Journal of Quality Measurement and Analysis *JQMA* 17(1) 2021, 129-139
e-ISSN: 2600-8602 *http://www.ukm.my/jqma*

USING THE EXPLICIT METHOD TO SOLVE PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS OF TEMPERATURE DISTRIBUTION IN THE CONDUCTOR OF A CRUDE CIRCUIT BREAKER

(Menggunakan Kaedah Tersurat untuk Menyelesaikan Persamaan Pembezaan Separa Parabola bagi Taburan Suhu dalam Konduktor Pemutus Litar Kasar)

SARATHA SATHASIVAM*, SHEHAB ABDULHABIB ALZAEEMI, MURALY VELAVAN, MUSTAFA MAMAT, TEOH POH LIN & WEE WEI SOON

ABSTRACT

In Malaysia, the circuit breaker is one of the most important and essential safety mechanisms in every building. When electricity enters a house, it goes to a circuit breaker box to be divided into a number of circuits. Each of these circuits is protected by a breaker or a fuse. For safety purposes, electrical appliances are designed to keep the current flow at a low level. However, whenever the current flow jumps above the safety level, the circuit breaker does its job by cutting off the circuit. This is done by the conductor's bending (a bimetallic strip or rod) in the circuit breaker when the wire temperature rises. If the conductor needs to bend upward when heated, the thermal conductivity of the lower metal rod must be higher than that of the upper rod. This paper focuses on the temperature distribution in the conductor of a crude circuit breaker in a fire alarm system by using different materials such as a bimetallic rod. The paper also describes how different materials can affect the efficiency of the fire alarm system. An implicit method makes it possible to tackle such problems. This can be achieved by the development of a simultaneous linear equations' system for temperature at a specified point in time for the entire interior nodes. Based on the simulation results, it was found that the explicit method (em) is a simple mechanism to solve parabolic partial differential equations (pde). The results can also be improved by using the implicit method to minimize errors.

Keywords: explicit method; parabolic partial differential equations; temperature distribution

ABSTRAK

Di Malaysia, pemutus litar merupakan satu daripada mekanisme keselamatan yang paling penting dan diperlukan pada setiap bangunan. Apabila elektrik mengalir ke sebuah rumah, ia mengalir ke kotak pemutus litar yang dibahagikan kepada beberapa litar. Setiap litar ini dilindungi oleh pemutus atau fius. Bagi tujuan keselamatan, peralatan elektrik dibuat untuk memastikan aliran arus adalah pada tahap rendah. Kemudian, setiap kali arus mengalir melebihi tahap yang selamat, pemutus litar melakukan tugasnya dengan memutuskan litar elektrik. Hal ini dilakukan dengan menlenturkan konduktor (jalur atau batang dwibesi) pada pemutus litar apabila suhu wayar meningkat. Dalam kes ini, sekiranya konduktor perlu membengkok ke atas ketika dipanaskan, kekonduksian terma rod logam bawah mestilah lebih tinggi daripada batang di sebelah atas. Dalam artikel ini, fokusnya adalah kepada taburan suhu pada konduktor pemutus litar anggaran dalam sistem penggera kebakaran dengan menggunakan bahan yang berbeza sebagai batang dwifungsi. Kemudiannya turut dibincangkan bagaimana bahan yang berbeza dapat mempengaruhi kecekapan sistem penggera kebakaran. Suatu kaedah tersirat memungkinkan kita untuk menyelesaikan masalah-masalah ini dengan menjanakan sistem persamaan linear serentak untuk suhu di semua nod dalaman pada waktu tertentu. Daripada hasil simulasi, keputusan menunjukkan bahawa kaedah tersurat adalah suatu mekanisme mudah untuk menyelesaikan persamaan pembezaan separa parabola. Hasilnya juga dapat ditingkatkan dengan menggunakan kaedah tersirat untuk meminimumkan ralat.

