right| \leq \left|\Delta T^{i}_{Nm(A)}(x_j) - \Delta T^{i}_{Nm(C)}(x_j)\right|$,

b) $\left|\Delta I^{i}_{Nm(A)}(x_j) - \Delta I^{i}_{Nm(B)}(x_j)\right| \leq \left|\Delta I^{i}_{Nm(A)}(x_j) - \Delta I^{i}_{Nm(C)}(x_j)\right|$,
$\left|\Delta I^{i}_{Nm(B)}(x_j) - \Delta I^{i}_{Nm(C)}(x_j)\right| \leq \left|\Delta I^{i}_{Nm(A)}(x_j) - \Delta I^{i}_{Nm(C)}(x_j)\right|$,

c) $\left|\Delta F^{i}_{Nm(A)}(x_j) - \Delta F^{i}_{Nm(B)}(x_j)\right| \leq \left|\Delta F^{i}_{Nm(A)}(x_j) - \Delta F^{i}_{Nm(C)}(x_j)\right|$,
$\left|\Delta F^{i}_{Nm(B)}(x_j) - \Delta F^{i}_{Nm(C)}(x_j)\right| \leq \left|\Delta F^{i}_{Nm(A)}(x_j) - \Delta F^{i}_{Nm(C)}(x_j)\right|$.

Hence, $S^{C}_{RNM}(A, C) \leq S^{C}_{RNM}(A, B)$ and $S^{C}_{RNM}(A, C) \leq S^{C}_{RNM}(B, C)$ since cosine function is a decreasing function within the interval $[0, \frac{\pi}{2}]$. For $S^{D}_{RNM}(A, B)$, the proof of can is made similarly.
Therefore, the proof is complete. □

4. Roughness and Similarity Measure Based Decision Making Under Rough Neutrosophic Multisets Environment

4.1 *Illustrative example*

By adapting the content of medical diagnosis from Ye and Ye (2014), we presented the following example to validate the proposed roughness and similarity measure of RNM theory via real life application which is in medical environment.

Example 1. Let B = {Viral Fever (Vf)} be a disease and S = {Temperature (T), Cough (C), Throat pain (Tp), Headache (H), Body pain (Bp)}$= \{x_1, x_2, x_3, x_4, x_5\}$ be a set of symptoms. The relation between patient and symptoms, and between disease and symptoms are considered in the same equivalence relation. For diagnosis, the patient A is kept under supervision for one day, and his symptoms pattern are monitored at the three-time inspection (morning: 6.00 am, noon: 12.00 pm, night: 6.00 pm). There are three specified degree of membership function for each time inspection which is for truth description of symptom, indeterminacy description symptom and falsity description symptom. The findings of the patient A can be summarized and represented with the RNM as follows:

$$A = \left\{ \begin{array}{l} \langle x_1, [((0.8, 0.6, 0.5), (0.3, 0.2, 0.1), (0.4, 0.2, 0.1))], [((0.8, 0.2, 0.1), (0.5, 0.4, 0.3), (0.6, 0.5, 0.5))] \rangle, \\ \langle x_2, [((0.5, 0.4, 0.3), (0.4, 0.4, 0.3), (0.6, 0.3, 0.4))], [((0.7, 0.1, 0.2), (0.3, 0.4, 0.1), (0.5, 0.3, 0.3))] \rangle, \\ \langle x_3, [((0.2, 0.1, 0.0), (0.3, 0.2, 0.2), (0.8, 0.7, 0.7))], [((0.6, 0.2, 0.2), (0.5, 0.3, 0.4), (0.6, 0.2, 0.4))] \rangle, \\ \langle x_4, [((0.7, 0.6, 0.5), (0.3, 0.2, 0.1), (0.4, 0.3, 0.2))], [((0.8, 0.0, 0.2), (0.9, 0.7, 0.4), (0.6, 0.8, 0.3))] \rangle, \\ \langle x_5, [((0.4, 0.3, 0.2), (0.6, 0.5, 0.5), (0.6, 0.4, 0.4))], [((0.6, 0.5, 0.5), (0.8, 0.3, 0.5), (0.6, 0.2, 0.2))] \rangle \end{array} \right\}$$

At 6.00 am, the truth membership degree for temperature (x_1) which surely belong, and which possibly belong to patient A is equal to 0.8. At 12.00 pm, the indeterminate membership degree for temperature (x_1) which surely belong to patient A is equal to 0.2 and which possibly belong to patient A is equal to 0.4. At 6.00 pm, the falsity membership degree for temperature (x_1) which surely belong to patient A is equal to 0.1 and which possibly belong to patient A is equal to 0.5.