Kata kunci: kaedah tersurat; persamaan pembezaan separa parabola; taburan suhu

1. Introduction

Thermal properties represent pivotal factors in the design of switch gears. A thermal design, which is reasonable, is essential for such products. Understanding the distribution of the temperature field inside a switchgear is important for selecting, assembling, and designing a specific material. The analysis of numerical thermal is utilized for calculating the internal temperature to disclose the distribution of the internal temperature, as well as the highest temperature spot's locations (Esa *et al.* 2020). According to Clemens *et al.* (2000), a simulation of a transient temperature field, as well as an electromagnetic field was carried out by using the method of finite-element combined with equations of heat conduction for analysing transient situations. The first analysis of the thermal network has been established at the Dutch Eindhoven University, Holland, followed by the establishment of the fuse thermal network's model by Meng (1995). Paulke *et al*. (2002) established the comprehensive temperature field's computer model, considering the thermal, as well as the contact resistance. The transformer's axisymmetric magnetic field was initially computed by Silvester and Ferrari (1996) by using the method of the 2-D element. In Buffa *et al.* (2000), the vortex of the dynamic structure was calculated by using the method of slide grid finite-element, followed by proposing the simulation models of the 2-D, as well as 3-D for the structures of nonstationary vortex. Kawase et al. (1999) analysed the thermal relay's temperature distribution by using the method of the 3-D finite-element. The single-phase circuit experimentation verified the calculation results. Carstea and Carstea (2003) simulated the temperature, as well as the electric field distribution are by using the analysis method of finite-element. Monnier *et al.* (2005) examined the behaviour of the spherical electrical contact in 2005 by utilizing the method of the ANSYS finite-element; then they established the model of the 2-D axisymmetric. The simplified arc model was implanted by Claessens *et al.* (1997) into the computational fluid dynamics (CFD) for simulating the hefty current arcing phenomenon. The temperature and electrical fields were simulated by Mateev *et al.* (2014) when using high-voltage vacuum circuit breakers using ANSYS. These fields' distribution was achieved when they treated the electric field as a heat source. Experienced values, in practical engineering, can be applied in boundary conditions for simulating a steady-state temperature field. Several differences were found between the simulation results, as well as the experimental results based on varied conditions. Several designated values do not possess a uniform standard, thereby affecting computational accuracy. For the purpose of achieving more efficient simulation results, a novel method has been implemented for calculating the rise in temperature, combining the flow and temperature field via the CFD method. According to fluid mechanics' equations of momentum (Amin *et al.* 2014; Sharma *et al.* 2021), mass, and energy, equations of partial differential have been established in the controlled body, displaying the parameters of the dynamic flow, including temperature, velocity, and density using different time and space. In Girgis *et al.* (1987) the transformer's winding loss was simulated. In Koppikar *et al*. (1999), a model of a single-phase transformer was established based on some assumptions. According to Liao *et al.* (2015), the analysis method was proposed on the basis of the method of finite-element for analysing the transformers of 35-kV oil-immersed. In Wang *et al.* (2016), the rise in temperature and the12-kV medium voltage switchgear's flow field distribution was analysed via a finite-element approach. In a fire alarm system, when the temperature rises, the conductors in the circuit breaker will expand. Since the lower rod expands faster than the upper rod of the conductor, the bimetallic rod conductor will bend upward, break the current flow of the electric circuit and connect to the other circuit which contains a buzzer to ring the alarm. Without this circuit breaker, the wire will heat up to unsafe levels and possibly cause a fire. So, an efficient crude circuit breaker is essential to prevent any unwanted circumstances. Hence, we want to learn more about the temperature distribution in the conductor of a crude circuit breaker by using different materials. Therefore, the objectives of this study are: To investigate the temperature distribution in the conductor of a crude circuit breaker by using the designated explicit method. To assess the distribution of temperature in the conductor of a crude circuit breaker by using different materials. To determine the efficiency of the crude circuit breaker by using different materials in the system. Sowmiya *et al.* (2021) have been proven the ability of the explicit method in solving the partial differential equation and ordinary differential equation which is required in computer application and physical processes as the unsteady heat equation can be solved explicitly. In this paper, the simulation methods are utilized for calculating temperature by using the explicit method to solve parabolic partial differential equations of the temperature distribution in the conductor of a crude circuit breaker.

2. Methodology of Simulation Model

2.1. *Information of the material of the rods in the crude circuit breaker*

This paper focus on four materials which are copper, aluminium, tungsten and iron. Thus, we need the information such as specific heat, density and thermal conductivity.

Table 1: The information of four materials

Materials	Specific Heat (*cal/g ℃*)	Density (*g/cm*^{3})	Thermal Conductivity (*cal/s cm ℃*)
Copper	0.0932	8.96	0.987
Aluminium	0.2170	2.60	0.566
Tungsten	0.0311	19.30	0.471
Iron	0.1080	7.87	0.225

From the information above, we can calculate the *k* for each material by using the formula:

$$k = \frac{Thermal\ Conductivity}{Density \times Specific\ Heat} \tag{1}$$

where *k* is the diffusivity constant *k* in the heat-conduction equation:

$$k\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial x} \tag{2}$$

So their respective value of the *k* is as follow:

Table 2: The respective value of *k*

Materials	*k (cm*^{2}*/s)*
Copper	1.1819
Aluminium	1.0032
Tungsten	0.7847
Iron	0.2647

2.2. *Application of the Explicit Method*

For the purpose of investigating the distribution of the temperature in the rods; knowing that it is a typical heat conduction problem, the heat conduction equation can be used here as follows:

$$k\frac{\partial^2 T}{\partial x^2}=\frac{\partial T}{\partial x} \tag{3}$$

By substituting:

$$\frac{\partial^2 T}{\partial x^2}=\frac{T_{i+1}^{l}-2T_i^{l}+T_{i-1}^{l}}{(\Delta x)^2} \tag{4}$$

and

$$\frac{\partial T}{\partial x}=\frac{T_i^{l+1}-T_i^{l}}{\Delta t} \tag{5}$$

we have

$$k\frac{T_{i+1}^{l}-2T_i^{l}+T_{i-1}^{l}}{(\Delta x)^2}=\frac{T_i^{l+1}-T_i^{l}}{\Delta t}$$

which can be solved for

$$T_i^{l+1}=T_i^{l}+\lambda(T_{i+1}^{l}-2T_i^{l}+T_{i-1}^{l}) \tag{6}$$

where

$$\lambda=k\frac{\Delta t}{(\Delta x)^2} \tag{7}$$

It is the equation of explicit method that we use to calculate the temperature of all interior nodes in the rods. Few assumptions are made:

1. The rods are with the length 30cm and the length interval, Δx is given by 6cm. It means that there are 6 nodes (exterior and interior nodes) in a rod.
2. The boundary conditions are fixed in the range, which are the left end of the rods has to be in the range of 200-400℃ meanwhile the right end of the rods has to be in the range of 20-40℃.
3. The initial temperature of the interior nodes of the rods is 20℃.
4. No other heat source in the rods except the ends of the rods.
5. Time interval, Δt is given by 10s and the distribution of the temperature in the rods was examined from t = 10s to t = 50s.

In order to design models by using mathematical equations interested reader can refer to some ideas from researchers (Mishra 2007; Mishra & Mishra 2012; Mishra 2017; Modh *et al.* 2015).

2.3. *Efficiency of the crude circuit breaker*

We conclude the efficiency of the crude circuit breaker depending on the expansion rate of the rods. As we know, the thermal conductivity of the lower rod has to be higher than the upper rod in order to make whole conductor bends upward.

Thus, we consider the temperature of the 4*th node* of the rods at t = 50s and their respective linear temperature expansion coefficient to get the value of expansion of the rods in length.

Table 3: The linear temperature expansion coefficient of each of the materials

Materials	Linear Temperature Expansion Coefficient (× 10^{-6} *cm/cm ℃*)
Copper	16.7
Aluminium	23
Tungsten	4.5
Iron	12.0

Since the linear temperature expansion coefficient is:

$$C = \frac{CL}{OL \times CT} \tag{8}$$

where C is the linear temperature expansion coefficient, CL is the change in length, OL is the original length, and CT is change in the temperature. And the value of expansion (change in length), VE, of the rods is given by:

$$VE = C \times OL \times CT \tag{9}$$

Next will compare which combination of the rods is the most efficient by taking the ratio of their value of expansion of the rods.

$$Ratio = \frac{VEL}{VEU} \tag{10}$$

where VEL is the value of expansion of the lower rod in length, and VEU is the value of expansion of the upper rod in length. The combination of the rods with the highest ratio is the most efficient among others.

3. Results and Discussion

This project using MATLAB Programming to visualize and calculate results. In this section check for the user input error before any calculations can be done on the temperature distribution of the conductor. There are four conditions to be checked including the choice of materials and the boundary conditions. Also, note that in the following outputs, the boundary temperature for the left end of the rods is fixed at 400℃ and the right end of the rods is fixed at 20℃. Each figure represents the combinations of materials as explained in Table 5.