Suppose the standard relation between disease and symptoms are represented by the following RNM as:

$$B = Vf$$
$$= \left\{ \begin{array}{l} \langle x_1, [((0.8, 0.1, 0.1), (0.6, 0.2, 0.2), (0.3, 0.4, 0.1))], [((0.4, 0.4, 0.4), (0.7, 0.5, 0.6), (0.5, 0.3, 0.2))] \rangle, \\ \langle x_2, [((0.2, 0.7, 0.1), (0.5, 0.8, 0.9), (0.4, 0.4, 0.4))], [((0.8, 0.0, 0.2), (0.9, 0.7, 0.4), (0.6, 0.2, 0.2))] \rangle, \\ \langle x_3, [((0.3, 0.5, 0.2), (0.5, 0.4, 0.3), (0.6, 0.5, 0.5))], [((0.8, 0.6, 0.7), (0.3, 0.5, 0.4), (0.4, 0.7, 0.5))] \rangle, \\ \langle x_4, [((0.5, 0.3, 0.2), (0.5, 0.5, 0.7), (0.5, 0.8, 0.9))], [((0.5, 0.3, 0.3), (0.6, 0.2, 0.4), (0.6, 0.8, 0.3))] \rangle, \\ \langle x_5, [((0.5, 0.4, 0.1), (0.5, 0.7, 0.6), (0.6, 0.2, 0.2))], [((0.6, 0.5, 0.5), (0.8, 0.3, 0.5), (0.7, 0.6, 0.5))] \rangle \end{array} \right\}$$

At 6.00 am, the truth membership degree for cough (x_2) which surely belong to viral fever is equal to 0.2, and which possibly belong to viral fever is equal to 0.8. At 12.00 pm, the indeterminate membership degree for cough (x_2) which surely belong to viral fever is equal to 0.8, and which possibly belong to viral fever is equal to 0.7. At 6.00 pm, the falsity membership degree for cough (x_2) which surely belong to viral fever is equal to 0.4, and which possibly belong to viral fever is equal to 0.2.

Then, the following algorithm is applied to diagnose whether a patient A is suffering from viral fever. This algorithm is first introduced by Ye and Ye (2014). The data involve for their research is involving single valued neutrosophic multisets. But in step three, the new approach is introducing for this algorithm which is to apply a roughness for lower and upper approximation which is suitable for RNM cases.

Step 1: Develop a model for RNM-sets for medical reports, determine from a medical person.
Step 2: Construct RNM for patient A.
Step 3: Calculate the roughness between the model RNM for patient A and roughness between the model for viral fever.
Step 4: Calculate the similarity measure between the model RNM for patient A and the model for viral fever.
Step 5: If the similarity measure is greater than 0.5, then the patient A may possibly suffer from the viral fever, and if the similarity measure is less than 0.5, then patient A may not possibly suffer from the viral fever.

Now, we can compute the similarity given roughness and similarity measure by vectorial models of information, which is cosine similarity measure and dice similarity measure. The comparison from the other existing method under NM environment is also discussed as shown in Table 1.

Table 1: Similarity measure values

Similarity Measure	Values
Proposed $S_{RNM}^{C}(A,B)$	0.9978
Proposed $S_{RNM}^{D}(A,B)$	0.9968
$S_{NM}^{C}(A,B)$ (Ye, 2015)	0.9443
$S_{NM}^{D}(A,B)$ (Ye &Ye, 2014)	0.7810

All the similarity result is greater than 0.5, indicate that the patient A is may possibly suffering from viral fever. But the proposed similarity measure of $S_{RNM}^{C}(A,B)$ and $S_{RNM}^{D}(A,B)$ give the closeness value for similarity measure (close to 1.0) which is 0.9978 and 0.9968. Therefore, the proposed method is more accurate compared to previous method. Besides that, the roughness between lower and upper approximation of RNM also give an important criterion for similarity measure especially for vectorial information model.

5. Conclusion

This paper has introduced the roughness the similarity measure of rough neutrosophic multisets (RNM) by the vectorial model of information which is cosine similarity measure and dice similarity measure. The lower and upper approximation of the RNM give a roughness value between the information given, and the similarity is used for the incomplete of the information. All the similarity measure properties are completely defined in this paper. The advantages of the proposed method are compared with the existing method under a neutrosophic environment in medical diagnosis, and the result showed that the proposed method is more acceptable because of the highest score involving the roughness approximation and similarity values. The measure proposed can be further extended to other similarity measure methods such as trigonometry similarity measure and the rough approximation will be added to find the roughness of the information given. As a conclusion, this paper proves that the RNM is valuable for a complicated situation involving lower and upper approximation and three multiple membership degree values.

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