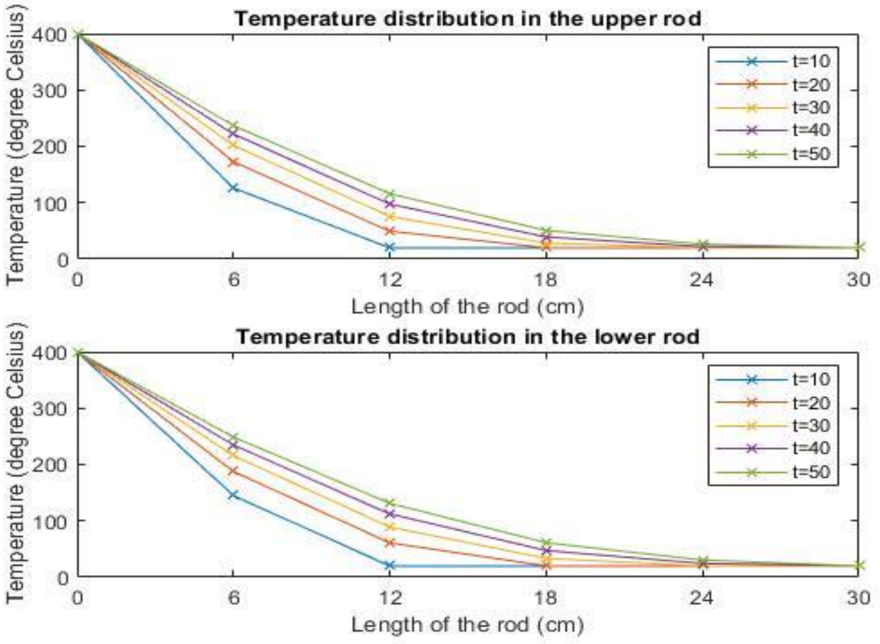

Figure 1 shows the temperature distribution in lower and upper rods at different time, *t*

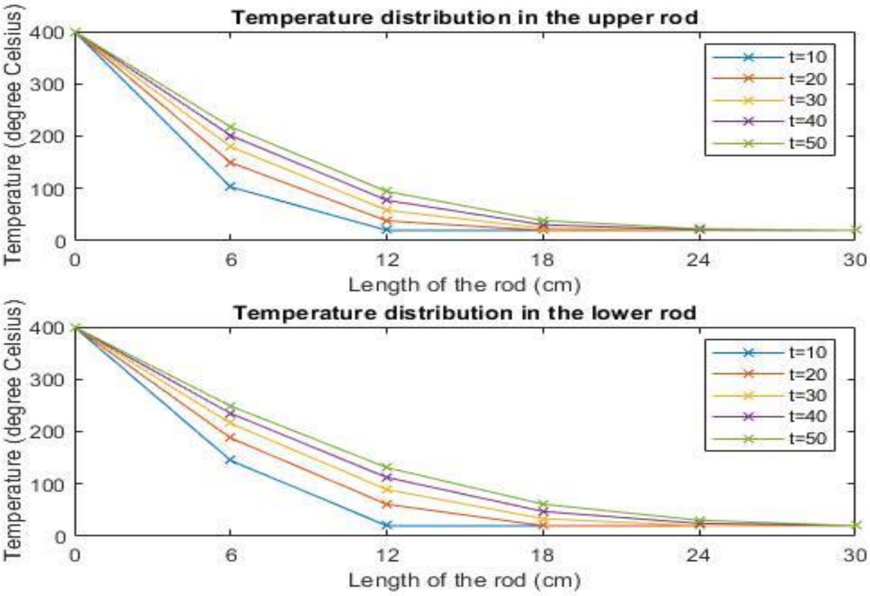

Figure 2 shows the temperature distribution in lower and upper rods at different time, *t*

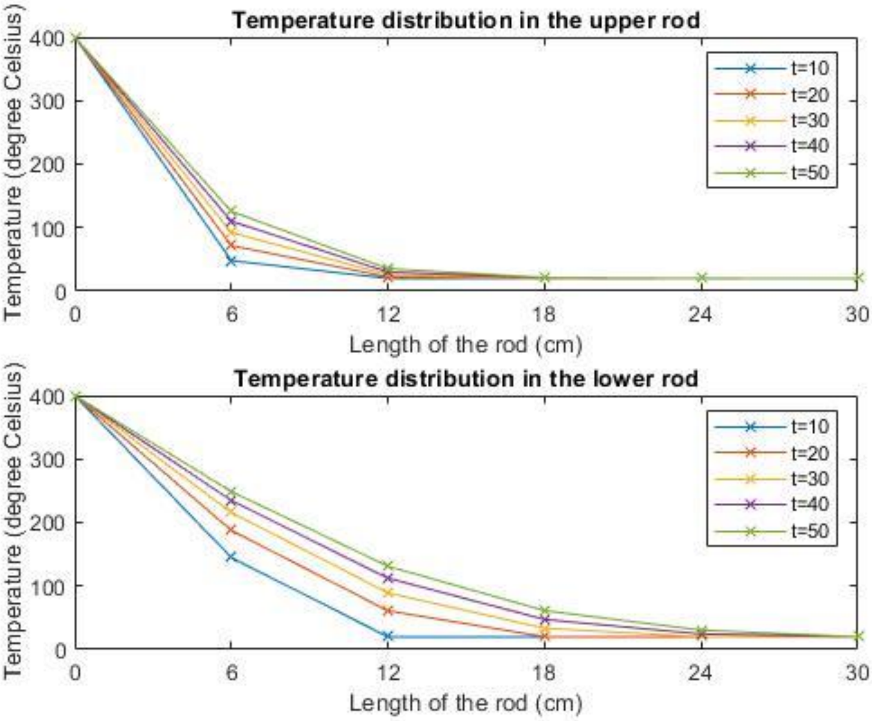

Figure 3 shows the temperature distribution in lower and upper rods at different time, *t*

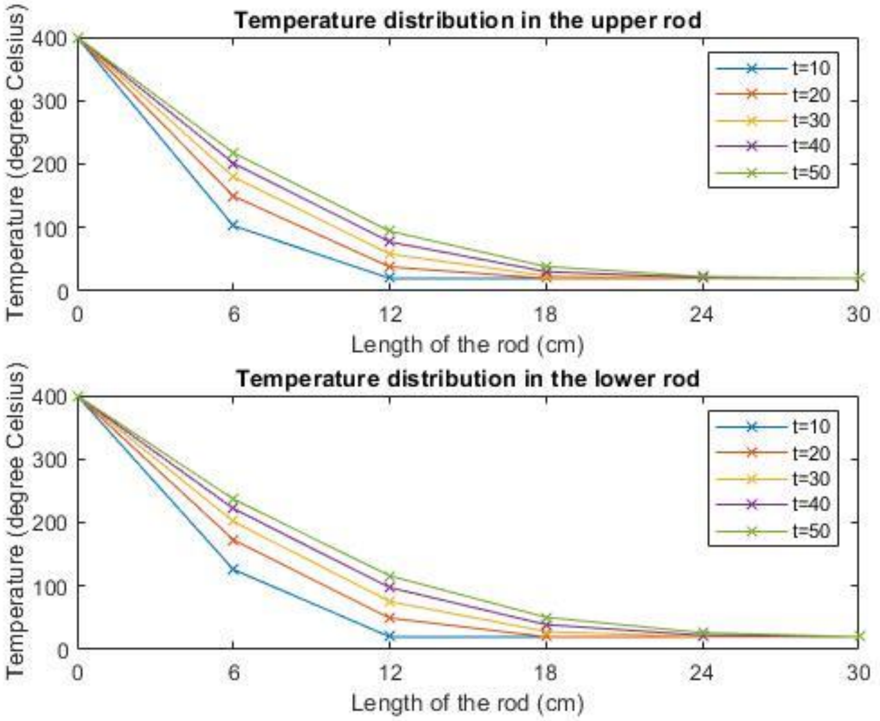

Figure 4 shows the temperature distribution in lower and upper rods at different time, *t*

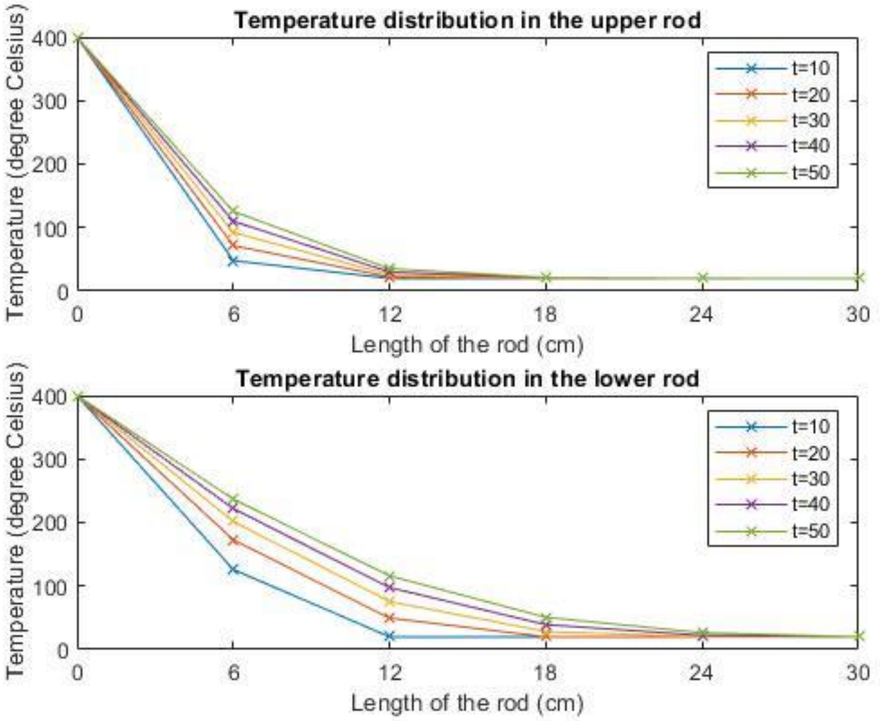

Figure 5 shows the temperature distribution in lower and upper rods at different time, *t*

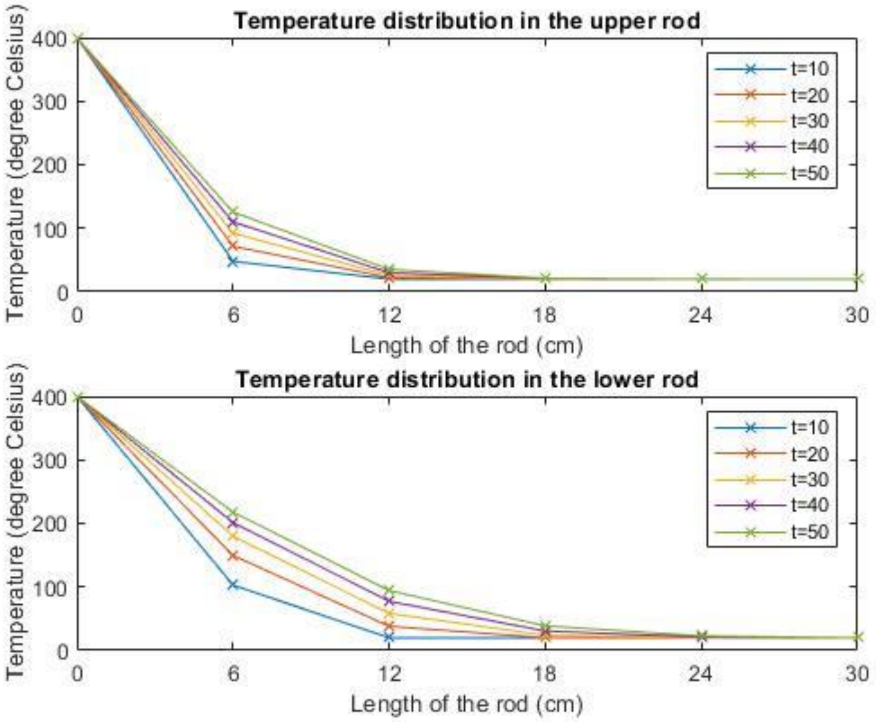

Figure 6 shows the temperature distribution in both rods at different time, *t*

Consider the temperature at the 4th *node* (length, l = 18*cm*) at time, t = 50*s*. We want to interpret and find out which pair of metal rods is the most efficient combination among the 6 pairs. Therefore, we calculate the value of expansion for each rod in length by using the *Equation* (1).

Table 4. the temperature of each material is isolated in the following table

Material	Temperature at 4th node for t = 50*s* (℃)	Initial Temperature, t_0 (℃)	Change in Temperature (℃)	Linear Temperature Expansion Coefficient ($\times 10^{-6}$*cm/cm* ℃)	Value of Expansion (*cm*)
Copper	61.1608	20.0	41.1608	16.7	12.373×10^{-3}
Aluminium	50.7274	20.0	30.7274	23.0	12.721×10^{-3}
Tungsten	38.6681	20.0	18.6681	4.5	1.512×10^{-3}
Iron	21.1994	20.0	1.1994	12.0	0.259×10^{-3}

Table 5. the efficiency of each pair of metal rods by taking the ratio of their value of expansion

Ratio		Upper Rod Material		
		Aluminium	Tungsten	Iron
Lower Rod Material	Copper	0.9726	8.1832	47.7722
	Aluminium	[redacted]	8.4134	49.1158
	Tungsten	[redacted]	[redacted]	5.8378

As we can see from the calculations above from Figure 1 to Figure 6 with Table 5 and Table 2, the combination of Aluminium and Iron has the biggest ratio (49.1158). This means that this combination as the bimetallic rod in the conductor of a crude circuit breaker is the most efficient match in a fire alarm system. On the other hand, the combination of Copper and Aluminium makes the least efficient match as the bimetallic rod due to its smallest ratio (0.9726).

Besides that, we can also interpret the results from the figures that we obtained in the previous section. By comparing Figure 1 and Figure 5, it is obvious that there is a substantial difference in the temperature distribution curves of the upper and lower rod for both the combination of Copper and Aluminium versus the combination of Aluminium and Iron.

From the calculations, we can see that the ratio of combination of Copper and Iron (47.7722) is close to that of Aluminium and Iron (49.1158), and also their respective Figures (i.e Figure 3 and Figure 5) look somehow similar. However, we should take note that the expansion of the metal rod not only depends on the temperature distribution in the rod but also the linear temperature expansion coefficient as well as the thermal conductivity. First and foremost, the explicit method has an error of $O[(\Delta x)^2]$ which is neglected in the computations of our results. This may cause a small error in solving for the temperature distribution of the rods.

Next, some of the assumptions that we made in our methodology may cause errors and inaccuracies in our results. For example:

a. The boundary temperature conditions for both ends of the rods are fixed in range.
    - ❖ In our calculations, we fixed the boundary temperature in the range of 200-400°C for the left end of both rods and a range of 20-40°C for the right end of both rods. However, in reality, the boundary temperature can constantly change over time.

b. There is no other heat source in the rods except at the ends of the rods.
    - ❖ In fact, this may or may not be true because the temperature in the lower rod can affect the temperature in the upper rod due to heat gain and vice versa.

In addition, the convergence and stability of the explicit method can affect our results. As the time increases, the temperature distribution curve of the figures will converge and become a straight line as shown in Figure 6. On the other hand, as the computation progresses, the errors at any stage of the computation which are not amplified but are attenuated can cause instability in the solution.

The errors caused by convergence and stability of the method can be minimized by halving Δx in order to improve the approximation of the calculations. However, the halving of Δx can double the total number of nodes in the equations to be computed, which result in a large computational burden. Finally, some of the external factors such as the sensitivity of the fire alarm system to detect the heat, the wire conditions, or the distance between the fire and the system can also affect the efficiency of the system. Since the explicit method can approximate the solutions, it may be the best method to use in computations.

4. Conclusion

In Malaysia, more than 5300 number of electrical fire cases were recorded from 2015 to 2017. Prevention is better than cure, hence adopting fire safety measures can prevent unwanted injuries or deaths to occur due to electrical fire cases. Therefore, an efficient fire alarm system with a highly conductive crude circuit breaker is essential. The explicit method can be used to investigate the temperature distribution in the conductor of the crude circuit breaker. We have also managed to compare the temperature distribution in the conductor by using different materials as the bimetallic rod. Based on our results, a combination of Aluminium and Iron as the bimetallic rod in the crude circuit breaker will make the perfect match in a fire alarm system.

To conclude, the explicit method is a simple way to solve for parabolic partial differential equations. The results can also be improved by using the implicit method to minimize the errors.

Acknowledgement

This research is supported by Research University Grant (RUI) (1001/PMATHS/8011131) by Universiti Sains Malaysia.

References

Amin N., Roslinda N. & Khamisah J. 2014. Stability analysis of stagnation-point flow past a shrinking sheet in a nanofluid. *Journal of Quality Measurement and Analysis* **10**(2): 51-63.

Buffa A., Maday Y. & Rapetti F. 2000. Calculation of eddy currents in moving structures by a sliding mesh-finite element method. *IEEE Transactions on Magnetics* **36**(4): 1356-1359.

Carstea D. & Carstea I. 2003. Simulation of coupled electric and thermal fields in coaxial cables. *6th International Conference on Telecommunications in Modern Satellite, Cable and Broadcasting Service*, pp. 697-700.

Claessens M., Von Starck R. & Thiel H. G. 1997. Simulation of gas flow phenomena in high-voltage self-blast circuit breakers at heavy fault current interruption. *IEEE Transactions on Plasma Science* **25**(5): 1001-1007.

Clemens M., Gjonaj E., Pinder P. & Weiland T. 2000. Numerical simulation of coupled transient thermal and electromagnetic fields with the finite integration method. *IEEE Transactions on Magnetics* **36**(4): 1448-1452.

Esa A.I.M., Halim S.A. & Abas N. 2020. Projection of temperature in relation to cardiovascular disease using bias correction method. *Journal of Quality Measurement and Analysis JQMA* **16**(2): 193-206.

Girgis R.S., Scott D.J., Yannucci D.A. & Templeton J.B. 1987. Calculation of winding losses in shell-form transformers for improved accuracy and reliability part-1: Calculation procedure and program description. *IEEE Transactions on Power Delivery* **2**(2): 398-403.

Kawase Y., Ichihashi T. & Ito S. 1999. Heat analysis of thermal overload relays using 3-D finite element method. *IEEE Transactions on Magnetics* **35**(3): 1658-1661.

Koppikar D.A., Kulkarni S.V., Srinivas P.N., Khaparde S.A. & Jain R. 1999. Evaluation of flitch plate losses in power transformers. *IEEE Transactions on Power Delivery* **14**(3): 996-1001.

Liao C., Ruan J., Liu C., Wen W., Wang S. & Liang S.Y. 2015. Comprehensive analysis of 3-D electromagnetic-fluid-thermal fields of oil-immersed transformer. *Electric Power Automation Equipment* **35**(9): 150-155.

Mateev V., Tanev R. & Marinova I. 2014. Simulation of electric and thermal fields of high voltage interrupter vacuum chamber. *2014 18th International Symposium on Electrical Apparatus and Technologies*, pp. 1-4.

Meng X.Z. 1995. Lifetime predictions of miniature fuses and semiconductor protection fuses. *Eindhoven University of Technology*. https://doi.org/10.6100/IR449473.

Mishra V.N. 2007. Some problems on approximations of functions in Banach spaces. PhD Thesis. Indian Institute of Technology.

Mishra V.N & Mishra L.N. 2021. Trigonometric approximation of signals (functions) in Lp− norm. *International Journal of Contemporary Mathematical Sciences* **7**(19): 909-918.

Mishra L.N. 2017. On existence and behavior of solutions to some nonlinear integral equations with applications. PhD Thesis. National Institute of Technology.

Modh A., Dabhi M., Mishra L.N. & Mishra V.N. 2015. Wireless network controlled robot using a website, android application or simple hand gestures. *Journal of Computer Networks* **3**(1): 1-5.

Monnier A., Froidurot B., Jarrige C., Meyer R. & Testé, P. 2005. A mechanical, electrical, thermal coupled-field simulation of a sphere-plane electrical contact. *Proceedings of the Fifty-First IEEE Holm Conference on Electrical Contacts*, pp. 224-231.

Paulke J., Weichert H. & Steinhaeuser P. 2002. Thermal simulation of switchgear. *IEEE Transactions on Components and Packaging Technologies* **25**(3): 434-439.

Silvester P.P. & Ferrari R.L. 1996. *Finite elements for electrical engineers*. 3rd Ed. New York: Cambridge University Press.

Sharma M.K., Dhiman N., Mishra L.N., Mishra V.N., Sahani S.K. 2021. Mediative fuzzy extension technique and its consistent measurement in the decision making of medical application. *Mathematical Problems in Engineering* **2021**: 1-9.

Sowmiya P., Revathi G.K. & Sakthipriya M. 2021. A study on finite difference explicit numerical method of symmetric hyperbolic partial differential equation. *IOP Conference Series: Materials Science and Engineering*, pp. 012063.

Wang L., Zheng W., Wang L., Lin J., Li X. & Jia S. 2016. Electromagnetic-thermal-flow field coupling simulation of 12-kV medium-voltage switchgear. *IEEE Transactions on Components, Packaging and Manufacturing Technology* **6**(8): 1208-1220.

School of Mathematical Sciences
Universiti Sains Malaysia
11800 USM, Penang, MALAYSIA
E-mail: saratha@usm.my, shehab_alzaeemi@yahoo.com, pohlin178@student.usm.my,*
wee_soon97@student.usm.my

School of General & Foundation Studies
AIMST University
08100 Bedong, Kedah DA, MALAYSIA
E-mail: muraly@aimst.edu.my

Faculty of Informatics and Computing
Universiti Sultan Zainal Abidin
21300 Kuala Terengganu
Terengganu DI, MALAYSIA
E-mail: must@unisza.edu.my

Received : 3 March 2021
Accepted : 30 June 2021

---

*Corresponding author